Problem List PRAGMATIC 2006

Universitá di Catania, Catania, 4 September/22 September 2006

Problems

(1) Reproduce Gordan-Noether proof of Hesse's Claim for $\mathbb{P}^3$ and their classification of hypersurfaces with vanishing hessian in $\mathbb{P}^4$ (study the relations with the work of Franchetta and Permutti). Classify all (irreducible or eventually only reduced) homaloidal surfaces in $\mathbb{P}^3$. Try to find some structure theorem for irreducible homaloidal hypersurfaces in $\mathbb{P}^r$, $r \geq 3$, see [Do], [CRS] and related references like [He1], [He2], [GN], [Fr1], [Fr2], etc. See [Lo] for a translation of the beginning of [GN].

(2) Classify special Cremona transformations of type $(2, 4)$ (or of type $(2, 2m)$, $m \geq 2$; or of type $(2, d)$, $d \geq 7$ odd; or of type $(3, d)$, $d \geq 2$), references: [ESB], [MR], [Ru1], [IR],[PR], [ST2], [ST3], [ST1], [HKS].

(3) Classification of smooth algebraic projective varieties with two (or more) unsplit families of rational curves of quasi-maximal dimension, references: [IR], [Oc], [OW].

(4) Generalizations and simpler proofs of the main results of [KS] via the techniques and ideas of [Ru1] and [IR];

(5) Let $X \subset \mathbb{P}^N$ be a smooth algebraic variety. Is it true that $\delta(X) = 2\dim(X) + 1 - \dim(SX) \geq 9$ implies $SX = \mathbb{P}^N$? (Lazarsfeld-Van de Ven, [LV]). Is it true for $LQEL$-manifolds? Is it true that if $SX \subsetneq \mathbb{P}^N$, then $\delta(X) \leq m$ with m not depending on $\dim(X) = n$? (Recall that $\delta(X) > \frac{n}{2}$ implies $SX = \mathbb{P}^N$, Zak's Linear Normality Theorem).

(6) Classification of $LQEL$-manifolds of type $\delta = \frac{n}{2} + 1$ and of type $\delta = \frac{n+1}{2}$ and their relations with smooth varieties whose dual has dimension $\dim(X^*) \leq \dim(X) + 2$. References: [IR], [Ru1], [Ei], [Mu1], [Mu2].

(7) Study of the tangential behaviour of algebraic varieties with applications to the Linear Normality Bound. Classification of the extremal cases. References: [Ru2].

(8) Classification of smooth threefolds $X \subset \mathbb{P}^{4k+3}$ such that through the general point of $\mathbb{P}^{4k+3}$ there passes a unique $\mathbb{P}^k$ which is $(k+1)$-secant to X. Eventually construction of examples. References: [CMR], [CR]

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