Basics of Digital Signal Processing

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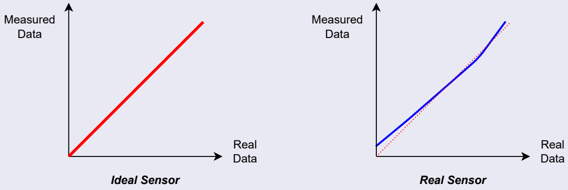
Robotic Systems

Measures and Errors

- Measure of state variables is a fundamental aspect of control systems
- If the measure is not precise or affected by a significant amount of noise, the whole control system cannot work properly
- However **any measurement** system is always affected by **errors**
- **Measurement errors** have different characteristics and depends on the kind of sensor used

Sensor Characteristics

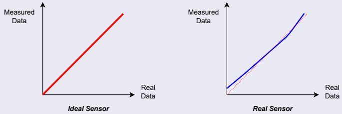
- The behaviour of a sensor is in general represented by its **characteristic curve**
- It is plot over a *XY* chart that reports in the *X* axis the **real data** and in the *Y* axis the **measured data**
- For an **ideal** sensor, the characteristic is a 45-degrees straight line
- But for a **real** sensor, the characteristic is a **curve** the is **close** to the 45-degrees straight line



Sensor Errors

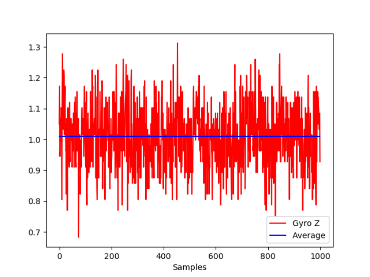
- **Offset:** it is the **non-zero value** given by the sensor when the real data is zero
- **Non-linearity:** it is the difference between the ideal and real characteristic
- **Noise:** it the variation of the measured data when the real data is **constant**

- Offset and Non-linearity can be reduced by means of **sensor calibration**
- Noise (that is harder to be removed) can be reduced by means of **digital filters**



Noise Errors

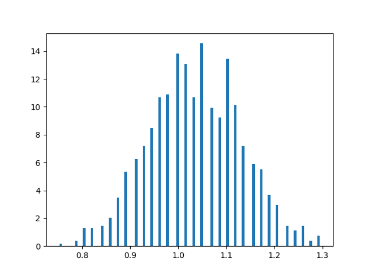
- Here is a plot of the data sampled by a gyroscope, Z-axis, when the system is **stopped**



- The **real data** should be **zero**, but we have an **offset** and a certain amount of **noise**
- The **blue line** is the **average**, if we subtract it, we can remove the offset but not the noise

Noise Characteristics

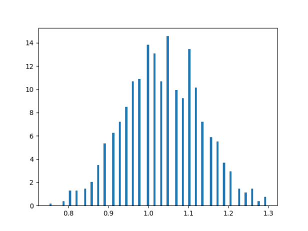
- If we plot the **histogram** of sampled data, we obtain the following chart



- Here we have, on *X* axis, the values of a sampled data, and, on *Y* axis, the number of times that value is got (data is organized in subintervals)
- The plot is the classical **Gaussian Curve** or **Normal Distribution** that is a common characteristic of noise errors

Noise Characteristics

- The plot is the classical **Gaussian Curve** or **Normal Distribution** that is a common characteristic of noise errors

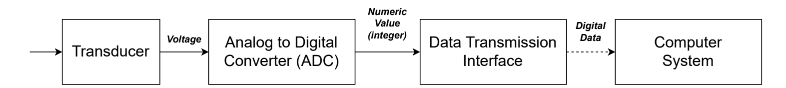


- This curve is characterised by the **average** and the **variance**
- Given that we have N measures, and given x_i the measures, we have:

$$\overline{x} = \frac{1}{N}\sum_i x_i$$

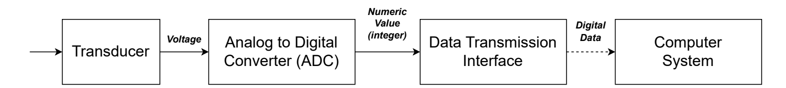
$$\sigma_x^2 = \frac{1}{N}\sum_i (x_i - \overline{x})^2$$

The Measurement Chain



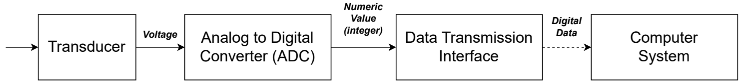
- **Transducer:** a electrical or mechanical system that converts the measure into an electrical magnitude (generally voltage)
- **ADC (Analog-to-Digital Converter):** an electronic circuit that converts a voltage signal into an n-bit binary representation
- **Data Interface:** a digital circuit (sometimes also CPU-equipped) that performs ADC data conversion and trasmission to a computer system

The Measurement Chain



- **Range:** the min and max value of the measure that the transducer is able to sample
- **Resolution:** the number of bits of the ADC, or the minimum variation of the measure that the ADC is able to detect
- **Sampling time:** the time required to perform a complete measure, include transducer measuring time and ADC conversion time

Resolution and Quantization Error



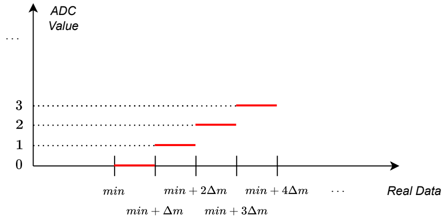
- Due to ADC, the measure value is discretised and the minimum variation that can be perceived is:

$$\Delta m = \frac{max - min}{2^n - 1}$$

- Where *min* and *max* represent measure range, and *n* the number of bits of the ADC
- Therefore, values allowed are $\{min, min + \Delta m, min + 2\Delta m, min + 3\Delta m, min + 4\Delta m, \ldots\}$

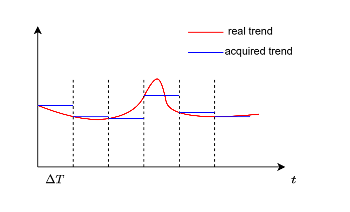
Sensors Characteristics

Resolution and Quantization Error



- By plotting the real value of the measure and the ADC number obtained the result is a stairway not a *straight line* (as we expect due to the proportional relationship)
- The phenomenon that produces that plot is called **quantization error**, and strongly depends on the resolution of the ADC

Choice of Sampling Time



- Another important factor of errors is the choice of **sampling time/interval ΔT**
- Given that any sensor has its **measuring time**, the choice of the *sampling interval* is a design choice
- The choice is made on the basis of what kind of signal variations we want to catch
- If ΔT is too low, the risk is to ignore important variations
- If ΔT is too high, the risk is to catch too noise
- What is the **right choice**?

Signals and Spectrum

The Fourier Series

Given any (periodic) signal of period T, it can be expressed as a linear combination of infinite sin and cos terms:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(\frac{2\pi}{T} nt) + b_n \sin(\frac{2\pi}{T} nt)]$$

- a_n, b_n are coeffients that represent the **contribution**, to the overall signal, of the sinusoid at frequency $\frac{n}{T}$
- They constitute the **spectrum** of the signal, i.e. a representation of the magnitude of the frequency components

Spectrum and Noise

- In any signal, the noise is a **high frequency** component in a spectrum, so it can be filtered
- But if the signal varies fast, it also provides high frequency components
- So it is important to understand when valid data terminate and noise begins
- If valid data end at a frequency $\overline{f}$, then set your sampling time at:

$$\Delta T = \frac{1}{2\overline{f}}$$

Simple Average

- Given x your sensor variable...
- For each ΔT, acquire N samples x'_i and compute the mean:

$$x = \frac{1}{N}\sum_{i=1}^{N} x'_i$$

- If t_s is the sensor's sampling time, it must be $N\ t_s < \Delta T$

Mobile Average

- Given x your sensor variable...
- For each ΔT, acquire one sample x'_i (at instant i), then compute the average of the last N past samples:

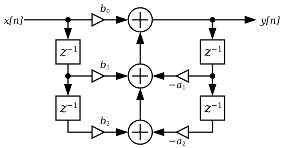
$$x = \frac{1}{N} \sum_{k=0}^{N-1} x'_{i-k}$$

Digital Filters

- A digital filter is discrete dynamic system characterised by the following relation:

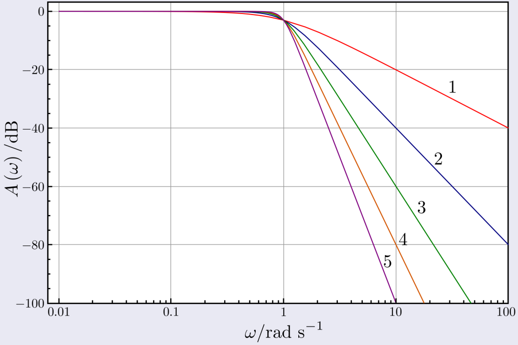
$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + \cdots - a_m y(k-m) + \\ + b_0 u(k) + b_1 u(k-1) + \cdots + b_n u(k-n)$$

- $u(k)$, input; $y(k)$, output
- $max(n, m)$ **filter order**
- Coefficients $a_i, b_i \in [0, 1] \subset \mathcal{R}$ determine the **filter type** and the **noise cut capability**
- The filter behaves as a **weighted average** of past inputs and outputs



Low-pass Filters and Orders

- A **low pass filter** is a filter that removes data at a frequency higher than the **cut-off freq**
- The **order** determines the slope of the cut line: the higher the more selective



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