$$\begin{cases} \dot{v} = -\frac{b}{M} v + \frac{1}{M} f \leftarrow \\ \dot{p} = v \end{cases}$$

$$\begin{bmatrix} \dot{v} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -\frac{b}{M} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \end{bmatrix} + \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix} f$$

$$x = \begin{bmatrix} v \\ p \end{bmatrix} \qquad A = \begin{bmatrix} -\frac{b}{M} & 0 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix}$$

$$A' = A \Delta T + I \qquad B' = B \Delta T$$

$$x(k+1) = A' x(k) + B' f(k) \leftarrow$$

$$A' = \begin{bmatrix} -\frac{b}{M} & 0 \\ 1 & 0 \end{bmatrix} \Delta T + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{b \Delta T}{M} + 1 & 0 \\ \Delta T & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix} \Delta T = \begin{bmatrix} \frac{\Delta T}{M} \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} v(k) \\ p(k) \end{bmatrix} \qquad x(k+1) = A' x(k) + B' f(k) \leftarrow$$

$$\begin{bmatrix} v(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} -\frac{b \Delta T}{M} + 1 & 0 \\ \Delta T & 1 \end{bmatrix} \begin{bmatrix} v(k) \\ p(k) \end{bmatrix} + \begin{bmatrix} \frac{\Delta T}{M} \\ 0 \end{bmatrix} f(k)$$

$$\begin{bmatrix} v(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} -\frac{b\Delta T}{M}+1 & 0 \\ \Delta T & 1 \end{bmatrix} \begin{bmatrix} v(k) \\ p(k) \end{bmatrix} + \begin{bmatrix} \frac{\Delta T}{M} \\ 0 \end{bmatrix} f(k)$$

$$v(k+1) = \left(-\frac{b}{M}\Delta T+1\right) v(k) + \frac{\Delta T}{M} f(k)$$

$$p(k+1) = \Delta T\, v(k) + p(k)$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad u \longrightarrow \boxed{A,B,C} \xrightarrow{\;y\;}$$

$$A' = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \Delta T + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \Delta T \\ -2\Delta T & -3\Delta T + 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \Delta T = \begin{bmatrix} 3\Delta T \\ 0 \end{bmatrix} \qquad C' = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{cases} (1)\; x(k+1) = A' x(k) + B' u(k) \\ (2)\; y(k) = C\, x(k) \end{cases} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(1)

$$x_1(k+1) = x_1(k) + x_2(k)\Delta T + 3\Delta T\, u(k)$$

$$x_2(k+1) = -2\Delta T\, x_1(k) + (1 - 3\Delta T)\, x_2(k)$$

(2)

$$y(k) = x_1(k)$$

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \Delta T + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \Delta T \\ 2\Delta T & 3\Delta T + 1 \end{bmatrix}$$

AUTOVALORI MATRICE A