Elements of Circuit Analysis

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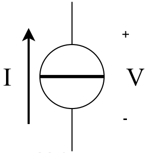
L.A.P. 1 Course

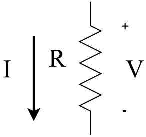
The Ohm's Law
The Kirchhoff Voltage Law (KVL)
The Kirchhoff Current Law (KCL)

Basic Elements of Direct Current (DC) Circuits

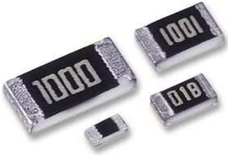
- *V*, voltage (Volt), difference of electrical potential
- *I*, current (Ampere), flow of electrons in circuit components
- *R*, resistance (Ohm), ability to "oppone" to electron flow



Voltage generator

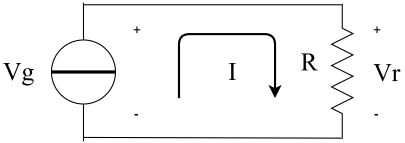


Resistor



The Ohm's Law

$$V = R\,I$$

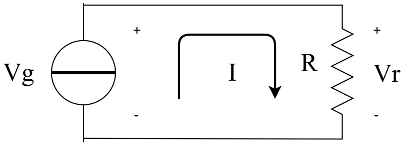


$$V_g = V_r$$
$$V_r = R\,I$$

The Ohm's Law

Given $V_g = 5V$ and $R = 10K\Omega$, calculate the current intensity

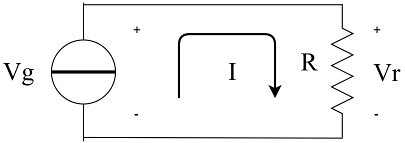
$$V = R\ I$$



$$\begin{aligned} I &= \frac{V_g}{R} = \\ &= \frac{5}{10 \cdot 10^3} = \\ &= 0.5 \cdot 10^{-3} A = \\ &= 0.5mA \end{aligned}$$

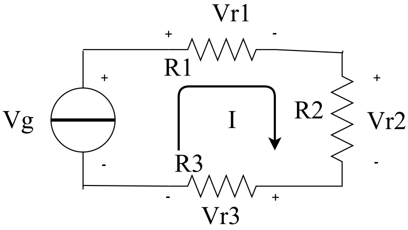
Given $V_g = 5V$, calculate the resistance to obtain a current of $3A$

$$V = R\,I$$



$$\begin{aligned} R &= \frac{V_g}{I} = \\ &= \frac{5}{3} = \\ &= 1.\overline{6}\Omega \end{aligned}$$

The algebraic sum of the voltages in a circuit **loop** is equal to 0

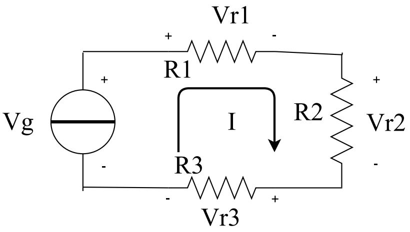


$$-V_g + V_{R1} + V_{R2} + V_{R3} = 0$$

$$V_{R1} + V_{R2} + V_{R3} = V_g$$

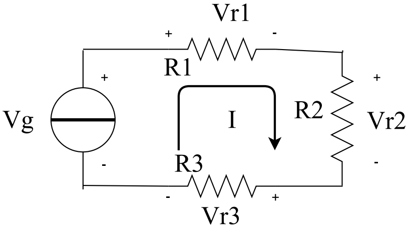
The Kirchhoff Voltage Law

Given $V_g = 5V$, $R1 = 220\Omega$, $R2 = 150\Omega$, $R3 = 18\Omega$, calculate V_{R1}, V_{R2} and V_{R3}.



$$
\begin{aligned}
V_g &= V_{R1} + V_{R2} + V_{R3} \\
V_g &= R1\ I + R2\ I + R3\ I \\
V_g &= (R1 + R2 + R3)\ I \\
I &= \frac{V_g}{R1 + R2 + R3} = \frac{5}{220 + 150 + 18} = 0.013A
\end{aligned}
$$

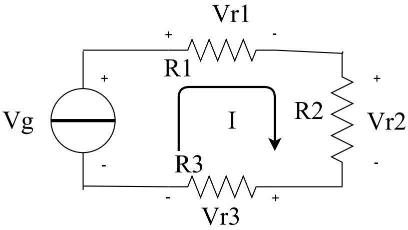
Given $V_g = 5V$, $R1 = 220\Omega$, $R2 = 150\Omega$, $R3 = 18\Omega$, calculate V_{R1}, V_{R2} and V_{R3}.



$$I = \frac{V_g}{R1 + R2 + R3} = \frac{5}{220 + 150 + 18} = 0.013A$$

$$V_{R1} = R1\ I = 220 \cdot 0.013 = 2.860\,V$$
$$V_{R2} = R2\ I = 150 \cdot 0.013 = 1.950\,V$$
$$V_{R3} = R3\ I = 18 \cdot 0.013 = 0.234\,V$$

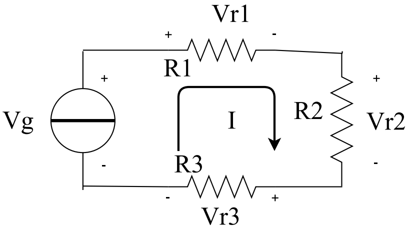
Given the circuit below, calculate e generic forumla that gives V_{R2} from V_g, $R1$, $R2$ and $R3$.



$$\begin{aligned} V_g &= V_{R1} + V_{R2} + V_{R3} \\ V_g &= R1\ I + R2\ I + R3\ I \\ V_g &= (R1 + R2 + R3)\ I \\ I &= \frac{V_{R2}}{R2} \\ V_g &= (R1 + R2 + R3)\frac{V_{R2}}{R2} \end{aligned}$$

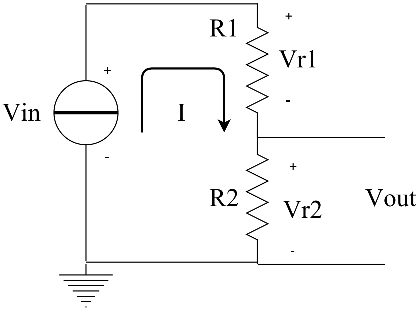
The Kirchhoff Voltage Law

Given the circuit below, calculate e generic forumla that gives V_{R2} from V_g, $R1$, $R2$ and $R3$.



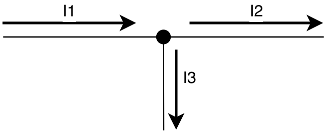
$$
\begin{aligned}
V_g &= (R1 + R2 + R3)\ I \\
I &= \frac{V_{R2}}{R2} \\
V_g &= (R1 + R2 + R3)\frac{V_{R2}}{R2} \\
V_{R2} &= \frac{R2}{R1 + R2 + R3} V_g
\end{aligned}
$$

The Voltage Divider



$$V_{out} = \frac{R2}{R1 + R2} V_{in}$$

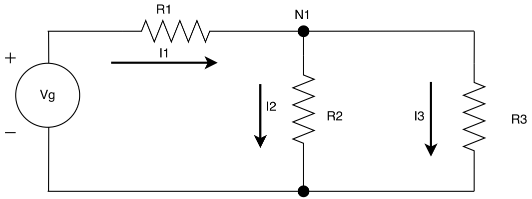
The algebraic sum of the currents insisting in a circuit **node** is equal to 0



$$I_1 - I_2 - I_3 = 0$$

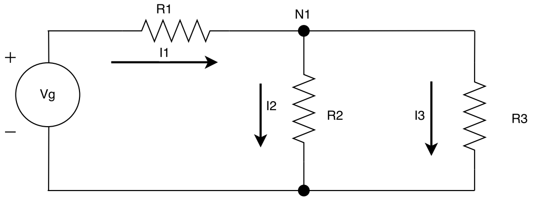
$$I_1 = I_2 + I_3$$

Given $V_g = 5V$, $R1 = 220\Omega$, $R2 = 150\Omega$, $R3 = 18\Omega$, calculate all the currents and voltages on the resistors



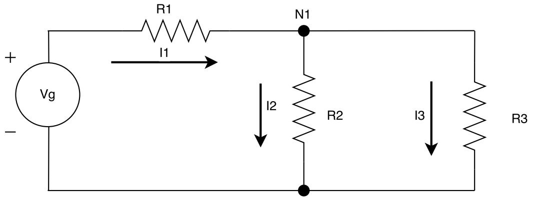
$$
\begin{aligned}
V_g &= V_{R1} + V_{R2} \\
V_{R2} &= V_{R3} \\
I_1 &= I_2 + I_3 \\
V_{R1} &= R1\ I_1 \\
V_{R2} &= R2\ I_2 \\
V_{R3} &= R3\ I_3
\end{aligned}
$$

Given $V_g = 5V$, $R1 = 220\Omega$, $R2 = 150\Omega$, $R3 = 18\Omega$, calculate all the currents and voltages on the resistors



$$\begin{cases} V_{R1} + V_{R2} = 5 \\ V_{R2} - V_{R3} = 0 \\ I_1 - I_2 - I_3 = 0 \\ V_{R1} - 220 I_1 = 0 \\ V_{R2} - 150 I_2 = 0 \\ V_{R3} - 18 I_3 = 0 \end{cases}$$

Given $V_g = 5V$, $R1 = 220\Omega$, $R2 = 150\Omega$, $R3 = 18\Omega$, calculate all the currents and voltages on the resistors



$$\left\{ \begin{array}{rcl} V_{R1} & = & 4.66V \\ V_{R2} & = & 0.35V \\ V_{R3} & = & 0.35V \\ I_1 & = & 21.2mA \\ I_2 & = & 2.3mA \\ I_3 & = & 18.9mA \end{array} \right.$$

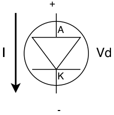
Rules of Circuit Analysis

1. Apply Kirchhoff Voltage Law to loops
2. Apply Kirchhoff Current Law to nodes
3. Apply Ohm's Law to elements (resistors)
4. Derive n (linear) equation in n unknown variables
5. Solve the derived linear system

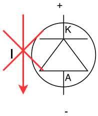
Semiconductors
Signal Diodes and Light Emitting Diodes (LEDs)

Diode

- A **diode** is an electronic component made of "semi-conductor" materiales (germanium, silicon, arsenic, gallium, ...)
- It has two wires **anode** and **catode**
- If it is **directly polarized**, it causes a **voltage fall** of V_d (˜0.7V in silicon diode, ˜2.0V in LEDs) and permits current flow
- If it is **inversely polarized**, it impedes current flow

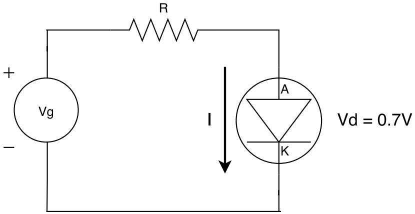


Direct Polarization



Inverse Polarization

Given $V_g = 5V$, $R = 220\Omega$, calculate the current I

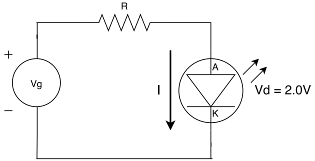


$$
\begin{aligned}
V_g &= V_R + V_d \\
5 &= V_R + 0.7 \\
V_R &= 4.3 \\
I &= \frac{V_R}{R} \\
I &= \frac{4.3}{220} = 0.02A = 20mA
\end{aligned}
$$

How to compute the limiting resistor for a LED

- LEDs have a **forward voltage** of 1.2–3.0 V
- LEDs have a **forward current** that depends on the luminosity, in general in the order of 20 mA

Given $V_g = 5V$ and $I = 20\ mA$, compute the limiting resistance

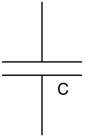


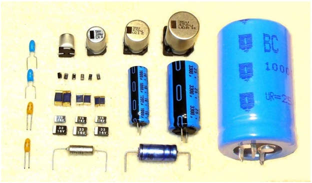
$$
\begin{aligned}
V_g &= V_R + V_d \\
5 &= V_R + 2.0 \\
V_R &= 3 \\
R &= \frac{V_R}{I} = \frac{3}{0.02} = 150\Omega
\end{aligned}
$$

Capacitors

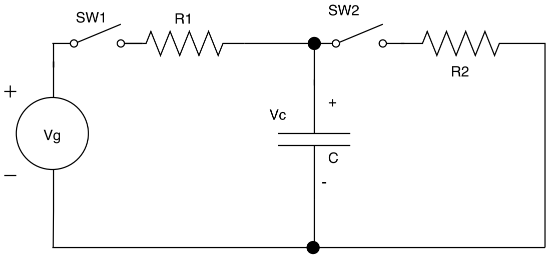
Capacitors

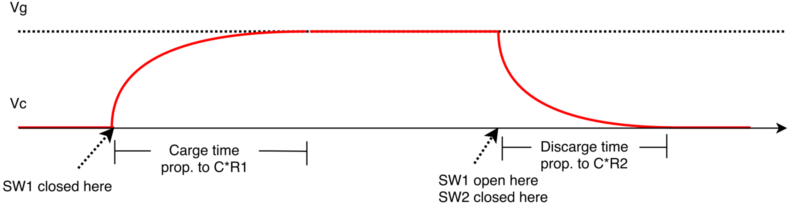
- A **capacitor** is a circuit element able to **gather/store electric charge**
- It is composed of two **plates** separated by a **dielectric** (insulator)
- The electric energy is stored in plates and depends on the size and material of plates and insulator
- The **capacity** (ability to store electric energy) is measured in **Farad** ($\mu F, nF, pF$)





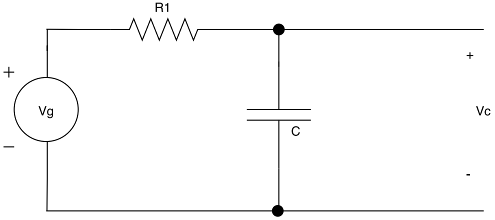
Dynamics of a capacitor

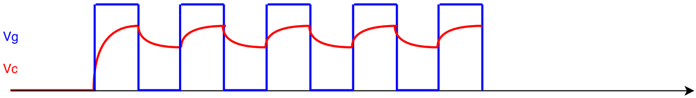




Capacitors as filters

- A capacitor with a resistor (**RC circuit**) act as a **filter**
- Here **Vg** is a generator of a **periodic signal** (a square wave)
- The RC circuit behaves by continuously charging and discharging the capacitor





Usage of Capacitors in digital circuits

- **By-pass capacitors**: placed between $V+$ and GND of a digital component, this capacitors are able to filter/remove spikes and other disturbances in the supply voltage
- **Debounce capacitors**: placed in parallel of push-buttons or switches are able to remove the "bouncing effect" of the mechanical parts

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