P.R.A.G.MAT.I.C. 2010

Mori theory in a variety of flavors

Scientific activity. Thirty years ago Shigefumi Mori published two ground breaking papers giving birth to a theory now bearing his name. In the first of these papers, devoted to a proof of the Frankel-Hartshorne conjecture, Mori proved that Fano manifolds are covered by rational curves. In the second paper, classical facts about the birational geometry of surfaces were extended to threefolds whose canonical divisor is not nef. To this end, Mori used rational curves to describe morphisms of such varieties and related them to the cone of effective one cycles.

These concepts were subsequently used in the Minimal Model Program (MMP) aimed at birational classification of complex projective varieties. Apart from its vital contribution to the success of MMP, Mori theory had enormous impact on biregular geometry and it changed geometers' view on many classical problems. The present course will be devoted to this latter field of applications of Mori theory. The aim of the course is to get the participants acquinted with basic tools of Mori theory and their applications in dealing with problems related to biregular geometry of projective manifolds.

Basics. Kleiman-Mori cone and contractions of its faces, adjunction, Fano manifolds, Mori's proof of the Hartshorne-Frankel conjecture, existence of rational curves and their deformations, schemes parametrizing rational curves, uniruled and rationally connected manifolds.

Topics. Mori theory for Fano manifolds of high index and manifolds whose tangent bundle is nef. Mori theory for embedded projective manifolds: manifolds covered by lines, defective manifolds and special cases of the Hartshorne Conjecture. Comparing the Kleiman-Mori cones for hyperplane sections. Dominating morphisms of Fano varieties, Remmert-Van de Ven problem. Mori Dream Spaces, cones of nef and movable divisors.