Efficient implementation of 2D Exner model using GPU

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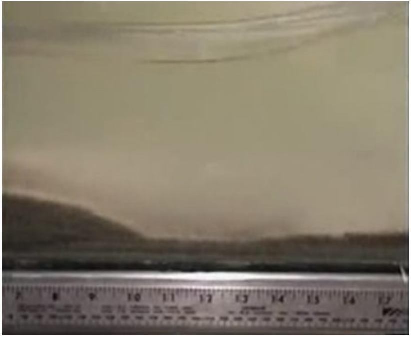
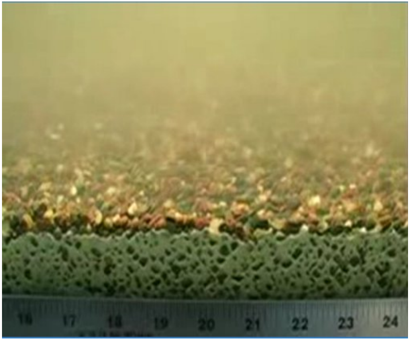
UNIVERSIDAD DE CÓRDOBA

Grupo EDANYA

Outline

Outline

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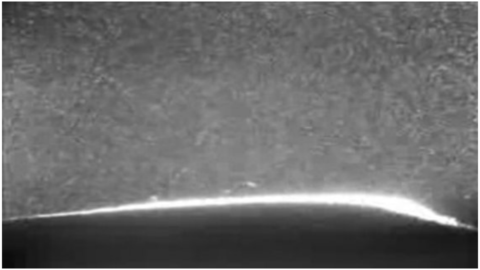
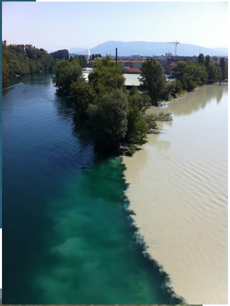


Real flows



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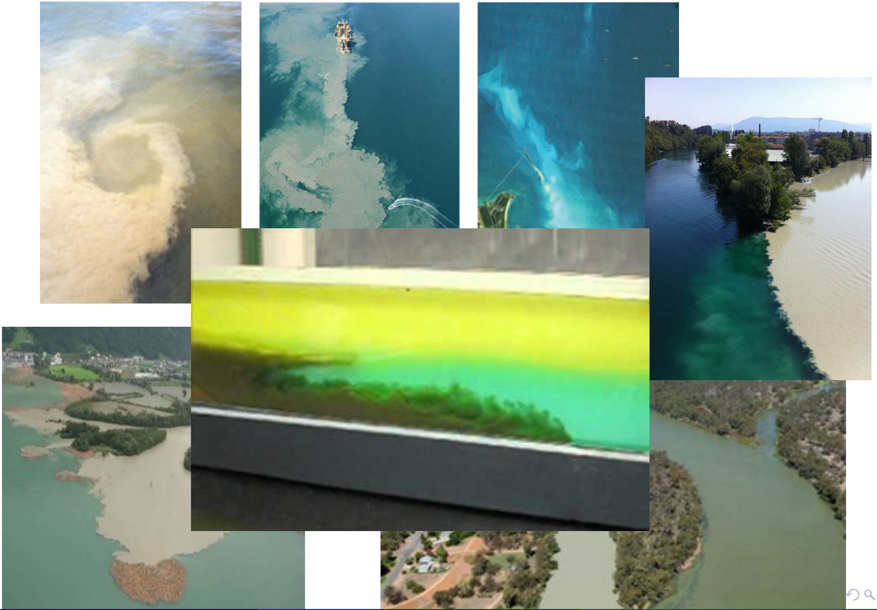


In this talk...

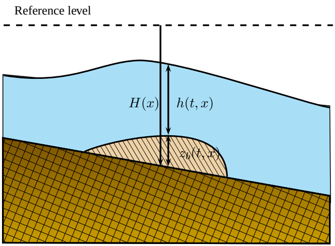
- bedload: [Grass 1981], [M&PM 1948], [Nielsen 1992], [Fowler 2007], [Fraccarollo 05,11], etc.
- suspension and hyperpycnal currents: [Parker 1986], [Kubo 2004], [Khan 2005], [Morales et al 2009], [Castro-Fernández-Morales 2015], etc.

Why sediment modeling is interesting?

- Profound impact on the morphology of continental shelves, lakes, artificial dams, etc...
- Destructive effect on pipelines, cables, submerged structures, ...

Outline

Exner equations



$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + g/2h^2) \\ \qquad\qquad = gh\partial_x(H - z_b) + S_f(W), \\ \partial_t z + \partial_x q_b = 0 \end{cases}$$

- $W = \begin{pmatrix} h & q & z_b \end{pmatrix}^T$
- $S_f(W)$ is the friction term
- $q(t,x) = h(t,x)\,u(t,x)$

Outline

Grass Model

$$q_b(h, hu) = \frac{1}{1-\varphi} A_g u|u|^{m_g - 1}, \quad 1 \le m_g \le 4$$

- A_g constant that depends on grain size, viscosity, etc.
- φ is the porosity
- Very simple model
- Critical shear stress set to zero (movement starts when $|u| > 0$)

Bottom shear stress

$$\tau_b = \rho_w g h S_{\mathrm{f}},$$

Darcy-Weisbach

$$S_{\mathrm{f}} = \frac{f u|u|}{8gh}$$

Manning

$$S_{\mathrm{f}} = \frac{n^2 u|u|}{h^{4/3}}$$

Shield's parameter

$$\theta = \frac{|\tau_b|}{(\rho_s - \rho_{\mathrm{w}})gd_s}$$

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Bedload transport flux

$$q_b = \frac{Q}{1-\varphi}\Phi(\theta)\,\mathrm{sgn}\,(\tau_b)$$

$$Q = \sqrt{\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{w}}} - 1\right) g d_{\mathrm{s}}^3}$$

Meyer-Peter&Müller model (1948)

$$q_b(h, hu) = \frac{Q}{1-\varphi} \cdot \Phi(\theta) \cdot \operatorname{sgn}(\tau_b) \qquad \text{(MPM)}$$

$$\Phi(\theta) = 8(\theta - \theta_c)_+^{3/2}$$

- More complex but gives good results
- Sediment transport starts if $\theta > \theta_c$
- Valid for small slopes (lower than 2%)

Fernández Luque & Van Beek (1976)

$$q_b(h, hu) = \frac{Q}{1-\varphi} \cdot \Phi(\theta) \cdot \operatorname{sgn}(\tau_b)$$

$$\Phi(\theta) = 5.7(\theta - \theta_c)_+^{3/2}$$

Ribberink (1998)

$$q_b(h, hu) = \frac{Q}{1-\varphi} \cdot \Phi(\theta) \cdot \operatorname{sgn}(\tau_b)$$

$$\Phi(\theta) = 11(\theta - \theta_c)_+^{1.65}$$

Nielsen (1992)

$$q_b(h, hu) = \frac{Q}{1-\varphi} \cdot \Phi(\theta) \cdot \operatorname{sgn}(\tau_b)$$

$$\Phi(\theta) = 12\sqrt{\theta}(\theta - \theta_c)_+$$

General formulation

$$q_b(h, hu) = \frac{cQ}{1-\varphi}(\theta)^{m_1}(\theta - \theta_c)_+^{m_2}\operatorname{sgn}(\tau_b),$$

Properties

- $\dfrac{\partial q_b}{\partial h} = -k\dfrac{q}{h}\dfrac{\partial q_b}{\partial q}$ with $k = 1$(Darcy-Weisbach) or $k = 7/6$ (Manning)

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- Grass and models based on Darcy-Weisbach always hyperbolic
- Manning models at least hyperbolic for $|u| < 6\sqrt{gh}$
- The models are only valid for small slopes, otherwise gravity effects should be included

Common problems to these expressions

- q_b depends only on the variables h and u and does not take into account sediment layer

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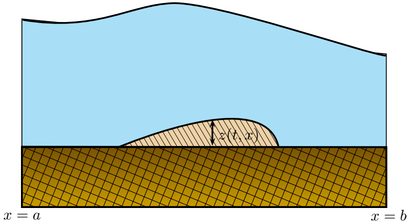

$$\partial_t z_b + \xi \partial_x q_b \qquad = 0$$

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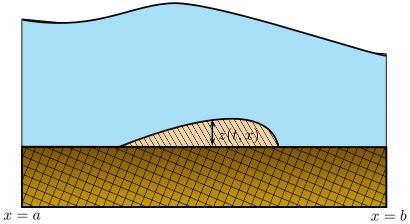

$$\int_0^T \int_a^b \left(\partial_t z_b + \xi \partial_x q_b\right) dxdt = 0$$

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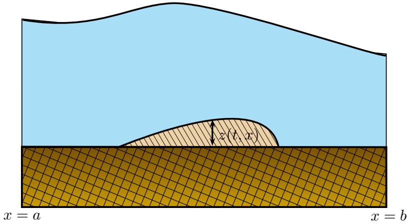


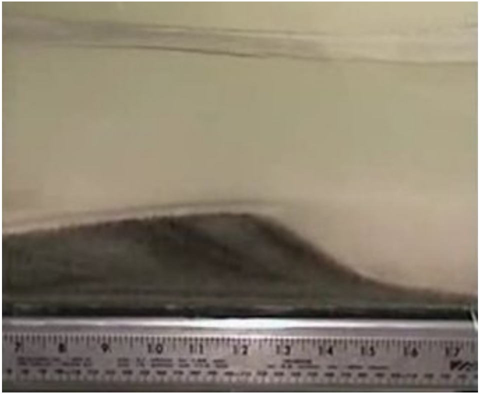
$$\int_a^b z_b|_{t=T}\,dx - \int_a^b z_b|_{t=0}\,dx = -\xi \int_0^T \left(q_b|_{x=b} - q_b|_{x=a}\right) dt$$

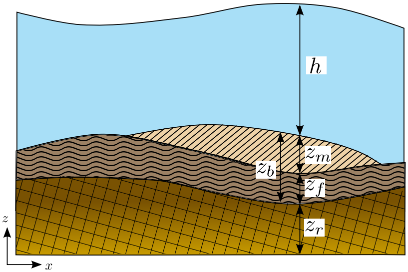
Sediment mass is not preserved!!

Modified models (Fowler type models)

$$q_b(h, hu, z_b) = Q\, c\, \frac{z_b}{z_b^0} \theta^{m_1} (\theta - \theta_{cr})_+^{m_2} \operatorname{sgn}(\tau_b) \qquad \text{(modFow)}$$

where z_b^0 is a characteristic thickness of the sediment layer.





$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) = -gh\partial_x(z_m + z_f + z_r), \\ \partial_t z_m + \partial_x \widetilde{q}_b = \dot{z}_e - \dot{z}_d, \\ \partial_t z_f = -\dot{z}_e + \dot{z}_d, \end{cases}$$

$$\widetilde{q}_b = z_m v_b,$$

[Fernández-Nieto, Lucas, Morales, Cordier 2014]

Extension to 2D

General formulation

$$\mathbf{q_b}(h, hu, hv) = \frac{cQ}{1-\varphi}(\theta)^{m_1}(\theta-\theta_c)_+^{m_2}\operatorname{sgn}(\boldsymbol{\tau_b}),$$

$$\theta = \frac{\|\boldsymbol{\tau_b}\|}{(\rho_s - \rho_{\mathrm{w}})gd_s}$$

$$\operatorname{sgn}(\boldsymbol{\tau_b}) = \frac{\boldsymbol{\tau_b}}{\|\boldsymbol{\tau_b}\|}$$

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$$\operatorname{sgn}(\boldsymbol{\tau_b}) = \frac{\boldsymbol{\tau_b}}{\|\boldsymbol{\tau_b}\|}$$

$$\frac{\partial \mathbf{q_b}}{\partial h} = -k\frac{q^x}{h}\frac{\partial \mathbf{q_b}}{\partial q^x} - k\frac{q^y}{h}\frac{\partial \mathbf{q_b}}{\partial q^y}$$

Outline

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Numerical Scheme

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{\Delta x}\left(\mathcal{D}^+_{i-1/2}\left(W_{i-1}^n, W_i^n, H_{i-1}, H_i\right) + \mathcal{D}^-_{i+1/2}\left(W_i^n, W_{i+1}^n, H_i, H_{i+1}\right)\right)$$

where

$$\mathcal{D}^{\pm}_{i+1/2}\left(W_i^n, W_{i+1}^n, H_i, H_{i+1}\right) = \frac{1}{2}\Big[\mathcal{F}\left(W_{i+1}^n\right) - \mathcal{F}\left(W_i^n\right) + B_{i+1/2}\left(W_{i+1}^n - W_i^n\right)$$
$$- S_{i+1/2}\left(H_{i+1} - H_i\right) \pm Q_{i+1/2}\left(W_{i+1}^n - W_i^n - \left(A_{i+1/2}\right)^{-1} S_{i+1/2}\left(H_{i+1} - H_i\right)\right)\Big].$$

being

$$\mathcal{F}(W) = \begin{pmatrix} q \\ \dfrac{q^2}{h} + \dfrac{g}{2}h^2 \\ \xi\, q_b \end{pmatrix}, \quad B_{i+1/2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & gh_{i+1/2} \\ 0 & 0 & 0 \end{pmatrix}, \quad S_{i+1/2} = \begin{pmatrix} 0 \\ gh_{i+1/2} \\ 0 \end{pmatrix},$$

Numerical Scheme

$$A_{i+1/2} = \begin{pmatrix} 0 & 1 & 0 \\ gh_{i+1/2} - u_{i+1/2}^2 & 2u_{i+1/2} & gh_{i+1/2} \\ \left(\dfrac{\partial q_b}{\partial h}\right)_{i+1/2} & \left(\dfrac{\partial q_b}{\partial q}\right)_{i+1/2} & 0 \end{pmatrix}.$$

- The Polynomial Viscosity Matrix $Q_{i+1/2}$ is defined using the PVM-IFCP method [Fernández-Nieto 2011]:

$$Q_{i+1/2} = \alpha_0 Id + \alpha_1 A_{i+1/2} + \alpha_2 \left(A_{i+1/2}\right)^2$$

  where α_0, α_1 and α_2 are defined from the eigenvalues of $A_{i+1/2}$.
- Finally, $S_f(W)$ is discretized in a semi-implicit way.

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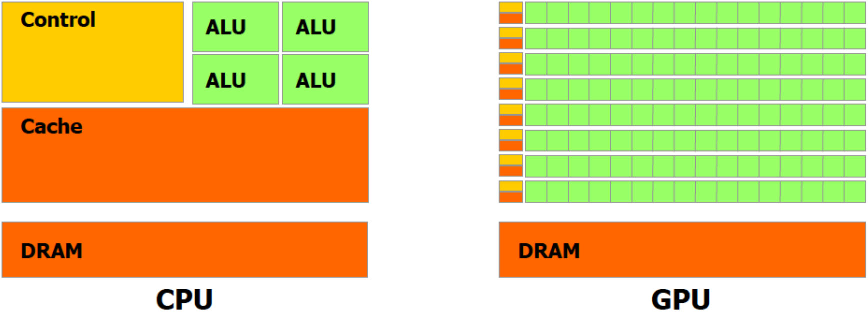
Second order numerical scheme

- TVD Runge-Kutta time discretization.
- MUSCL-type reconstruction operator for triangular meshes.

Outline

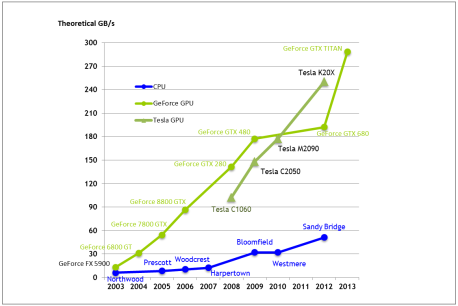
What is a GPU?

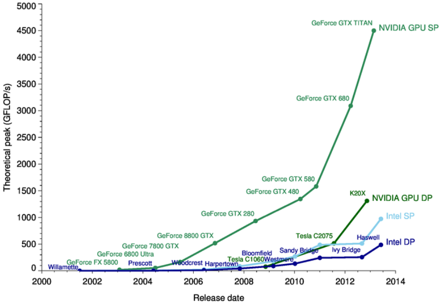
CPU vs GPU architectures



- GPUs are specialized circuits designed to rapidly manipulate the building of images in a a frame buffer intended for output to a display.
- GPUs are specialized for compute-intensive highly parallel computations.
- GPU are especially well-suited to address problems that present a high data-parallel paradigm.
- The same program is executed for each data element in GPUs, so the flow control of the code is simple.

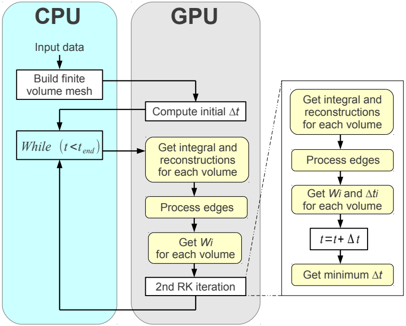
Why GPUs?





- Also price!!!

CUDA Implementation



- Each GPU step is assigned to a CUDA kernel.

- Full code in double precision.
- Edge data (normals and volume indexes) in global memory.
- Volume data in 1D textures.
- Stencil data (volume indexes) in 1D texture.
- In global memory: an auxiliary state, the barycenters of the volumes, 12 vectors for the reconstruction coefficients, and all the reconstructed values.
- The creation of the stencils and the computation of the reconstruction coefficients is performed at the beginning.
- 6 accumulators in global memory to write the edge contributions.

Outline

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Conical Dune

- Test proposed in [Hudson 2001].
- Grass model.
- Mesh size: 64000 triangles.
- Simulation time: 100 hours.
- Incoming $u_x = 1$ m/s
- CFL $= 0.7$
- $\rho_0 = 0.4$
- $A_g = 10^{-3}$
- Spread angle of 22.3° using a mesh of 13755 triangles (asymptotic $\approx 21.79°$)

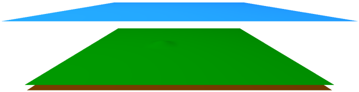
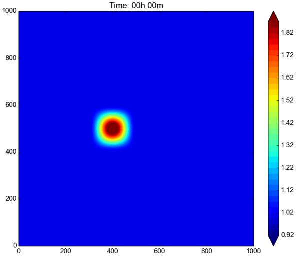


Figure: Initial state



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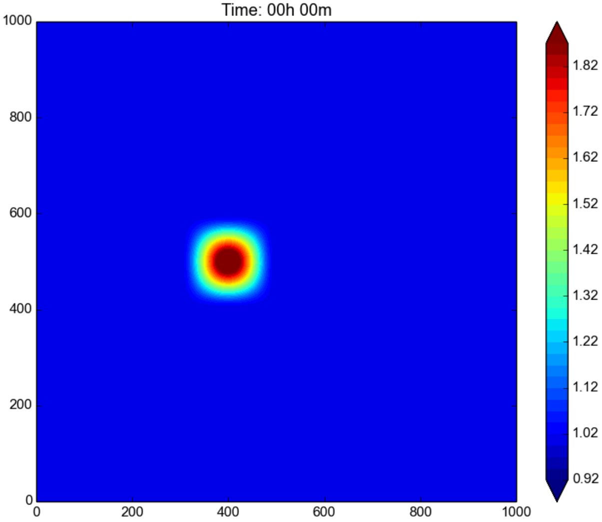

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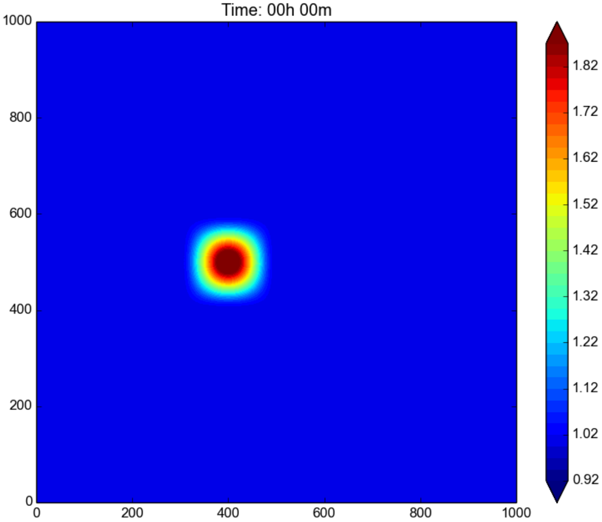


Analysis of Efficiency

- GeForce GTX Titan Black.
- 64000 triangles, 100 hours.

Order	Iterations	Runtime	MVols/s
1st	5755088	2h 18m 39s	44.3
2nd	5754913	5h 19m 08s	19.2

Order	Most costly kernel
1st	Edge processing (**90** % of runtime)
2nd	Edge processing (**78** % of runtime)

Outline

L-shaped channel

- MPM model.
- Mesh size: 128002 triangles.
- Simulation time: 1 hour.
- Incoming $u_y = -0.5$ m/s
- CFL $= 0.9$
- $\rho_0 = 0.4$
- $\rho_{\rm w} = 1000$ kg/m^3
- $\rho_{\rm s} = 2600$ kg/m^3
- $d_{\rm s} = 10^{-3}$ m

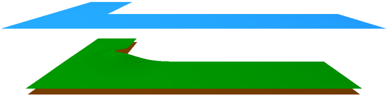
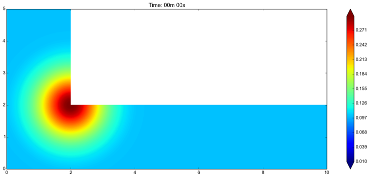


Figure: Initial state



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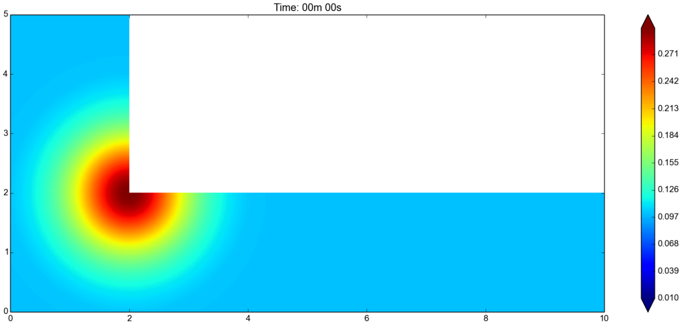

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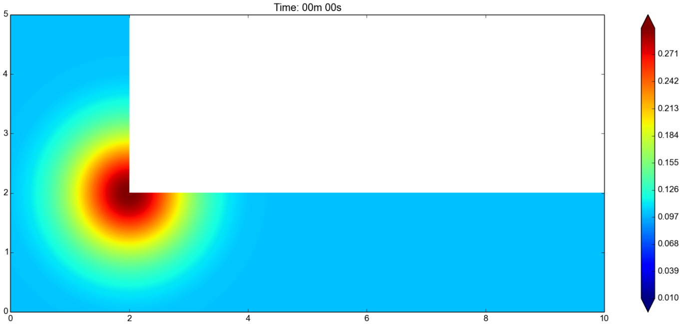


Analysis of Efficiency

- GeForce GTX Titan Black.
- 128002 triangles, 1 hour.

Order	Iterations	Runtime	MVols/s
1st	4521681	7h 03m 36s	22.8
2nd	4612758	15h 49m 17s	10.4

Order	Most costly kernel
1st	Edge processing (**95** % of runtime)
2nd	Edge processing (**87** % of runtime)

Outline

Scour Around a Spur Dike

- Laboratory experiment [Mioduszewski 2003].
- MPM model.
- Mesh size: 31016 triangles.
- Simulation time: 40 seconds
- Incoming $q_x = 0.005$ m^3/s
- CFL $= 0.5$
- $\rho_0 = 0.4$
- $\rho_s/\rho_w = 1.4$
- $d_s = 1.28$ mm
- Preliminary results adjust well with laboratory results.

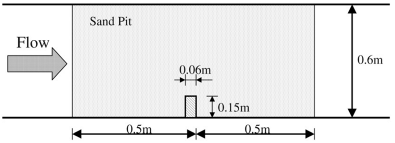
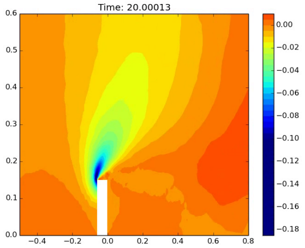


Figure Source: [Liu 2008]



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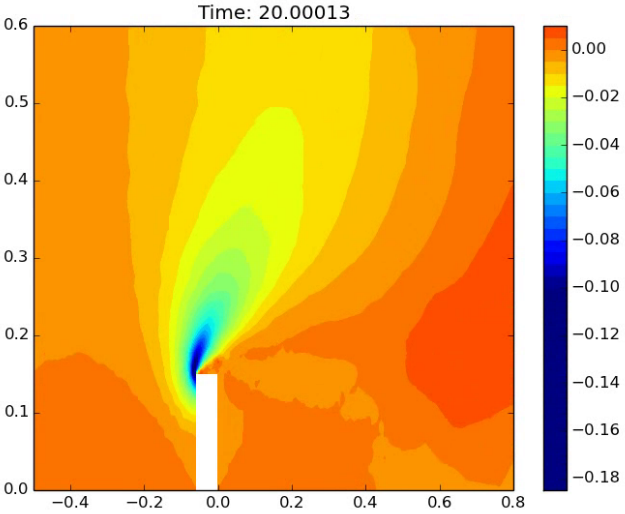

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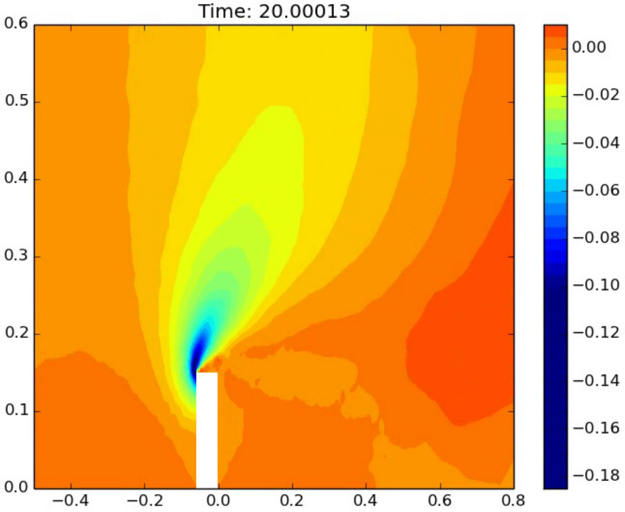


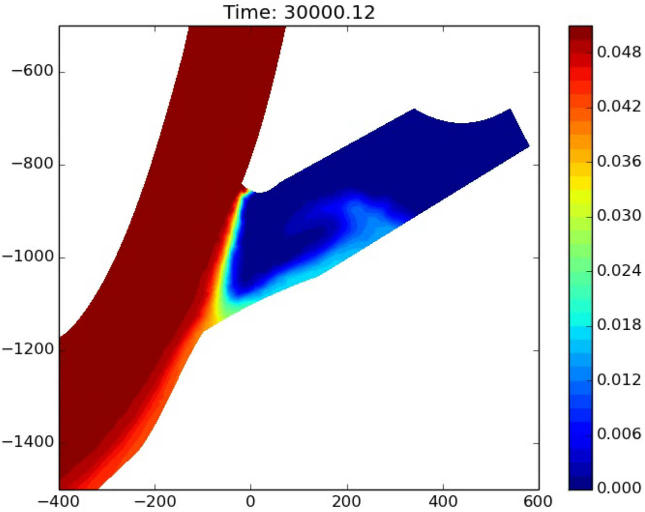
Outline

Conclusions

- First and second order GPU implementations of the Grass and MPM bedload sediment transport models using triangular meshes and double numerical precision.
- The numerical scheme is robust and preliminary results adjust well with the angle of evolution of a dune and some laboratory experiments.

Future Work

- Work with real domains and more complex models.
- Hyperpycnal model example (Guadalquivir river):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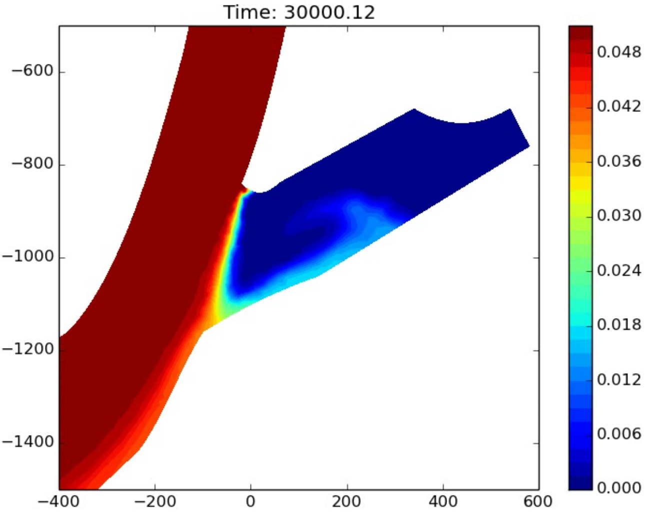
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Thank you for your attention

Tomas.Morales@uco.es