Canonica Signals
System Responses

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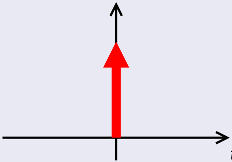
Robotic Systems

Canonical Signals

Canonical Signals

System Theory often uses some specific input signals, called **canonical signals**, to study the behaviour of a system:

- **Impulse** or **Dirac Delta**
- **(Unitary) Step**
- **Ramp**

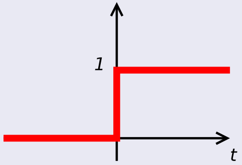


The **Dirac Delta** $\delta(t)$ is an impulsive signal that, from the mathematical point of view, is defined as:

$$\begin{cases} \delta(t) & = & 0, \forall t \neq 0 \\ \delta(t) & = & +\infty, t = 0 \\ \int_{-\infty}^{+\infty} \delta(t) dt & = & 1 \end{cases}$$

It is used to represent a physical fenomena with a great intensity but with an infinitesimal duration

Unitary Step

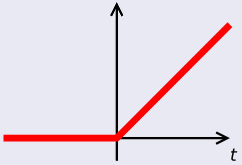


The **step** $u(t)$ is a signal defined as follows:

$$\begin{cases} u(t) = 0, \forall t < 0 \\ u(t) = 1, \forall t \geq 0 \end{cases}$$

It is used to model the application, at time 0, of a constant stimulus to a system

Ramp



The **ramp** $r(t)$ is an increasing signal defined as follows:

$$\begin{cases} r(t) = 0, \forall t < 0 \\ r(t) = t, \forall t \geq 0 \end{cases}$$

It is used to model the application to a system, at time 0, of a simulus that grows indefinitely

Relationship between Canonical Signals

Signals

$$u(t) = \int_0^t \delta(\tau) d\tau \qquad \frac{du(t)}{dt} = \delta(t)$$

$$r(t) = \int_0^t u(t) d\tau \qquad \frac{dr(t)}{dt} = u(t)$$

Responses

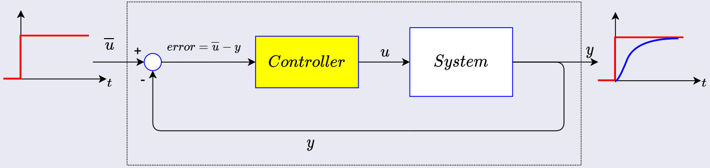
- Given a **linear system**, if $y_d(t)$ is the impulse response, then the **step response** is:

$$y_s(t) = \int_0^t y_d(\tau) d\tau$$

- and the **ramp response** is:

$$y_r(t) = \int_0^t y_s(\tau) d\tau$$

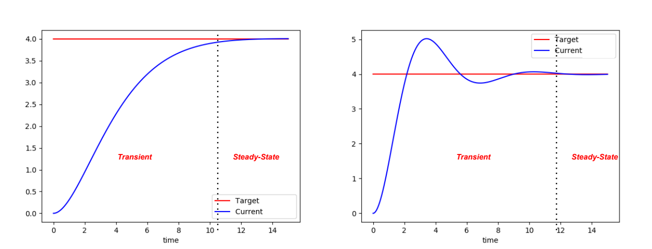
Canonical Signals and Control Systems



- Given a control system, its performaces are measured on the basis of **canonical inputs**
- The step represents a **constant reference** that is suddenly applied
- The ramp represents a **moving reference**, thus making it possible to measure the ability of the control system to follow changing references

Canonical Signals and Control Systems

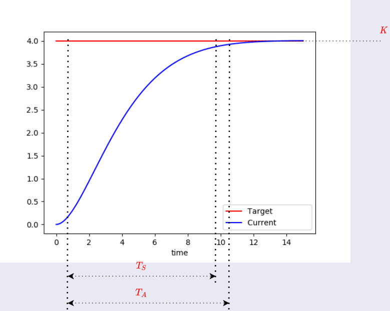
Transient and Steady-State Regimes



The response of a (asymptotically stable) system to a step (or a pulse) is composed of two parts:

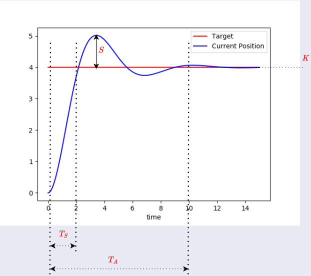
- **Transient**: initial part of the response; the output changes substantially during time
- **Steady-State**: when the transient is over, the output features small or no changes and stabilise to a specific value
- According to the response type (left or right figures above), the transient features some specific characteristics

Transient Characteristics



- **Steady-State Value**: $K = \lim_{t \to \infty} y(t)$
- **Rise Time** T_S ("tempo di salita"): the time required to go from 10% of K to 90% of K
- **Set-up Time** T_A ("tempo di assestamento"): the time required to have the output around the 98% of K

Transient Characteristics



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- **Overshoot** S ("sovraelongazione"): the percentage w.r.t. K of the first peak $S = \frac{peak-K}{K}$

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