# Multirotor Control (Simplified Model)

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Robotic Systems

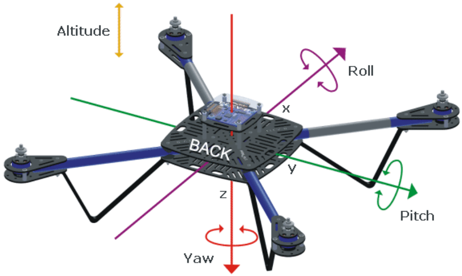
# Multirotor: definitions

A **multirotore** (a.k.a. “drone”) is a flying object characterised by:

- An **even set** of equal *horizontal propellers* (and motors), $\geq 4$, symmetrically placed in a circular shape
- A **symmetric/balanced** airframe (even if not strictly mandatory)
- **VTOL** (Vertical Take-off and Landing) capabilities
- **Four degrees of freedom**, $XYZ + Heading$
- **No critical issues** from the mechanical/aerodynamic point of view
- Total control in **software**, no mechanical parts

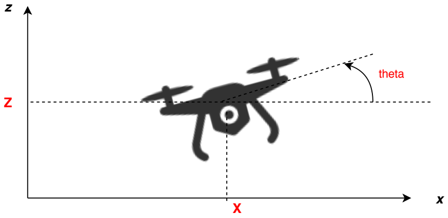
# Reference System

- The **body reference system** usually employed is the one in figure
- The system also define the **Euler angles** that represents the **attitude**:
  - **roll**, $\phi$
  - **pitch**, $\theta$
  - **yaw**, $\psi$
- The **pose** of the multirotor is represented by:
  - $\{X, Y, Z, \phi, \theta, \psi\}$, in the **Earth frame**



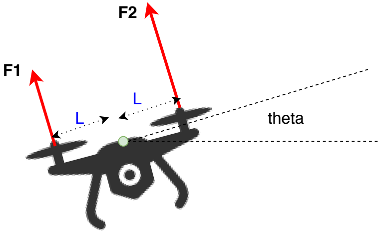
# Simplified Dynamics

- To understand the dyamics we adopt a **simplified bidimensional model**
- The **pose** is here represented by the tuple $\{X, Z, \theta\}$:
  - $X$
  - $Z$, altitude
  - $\theta$, inclination w.r.t. the horizon

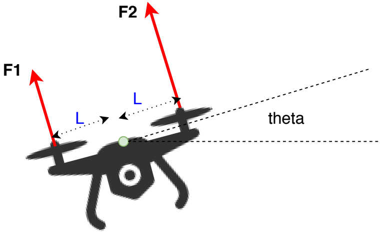


# Rotational Dynamics

- Each propeller (we suppose only 2) produces a force perpendicular to its rotation plane
- If the forces are not the same, the result is a **rotation** along the mass center; the **rotation speed** is dependent on the force's difference
- To model the motion we use the **rotational second Newton's Law**: the sum of the **torques** is equal to the **moment of intertia** of the body times the **angular acceleration**



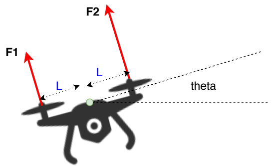
# Rotational Dynamics




- Say:
  - $M$, the mass of the multirotor
  - $I = \frac{1}{12}M(2L)^2$, the moment of inertia of the multirotor (we are supposing a rigid bar with a length of $2L$ and mass $M$)
  - $-bL\dot{\theta}$, the friction torque
- we have:

$$F_2 L - F_1 L - bL\dot{\theta} = I\ddot{\theta}$$

# Rotational Dynamics

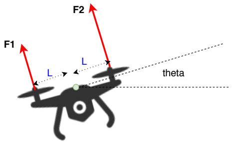



$$F_2 L - F_1 L - bL\dot{\theta} \quad = \quad I\ddot{\theta}$$

- Let's include the angular speed $\omega = \dot{\theta}$, we obtain:

$$\begin{cases} \dot{\theta} & = & \omega \\ \dot{\omega} & = & -\frac{bL}{I}\omega + \frac{L}{I}(F_2 - F_1) \end{cases}$$

# Rotational Dynamics

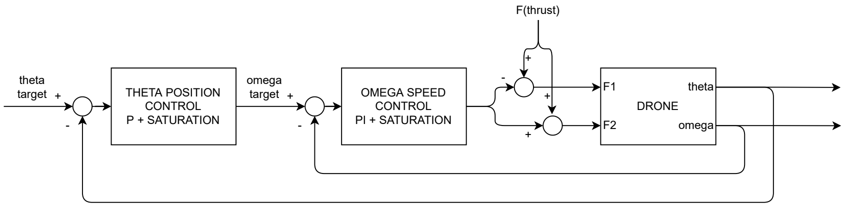



$$\begin{cases} \dot{\theta} & = & \omega \\ \dot{\omega} & = & -\frac{bL}{I}\omega + \frac{L}{I}(F_2 - F_1) \end{cases}$$

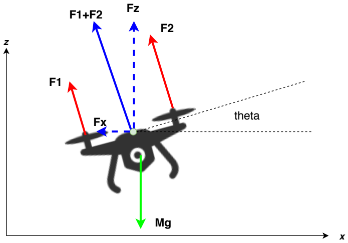
Discretization:

$$\begin{cases} \theta(k+1) & = & \theta(k) + \Delta T \omega(k) \\ \omega(k+1) & = & \omega(k) - \frac{bL\Delta T}{I}\omega(k) + \Delta T \frac{L}{I}(F_2 - F_1) \end{cases}$$

# Rotation Control



# Translation Dynamics (along X)

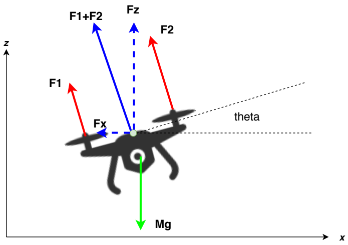



$$(F_1 + F_2) sin(-\theta) - b v_x \quad = \quad M \dot{v}_x$$

- here $-b v_x$ is the friction force
- we have:

$$\begin{cases} \dot{x} & = & v_x \\ \dot{v}_x & = & -\frac{b}{M} v_x + \frac{F_1 + F_2}{M} sin(-\theta) \end{cases}$$

# Translation Dynamics (along X)

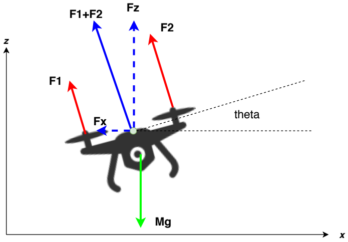



$$\begin{cases} \dot{x} = v_x \\ \dot{v}_x = -\frac{b}{M} v_x + \frac{F_1+F_2}{M} sin(-\theta) \end{cases}$$

Let's discretize:

$$\begin{cases} x(k+1) = x(k) + \Delta T v_x(k) \\ v_x(k+1) = (1 - \Delta T \frac{b}{M}) v_x(k) + \Delta T \frac{F_1+F_2}{M} sin(-\theta) \end{cases}$$

# Translation Dynamics (along Z)

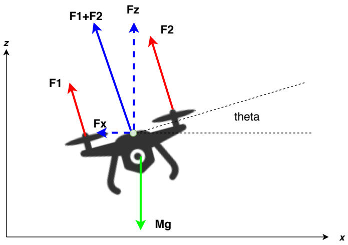



$$(F_1 + F_2)cos\theta - bv_z - Mg = M\dot{v}_z$$

- We have:

$$\begin{cases} \dot{z} = v_z \\ \dot{v}_z = -\frac{b}{M}v_z + \frac{F_1+F_2}{M}cos\theta - g \end{cases}$$

# Translation Dynamics (along Z)

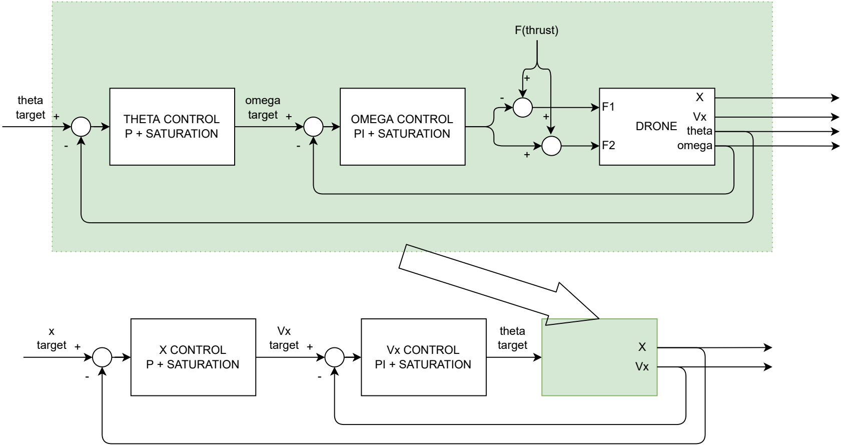



$$\begin{cases} \dot{z} = v_z \\ \dot{v}_z = -\frac{b}{M} v_z + \frac{F_1+F_2}{M} cos\theta - g \end{cases}$$

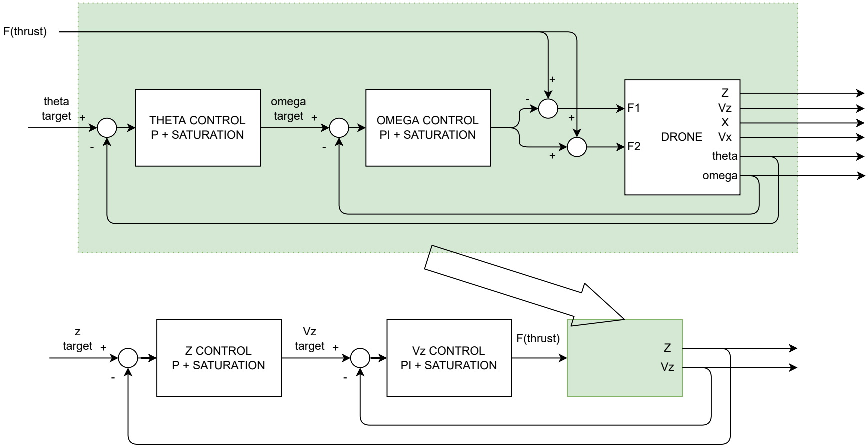
Let's discretize:

$$\begin{cases} z(k+1) = z(k) + \Delta T v_z(k) \\ v_z(k+1) = (1 - \Delta T \frac{b}{M}) v_z(k) + \Delta T \frac{F_1+F_2}{M} cos\theta - \Delta T g \end{cases}$$

# Horizontal Motion Control Model



# Vertical Motion Control Model



# Dati da usare

- Massa, $M = 1\ Kg$
- Lunghezza bracci, $L = 0.25\ m$
- Coefficiente di attrito viscoso, $b = 7 \cdot 10^{-5}$
- Forza massima di spinta motori, $15\ N$
- Inclinazione massima, $\theta_{max} = 0.52\ rad$ (circa 30 gradi)
- Velocità di rotazione massima, $\omega_{max} = 1.57\ rad/s$ (circa 90 gradi al secondo)
- Velocità di traslazione massima (sia X che Z), $V_{max} = 2\ m/s$

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