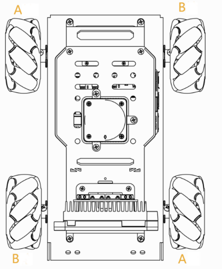
Locomotion of a Mobile Robot in a 2D Space
Mecanum Drive

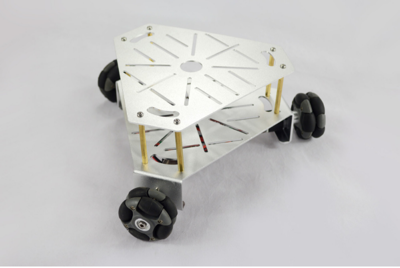
Corrado Santoro

ARSLAB - Autonomous and Robotic Systems Laboratory
Dipartimento di Matematica e Informatica - Università di Catania, Italy
`santoro@dmi.unict.it`

Robotic Systems

Mecanum Drive

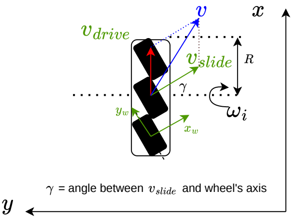




- It is a locomotion model based on special kind of wheels
- Wheels are equipped with free rollers that are able to make robot motion as **omnidirectional**
- It makes it possible to control v_x, v_y and ω **independently** by properly modulating the rotation speeds of the wheels

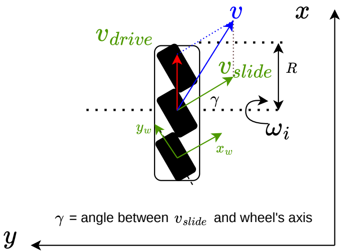
Kinematic Model

Mecanum Wheel Basics



- We consider a wheel placed in the reference frame of the robot
- We have:
  - ω: angular velocity of the motor driving the wheel
  - R: radius of the wheel
  - v_{drive}: the linear velocity resulting from motor motion
  - v_{slide}: the velocity of wheel sliding due to rollers (perpendicular to roller's axis)
  - γ: the inclination of roller's axis with respect to x
  - v: the overall velocity of the wheel composed by v_{drive} and v_{slide} vectors

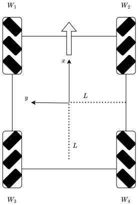
Mecanum Wheel Basics



Let's decompose v into v_x and v_y

$$
\begin{aligned}
v_x &= v_{drive} + v_{slide} \sin\gamma \\
v_y &= -v_{slide} \cos\gamma \\
v_{drive} &= v_x + v_y \frac{\sin\gamma}{\cos\gamma} \\
\omega R &= v_x + v_y \tan\gamma
\end{aligned}
$$

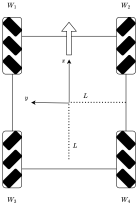
Mecanum 4-Wheel Drive



$$\begin{aligned}
\omega_1 R &= v_{1,x} + v_{1,y} \tan \gamma_1 \\
\omega_2 R &= v_{2,x} + v_{2,y} \tan \gamma_2 \\
\omega_3 R &= v_{3,x} + v_{3,y} \tan \gamma_3 \\
\omega_4 R &= v_{4,x} + v_{4,y} \tan \gamma_4
\end{aligned}$$

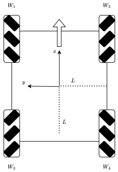
$$v_x = \frac{\sum v_{i,x}}{4} \qquad v_y = \frac{\sum v_{i,y}}{4}$$

Mecanum 4-Wheel Drive



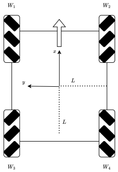
$$
\begin{aligned}
\omega_1 R &= v_{1,x} - v_{1,y}, \quad \gamma_1 = -\frac{\pi}{4} \\
\omega_2 R &= v_{2,x} + v_{2,y}, \quad \gamma_2 = \frac{\pi}{4} \\
\omega_3 R &= v_{3,x} + v_{3,y}, \quad \gamma_3 = \frac{\pi}{4} \\
\omega_4 R &= v_{4,x} - v_{4,y}, \quad \gamma_4 = -\frac{\pi}{4}
\end{aligned}
$$

Mecanum 4-Wheel Drive



Transformation Matrix, Inverse Kinematics

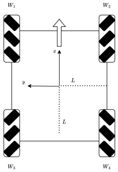
$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$



Transformation Matrix, Direct Kinematics

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{R}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

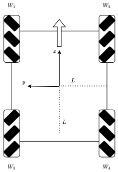
Mecanum 4-Wheel Drive



Angular Velocity

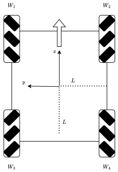
$$\dot{\theta} = \frac{R}{4}(-\omega_1 + \omega_2 - \omega_3 + \omega_4)\frac{1}{2L}$$

Mecanum 4-Wheel Drive



Final Transformation Matrix, Inverse Kinematics

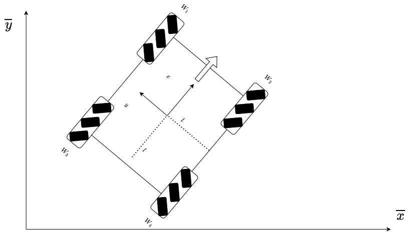
$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 1 & -1 & -2L \\ 1 & 1 & 2L \\ 1 & 1 & -2L \\ 1 & -1 & 2L \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix}$$



Final Transformation Matrix, Direct Kinematics

$$\begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} = \frac{R}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{1}{2L} & \frac{1}{2L} & -\frac{1}{2L} & \frac{1}{2L} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

Mecanum 4-Wheel Drive



Global Reference Frame

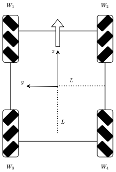
Given that the pose of the robot is X_R, Y_R, θ_R and $\{\overline{x}, \overline{y}\}$ the global reference frame, we have:

$$\begin{bmatrix} \overline{v}_x \\ \overline{v}_y \end{bmatrix} = \begin{bmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{bmatrix} \begin{bmatrix} \overline{v}_x \\ \overline{v}_y \end{bmatrix}$$

Dynamic Model

Mecanum 4-Wheel Drive



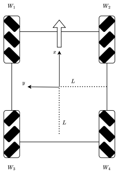
We must consider the Second Newton's Law in a 2D space:

$$\vec{F} - b\vec{v} = M\vec{a}$$

$$\begin{aligned} F_x - bv_x &= Ma_x \\ F_y - bv_y &= Ma_y \end{aligned}$$

$$\begin{aligned} \dot{v}_x &= -\frac{b}{M}v_x + \frac{1}{M}F_x \\ \dot{v}_y &= -\frac{b}{M}v_y + \frac{1}{M}F_y \end{aligned}$$

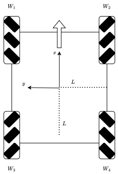
Mecanum 4-Wheel Drive



We must consider also the rotational Second Newton's Law:

$$
\begin{aligned}
T - \beta\dot{\theta} &= I\ddot{\theta} \\
\ddot{\theta} &= -\frac{\beta}{I}\dot{\theta} + \frac{1}{I}T
\end{aligned}
$$

($I =$ moment of inertia of the robot)



Transformation Matrix, Direct Dynamics

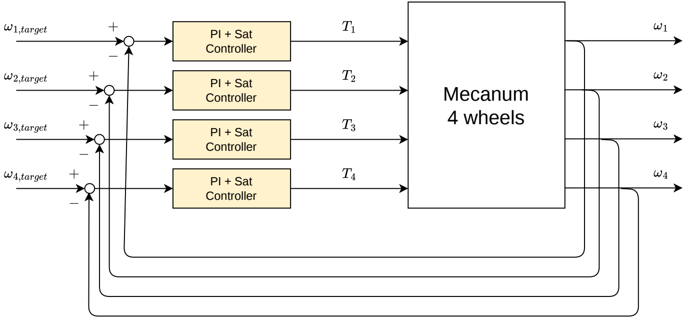
Given that T_i is the **torque** generated by i^{th} wheel:

$$\begin{bmatrix} F_x \\ F_y \\ T \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\sqrt{2}L & \sqrt{2}L & -\sqrt{2}L & \sqrt{2}L \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

Control

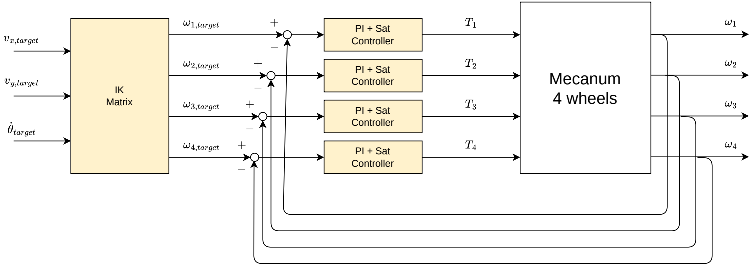
Wheel Control

- First of all we must control the speed of **each whell**
- This can be done by using **four** independent PI(+Sat) controllers



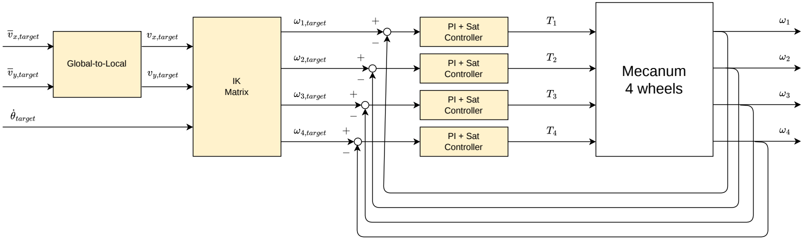
Robot's Frame Speed Control

- By adding the Inverse Kinematics matrix transformation, we can control linear and angular speed of the robot in **its frame**



Global Frame Speed Control

- By adding the global-to-local rotation we can control linear and angular speed of the robot in the **global frame**



Locomotion of a Mobile Robot in a 2D Space
Mecanum Drive

Corrado Santoro

ARSLAB - Autonomous and Robotic Systems Laboratory
Dipartimento di Matematica e Informatica - Università di Catania, Italy
`santoro@dmi.unict.it`

Robotic Systems