A multirotor (a.k.a. “drone”) is a flying object characterised by:

- An even set of equal horizontal propellers (and motors), $\geq 4$, symmetrically placed in a circular shape
- A symmetric/balanced airframe (even if not strictly mandatory)
- VTOL (Vertical Take-off and Landing) capabilities
- Four degrees of freedom, $XYZ + Heading$
- No critical issues from the mechanical/aerodynamic point of view
- Total control in software, no mechanical parts
The **body reference system** usually employed is the one in figure.

The system also define the **Euler angles** that represents the **attitude**:

- roll, $\phi$
- pitch, $\theta$
- yaw, $\psi$

The **pose** of the multirotor is represented by:

- $\{X, Y, Z, \phi, \theta, \psi\}$, in the **Earth frame**
To understand the dynamics we adopt a simplified bidimensional model.

The pose is here represented by the tuple $\{X, Z, \theta\}$:
- $X$
- $Z$, altitude
- $\theta$, inclination w.r.t. the horizon
Rotational Dynamics

- Each propeller (we suppose only 2) produces a force perpendicular to its rotation plane.

- If the forces are not the same, the result is a rotation along the mass center; the rotation speed is dependent on the force’s difference.

- To model the motion we use the rotational second Newton’s Law: the sum of the torques is equal to the moment of inertia of the body times the angular acceleration.

\[ F_1 \times L = I \times \ddot{\theta} \]

\[ F_2 \times L = I \times \ddot{\theta} \]

\[ \sum \tau = I \ddot{\theta} \]

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Control of a simplified multirotor
Say:

- $M$, the mass of the multirotor
- $I = \frac{1}{12}M(2L)^2$, the moment of inertia of the multirotor (we are supposing a rigid bar with a length of $2L$ and mass $M$)
- $-bL\dot{\theta}$, the friction torque

we have:

$$F_2L - F_1L - bL\dot{\theta} = I\ddot{\theta}$$
Let's include the angular speed \( \omega = \dot{\theta} \), we obtain:

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= -\frac{bL}{I} \omega + \frac{L}{I} (F_2 - F_1)
\end{align*}
\]
\[ \begin{align*} \dot{\theta} &= \omega \\
\dot{\omega} &= -\frac{bL}{I}\omega + \frac{L}{I}(F_2 - F_1) \end{align*} \]

Discretization:

\[ \begin{align*} \theta(k + 1) &= \theta(k) + \Delta T \omega(k) \\
\omega(k + 1) &= \omega(k) - \frac{bL\Delta T}{I}\omega(k) + \Delta T \frac{L}{I}(F_2 - F_1) \end{align*} \]
Translation Dynamics (along X)

\[ (F_1 + F_2) \sin(-\theta) - b v_x = M \dot{v}_x \]

- here \(-b v_x\) is the friction force
- we have:

\[
\begin{align*}
\dot{x} & = v_x \\
\dot{v}_x & = -\frac{b}{M} v_x + \frac{F_1 + F_2}{M} \sin(-\theta)
\end{align*}
\]
Translation Dynamics (along X)

\[ \begin{align*}
\dot{x} &= v_x \\
\dot{v}_x &= -\frac{b}{M} v_x + \frac{F_1 + F_2}{M} \sin(-\theta)
\end{align*} \]

Let's discretize:

\[ \begin{align*}
x(k+1) &= x(k) + \Delta T v_x(k) \\
 v_x(k+1) &= (1 - \Delta T \frac{b}{M}) v_x(k) + \Delta T \frac{F_1 + F_2}{M} \sin(-\theta)
\end{align*} \]
Translation Dynamics (along Z)

\[(F_1 + F_2)\cos \theta - bv_z - Mg = M \ddot{v}_z\]

We have:

\[\begin{align*}
\dot{z} &= v_z \\
\dot{v}_z &= -\frac{b}{M} v_z + \frac{F_1 + F_2}{M} \cos \theta - g
\end{align*}\]
Translation Dynamics (along Z)

\[
\begin{align*}
\dot{z} &= v_z \\
\dot{v}_z &= -\frac{b}{M} v_z + \frac{F_1 + F_2}{M} \cos \theta - g
\end{align*}
\]

Let's discretize:

\[
\begin{align*}
z(k+1) &= z(k) + \Delta T v_z(k) \\
v_z(k+1) &= (1 - \Delta T \frac{b}{M}) v_z(k) + \Delta T \frac{F_1 + F_2}{M} \cos \theta - \Delta T g
\end{align*}
\]
Horizontal Motion Control Model

THETA CONTROL
P + SATURATION

OMEGA CONTROL
P + SATURATION

DRONE
Vx

F1
F2

X CONTROL
P + SATURATION

Vx CONTROL
P + SATURATION

Vx

X

Control of a simplified multirotor
Vertical Motion Control Model

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Control of a simplified multirotor
Dati da usare

- Massa, $M = 1 \text{ Kg}$
- Lunghezza bracci, $L = 0.25 \text{ m}$
- Coefficiente di attrito viscoso, $b = 7 \cdot 10^{-5}$
- Forza massima di spinta motori, $15 \text{ N}$
- Inclinazione massima, $\theta_{\text{max}} = 0.52 \text{ rad}$ (circa 30 gradi)
- Velocità di rotazione massima, $\omega_{\text{max}} = 1.57 \text{ rad/s}$ (circa 90 gradi al secondo)
- Velocità di traslazione massima (sia X che Z), $V_{\text{max}} = 2 \text{ m/s}$
Multirotor Control
(Simplified Model)

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Robotic Systems

Control of a simplified multirotor