Controlling Position and Speed using Profiles

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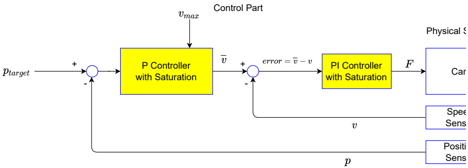
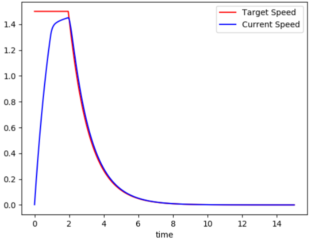
Robotic Systems

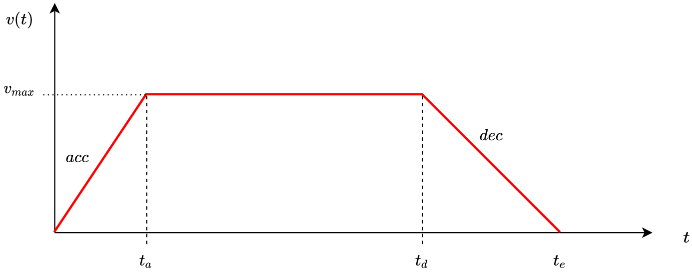
The Simple "P" Controller





- In the "simple-P" controller, the (real) speed shows a specific trend:
  - It has an initial acceleration phase
  - then there is a "cruise" phase at the maximum speed (saturation, v_{max}
  - And, when the P controller exits from saturation, the speed gradually decreases (deceleration phase)
- While the controller works (i.e. the target position is reached), we have **no control** over acceleration and deceleration: in some cases this is undesirable!

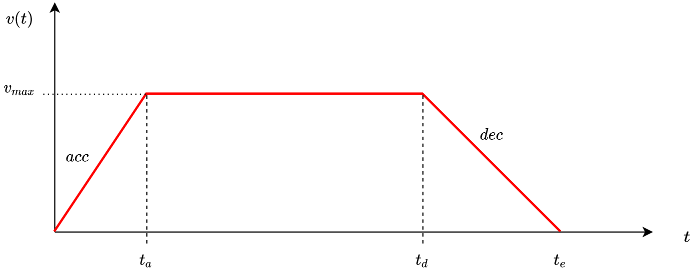
The Speed Profile



- Indeed, a more desirable situation is the one in which we can decide:
  - The final/target position p_{target}
  - The value of the acceleration *acc*
  - The maximum/cruise speed v_{max}
  - The value of decelration *dec* (that could be even equal to acceleration)
- In such a case, the aim of the controller is to ensure that when the deceleration phase ends the robot is **exactly in position** p_{target}

The Virtual Robot

The Virtual Robot



- Rather than dealing with the problem of "control", let us concentrate on how to create the profile above
- To this aim, let us consider an "ideal" (virtual) robot that has to travel a certain distance p_{target} by following that speed profile
- To model such a motion, we consider the cinematic equations related to uniform motion and uniformly accelerated motion

The Virtual Robot

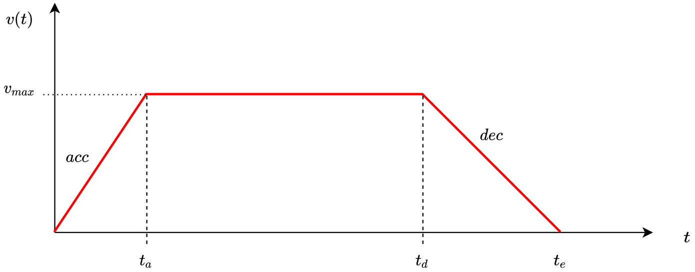
Uniformly Accelerated Motion

$$
\begin{aligned}
a(t) &= a \quad (= const) \\
v(t) &= v(t_0) + a \cdot (t - t_0) \\
p(t) &= p(t_0) + v(t_0) \cdot (t - t_0) + \frac{1}{2} \cdot a \cdot (t - t_0)^2
\end{aligned}
$$

Uniform Motion

$$
\begin{aligned}
v(t) &= v \quad (= const) \\
p(t) &= p(t_0) + v \cdot (t - t_0)
\end{aligned}
$$

The Virtual Robot



- We must simulate the motion of the ideal robot by applying the equation above
- However, we must identify **when to change the motion** (from acceleration to cruise, and from cruise to deceleration)
- In other words, we should determine the time instants t_a and t_d in which the regime changes
- This can be done by using the equations, however we must remember that we then act in a “discretized” world!!

The Virtual Robot

Let's implement the virtual robot

- We can write a class that receives the desired parameters of the motion and acts accordingly to the speed profile
- The class embeds, in its attributes, the current speed and position of the robot
- Moreover, we need to somehow encode the phase in which our motion is

```
class VirtualRobot:
   ACCEL = 0
   CRUISE = 1
   DECEL = 2
   TARGET = 3
   def __init__(self, _p_target, _vmax, _acc, _dec):
       self.p_target = _p_target
       self.vmax = _vmax
       self.accel = _acc
       self.decel = _dec
       self.v = 0 # current speed
       self.p = 0 # current position
       self.phase = VirtualRobot.ACCEL
```

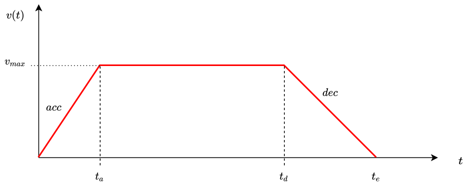
The Virtual Robot

Let's implement the virtual robot

- In the **evaluate** method, let's implement the behaviour of the motion
- acceleration and cruise phases are easy to implement, and also their transition can be easily idenfied
- but... when we should start the deceleration?

```
def evaluate(self, delta_t):
    if self.phase == VirtualRobot.ACCEL:
        self.p = self.p + self.v * delta_t \
                 + self.accel * delta_t * delta_t / 2
        self.v = self.v + self.accel * delta_t
        if self.v >= self.vmax:
            self.v = self.vmax
            self.phase = VirtualRobot.CRUISE

    elif self.phase == VirtualRobot.CRUISE:
        self.p = self.p + self.vmax * delta\_t
        if ?????:
            self.phase = VirtualRobot.DECEL
    ...
```

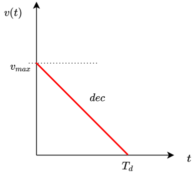
The Deceleration Distance



- Let's consider the final part of the motion, from t_d to the end t_e
- We start at speed v_{max}, at time t_d
- We end at speed 0, at time t_e
- Let us apply the formulae of the uniformly accelerated (decelerated) motion (let's suppose that *dec* is positive)

$$\begin{aligned} v(t) &= v(t_0) + a \cdot (t - t_0) \\ v(t_e) &= v(t_d) - dec \cdot (t_e - t_d) \\ 0 &= v_{max} - dec \cdot (t_e - t_d) \\ (t_e - t_d) &= \frac{v_{max}}{dec} \end{aligned}$$

The Deceleration Distance



- Now let's everything but final part of the motion
- Its duration is $T_d = t_e - t_d = \frac{v_{max}}{dec}$
- Let's suppose that it starts at position 0 and ends a position D

$$
\begin{aligned}
p(t) &= p(t_0) + v(t_0) \cdot (t - t_0) + \frac{1}{2} \cdot a \cdot (t - t_0)^2 \\
D &= 0 + v_{max} \cdot T_d - \frac{1}{2} \cdot dec \cdot T_d^2 \\
D &= v_{max} \cdot \frac{v_{max}}{dec} - \frac{1}{2} \cdot dec \cdot \frac{v_{max}^2}{dec^2} \\
D &= \frac{1}{2} \cdot \frac{v_{max}^2}{dec}
\end{aligned}
$$

The Deceleration Distance

$$D = \frac{1}{2} \cdot \frac{v_{max}^2}{dec}$$

- We obtained the **deceleration distance**
- It is the **distance from the target** at which we must start the deceleration phase
- Therefore, if $p_{target} - p_{current} \leq D$, we are in the deceleration phase

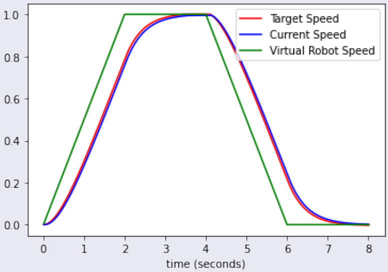
```
class VirtualRobot:
   ...
   def __init__(self, _p_target, _vmax, _acc, _dec):
       ...
       self.decel_distance = 0.5 * _vmax * _vmax / _dec

   def evaluate(self, delta_t):
       ...
       elif self.phase == VirtualRobot.CRUISE:
           self.p = self.p + self.vmax * delta\_t
           if self.p_target - self.p <= self.decel_distance:
               self.phase = VirtualRobot.DECEL
       ...
```

- And finally let's implement the deceleration phase

```
def evaluate(self, delta_t):
    ...
    elif self.phase == VirtualRobot.DECEL:
        self.p = self.p + self.v * delta_t \
                 - self.decel * delta_t * delta_t / 2
        self.v = self.v - self.decel * delta_t
        if self.p >= self.p_target:
            self.v = 0
            self.p = self.p_target
            self.phase = VirtualRobot.TARGET
    ...
```

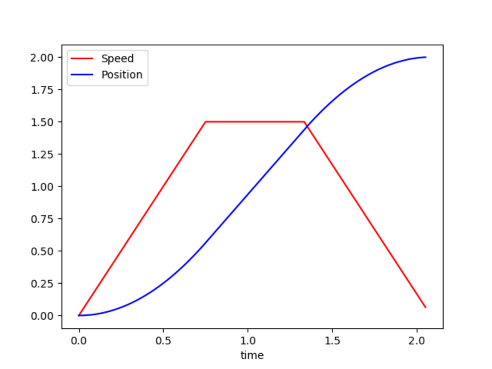
Testing the Code

```
rob =   VirtualRobot(   4, # distance 4 m
                        1.5, # max speed 1.5 m/s
                        2.0, # accel 2 m/s2
                        2.0) # decel 2 m/s2
```



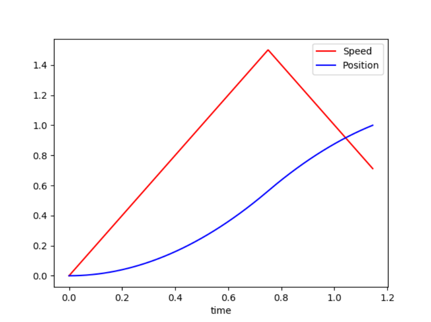
Testing the Code

```
rob =   VirtualRobot(   2, # distance 2 m
                        1.5, # max speed 1.5 m/s
                        2.0, # accel 2 m/s2
                        2.0) # decel 2 m/s2
```



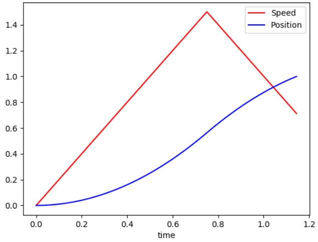
Phase Overappling

```
rob =   VirtualRobot(   1, # distance 2 m
                        1.5, # max speed 1.5 m/s
                        2.0, # accel 2 m/s2
                        2.0) # decel 2 m/s2
```



The target is reached but the final speed **is not 0**!!

Virtual Robot



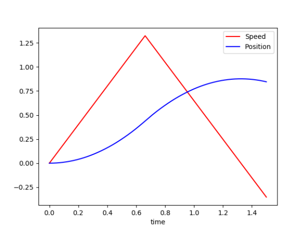
Phase Overlapping

- When the distance is too short, phases may overlap
- The deceleration distance is such that the deceleration phase should begin **before** the acceleration phase is ended
- So we should consider this particular case in our code

Testing the Code

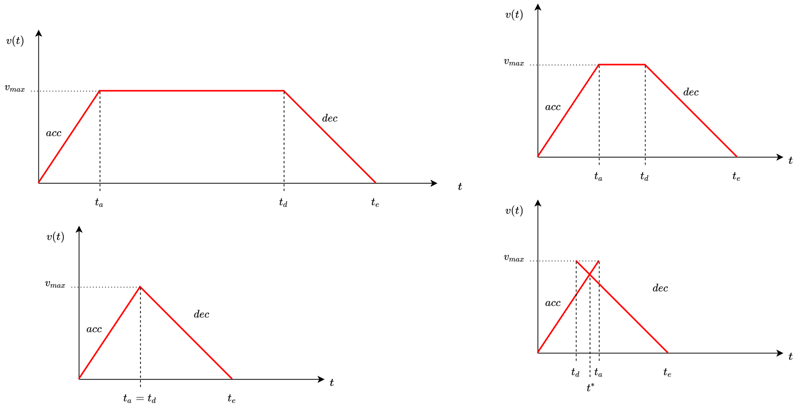
- At first sight, the code should be patched as follows:

```
def evaluate(self, delta_t):
    if self.phase == VirtualRobot.ACCEL:
        self.p = self.p + self.v * delta_t \
                 + self.accel * delta_t * delta_t / 2
        self.v = self.v + self.accel * delta_t
        if self.v >= self.vmax:
            self.v = self.vmax
            self.phase = VirtualRobot.CRUISE
        elif self.p_target - self.p <= self.decel_distance:
            self.phase = VirtualRobot.DECEL
```



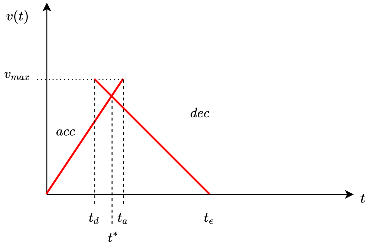
The target is never reached!! Why??

Speed Profile and Distance



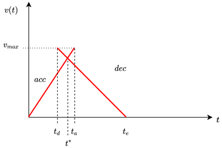
- As soon as the the target distance decreases, the cruise phase is shortened and the deceleration phase “approaches” the acceleration phase
- Until the acceleration and deceleration phases **overlap!**

Speed Profile and Distance



- In this case, the deceleration distance **is not** the one computed before
- But we must find the place in which the acceleration and deceleration lines **meet**

Speed Profile and Distance



Where do the acc and dec phases meet?

- Let's consider once again only the deceleration phase
- Let us suppose that, at a certain time instant, we are at a distance d from the target
- Here we will start travelling at a certain speed v_d and we will have the distance d to cover
- According to dec that distance will be covered in certain time t'
- We have:

$$d = 0 + v_d \cdot t' - \frac{1}{2} \cdot dec \cdot t'^2$$

Speed Profile and Distance

Where do the acc and dec phases meet?

- We have:

$$d = 0 + v_d \cdot \Delta t' - \frac{1}{2} \cdot dec \cdot \Delta t'^2 \qquad (1)$$

- In the same time interval $\Delta t'$, our speed will go from v_d (unknown) to 0, so:

$$0 = v_d - dec \cdot \Delta t' \qquad (2)$$

- Let's compute $\Delta t'$ from (2) and substitute in (1):

$$d = v_d \cdot \frac{v_d}{dec} - \frac{1}{2} \cdot dec \cdot (\frac{v_d}{dec})^2 \qquad (3)$$

Speed Profile and Distance

Where do the acc and dec phases meet?

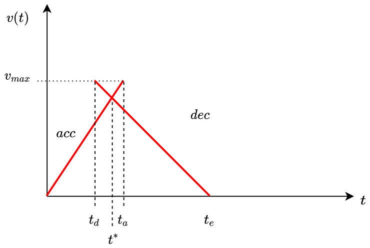
$$d = v_d \cdot \frac{v_d}{dec} - \frac{1}{2} \cdot dec \cdot \frac{v_d^2}{dec^2} \quad (4)$$

- Let's determine v_d from (4):

$$v_d = \sqrt{2 \cdot dec \cdot d} \quad (5)$$

- Formula (5) gives the **expected speed** v_d when we are at a distance d from the end of the motion

Speed Profile and Distance



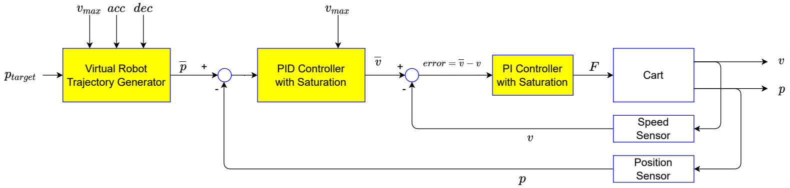
Resolving the Overlapping

- Now, we are in the acceleration phase, and our speed is v
- According to our initial computation of the **deceleration distance** we have that, **hypothetically**, our deceleration should start at t_d, can we really enter in that phase?
- Since we know the distance to be travelled d, let's determine the expected speed v_d
- if $v_d > v$, **we are still in the acceleration phase**, so continue to accelerate until the condition becomes false

The final code

```
def evaluate(self, delta_t):
    if self.phase == VirtualRobot.ACCEL:
        self.p = self.p + self.v * delta_t \
                 + self.accel * delta_t * delta_t / 2
        self.v = self.v + self.accel * delta_t
        distance = self.p_target - self.p
        if self.v >= self.vmax:
            self.v = self.vmax
            self.phase = VirtualRobot.CRUISE
        elif distance <= self.decel_distance:
            v_exp = math.sqrt(2 * self.decel * distance)
            if v_exp < self.v:
                self.phase = VirtualRobot.DECEL

    elif self.phase == VirtualRobot.CRUISE:
        self.p = self.p + self.vmax * delta_t
        distance = self.p_target - self.p
        if distance <= self.decel_distance:
            self.phase = VirtualRobot.DECEL

    elif self.phase == VirtualRobot.DECEL:
        self.p = self.p + self.v * delta_t \
                 - self.decel * delta_t * delta_t / 2
        self.v = self.v - self.decel * delta_t
        if self.p >= self.p_target:
            self.v = 0
            self.p = self.p_target
            self.phase = VirtualRobot.TARGET
```

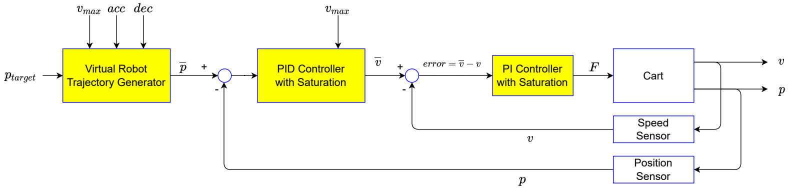
Back to Position Control



From Virtual to Real

- Now we have our virtual robot that travels according to a “path” generated from our initial requirements (distance, maximum speed, acceleration and deceleration)
- How can we use it in our real **position control**?
- The idea is to let the real robot “catch” the virtual robot

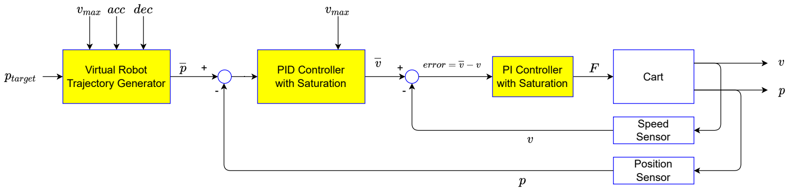
Back to Position Control



Catching the Virtual Robot

- The trajectory generator (our VirtualRobot class) gives the position $\overline{p}$ of the virtual robot time-by-time
- $\overline{p}$ is the position in which we expect to find also the real robot, but this will not be the case
- Let's determine the error $\overline{p} - p$ between expected and real position of the real robot and use a PID controller to compute the speed needed to reach $\overline{p}$
- In other words, the control system works in order to keep the error $\overline{p} - p$ as **non-zero** in order to output a travelling speed (until $\overline{p} = p_{target}$)

Catching the Virtual Robot



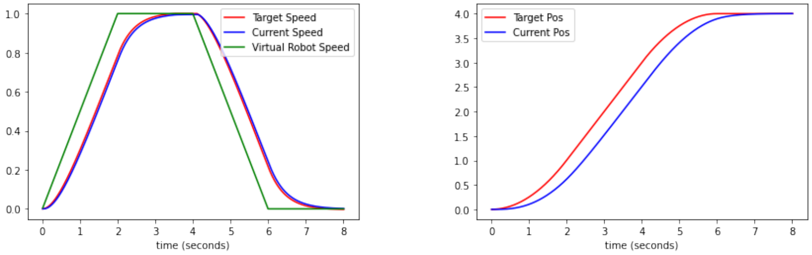
The Code

```
    ...
trajectory = VirtualRobot(4.0,     # 4 meters
                          1.0,     # vmax = 1 m/s
                          0.5, 0.5) # acc = dec = 0.5 m/s

ctrl = PositionSpeedControl(2.0, 1.0,     # PositionController: sat = 1 m/s
                            20.0, 10.0, 20.0)
                           # Speed Controller: kp =20, ki = 10, sat = 20 N
    ...
```

(see examples/position_control/cart_position_control_virtual_robot.ipynb)

Catching the Virtual Robot

$K_P = 2.0$

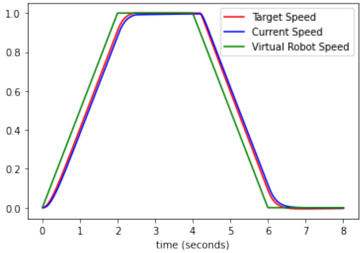


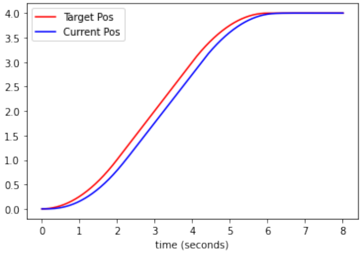
The Role of Constants of the Position Controller

- K_P controls the **delay** of the real robot with respect to the virtual robot
- It is only a delay not an error, since the target position is (sooner or later) reached

Catching the Virtual Robot

$$K_P = 4.0$$

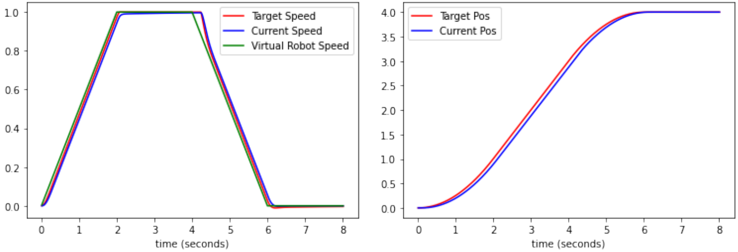




The Role of Constants of the Position Controller

- Interesting.... but still slow

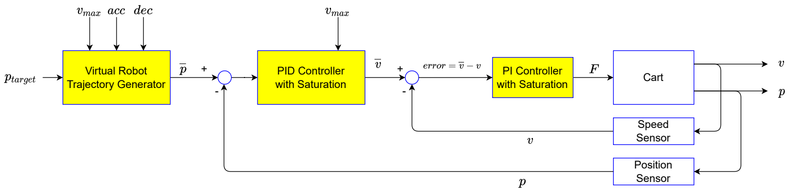
$K_P = 8.0$



The Role of Constants of the Position Controller

- Very nice!!

The Virtual Robot



Lesson Learned

- The virtual robot is indeed a generator of the **theoretical trajectory** that, during time, must be followed by the real system
- Here we have a case with mono-dimensional motion and thus a single (position) variable to control
- However the same concepts can be applied when the trajectory is in a plane or in space

Controlling Position and Speed using Profiles

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