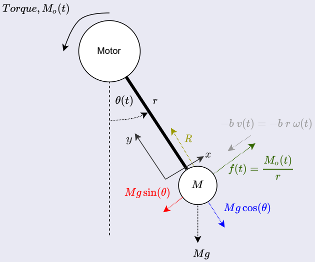
Modelling an Arm

Corrado Santoro

ARSLAB - Autonomous and Robotic Systems Laboratory
Dipartimento di Matematica e Informatica - Università di Catania, Italy
santoro@dmi.unict.it

Robotic Systems

An Arm

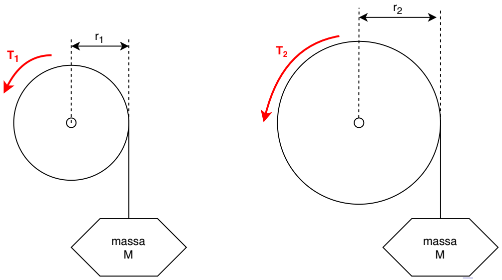


- Let's consider an **arm** with a single joint and a bar
- At the end of the bar there is a mass M
- We can ignore the mass of the bar since we soppose it is sensibly less that M
- We surely consider the friction
- The arm is moved by a motor that generates a certain **torque** $M_o(t)$
- We want to determine the angle $\theta(t)$ and the (angular) velocity $\omega(t)$

An Arm

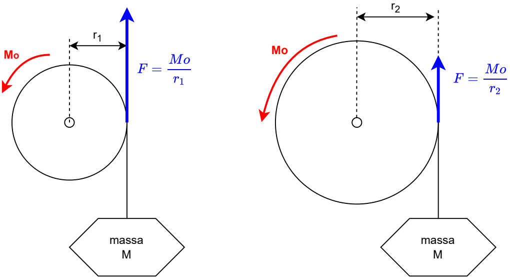
Why the **Torque**?

- In rotational systems, it's better to consider the concept of **couple of forces** rather than a single force
- In the figure below, given that the mass is the same, for the case 2 we need a motor with a power greather than the motor used in case 1
- This is because, in case 2, the distance of the mass from the rotation point is greather than the one in case 1: $r_2 > r_1$
- Therefore, in rotational system, in determining the force needed to move a certain mass **the distance matters!**



Why the **Torque**?

- Therefore, in rotational system, in determining the force needed to move a certain mass **the distance matters!**
- For this reason, in rotation systems, we consider the **mechanical momentum** or **torque** which is the cross-product of the force with the distance: $\vec{M_o} = \vec{F} \times \vec{r}$
- In scalar terms, given that often the force and the distance are perpendicular, we have: $M_o = F\ r$



An Arm

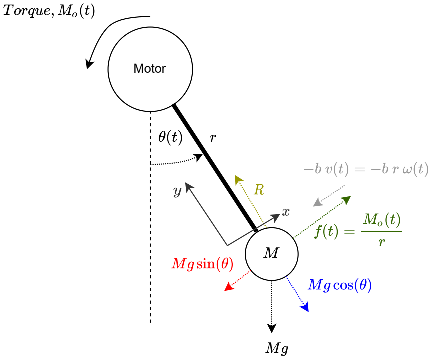
- We consider the reference system xy centered in the mass
- The weight of the mass is split in the two components, according to the angle $\theta(t)$ (in radians)
- Along $y-$axis, the component is compensated by reaction R:

$$R = Mg\cos(\theta)$$

- Along $x-$axis, we apply the second Newton's law:

$$-Mg\sin(\theta) + \frac{M_o}{r} - br\omega = M\dot{\omega}$$

- Here θ is the angular position, $\omega = \dot{\theta}$ is the angular speed and $\dot{\omega}$ is the angular acceleration



Arm Model

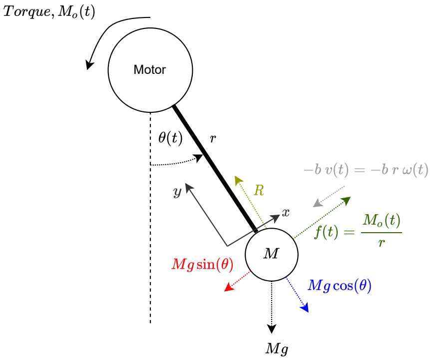
Given that:

$$-Mg\sin(\theta) + \frac{M_o}{r} - br\omega = M\dot{\omega}$$

the system model is:

$$\begin{cases} \dot{\omega} = -\frac{br}{M}\omega - g\sin\theta + \frac{1}{Mr}M_o \\ \dot{\theta} = \omega \end{cases}$$

It is **non linear!!**



An Arm

Arm Model Linearization

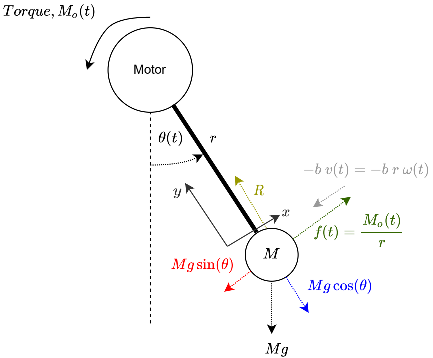
$$\begin{cases} \dot{\omega} & = & -\frac{br}{M}\omega - g\sin\theta + \frac{1}{Mr}M_o \\ \dot{\theta} & = & \omega \end{cases}$$

If we consider small values of $\theta(t)$, we can apply the following approximation:

$$\sin\theta \simeq \theta$$

therefore:

$$\begin{cases} \dot{\omega} & = & -\frac{br}{M}\omega - g\theta + \frac{1}{Mr}M_o \\ \dot{\theta} & = & \omega \end{cases}$$



An Arm

Arm Model Discretization

$$\begin{cases} \dot{\omega} &= -\frac{br}{M}\omega - g\theta + \frac{1}{Mr}M_o \\ \dot{\theta} &= \omega \end{cases}$$

$$\begin{cases} \frac{\omega(t+\Delta T)-\omega(t)}{\Delta T} &= -\frac{br}{M}\omega(t) - g\theta(t) + \frac{1}{Mr}M_o(t) \\ \frac{\theta(t+\Delta T)-\theta(t)}{\Delta T} &= \omega(t) \end{cases}$$

$$\begin{cases} \omega(t+\Delta T) &= \omega(t) - \frac{br}{M}\Delta T\omega(t) - g\Delta T\theta(t) + \frac{\Delta T}{Mr}M_o(t) \\ \theta(t+\Delta T) &= \theta(t) + \omega(t)\Delta T \end{cases}$$

Implementing the Arm

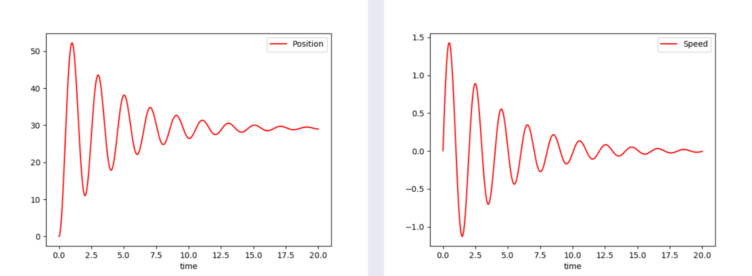
```
G = 9.81

class Arm:

    def __init__(self, _mass, _friction, _length):
        self.M = _mass
        self.b = _friction
        self.r = _length
        self.omega = 0
        self.theta = 0

    def evaluate(self, delta_t, _torque):
        new_omega = self.omega \
          - ((self.b * self.r) / self.M) * delta_t * self.omega \
          - G * delta_t * self.theta + delta_t/(self.M * self.r) * _torque
        new_theta = self.theta + self.omega * delta_t
        self.omega = new_omega
        self.theta = new_theta
```

$$M = 1\ Kg, b = 0.8, r = 0.6\ m, M_o = 3\ Nm$$



Arm Model Discretization without appromation (non-linear)

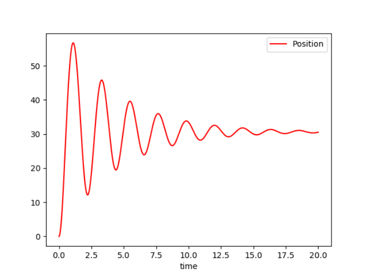
$$\begin{cases} \dot{\omega} &= -\frac{br}{M}\omega - g\sin\theta + \frac{1}{Mr}M_o \\ \dot{\theta} &= \omega \end{cases}$$

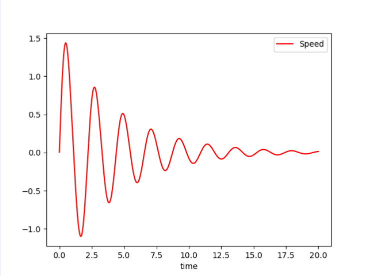
$$\begin{cases} \frac{\omega(t+\Delta T)-\omega(t)}{\Delta T} &= -\frac{br}{M}\omega(t) - g\sin\theta(t) + \frac{1}{Mr}M_o(t) \\ \frac{\theta(t+\Delta T)-\theta(t)}{\Delta T} &= \omega(t) \end{cases}$$

$$\begin{cases} \omega(t+\Delta T) &= \omega(t) - \frac{br}{M}\Delta T\omega(t) - g\Delta T\sin\theta(t) + \frac{\Delta T}{Mr}M_o(t) \\ \theta(t+\Delta T) &= \theta(t) + \omega(t)\Delta T \end{cases}$$

Simulating the Arm without Approximation

$$M = 1\ Kg, b = 0.8, r = 0.6\ m, M_o = 3\ Nm$$





Modelling an Arm

Corrado Santoro

ARSLAB - Autonomous and Robotic Systems Laboratory
Dipartimento di Matematica e Informatica - Università di Catania, Italy
santoro@dmi.unict.it

Robotic Systems