Section 1.3: A Simple Inventory System

Discrete-Event Simulation: A First Course

 0-13-142917-5

Section 1.3: A Simple Inventory System

- Distributes items from current inventory to customers
- Customer demand is discrete
- Simple $\Longleftrightarrow$ one type of item

Inventory Policy

- *Transaction Reporting*
  - Inventory review after *each* transaction
  - Significant labor may be required
  - Less likely to experience shortage
- *Periodic Inventory Review*
  - Inventory review is periodic
  - Items are ordered, if necessary, only at review times
  - (s, S) are the min,max inventory levels, $0 \leq s < S$
- We assume periodic inventory review
- Search for (s, S) that minimize cost

Conceptual Model

Inventory System Costs

• *Holding cost:*	*for items in inventory*
• *Shortage cost:*	*for unmet demand*
• *Setup cost:*	*fixed cost when order is placed*
• *Item cost:*	*per-item order cost*
• *Ordering cost:*	*sum of setup and item costs*

Additional Assumptions

- *Back ordering is possible*
- *No delivery lag*
- *Initial inventory level is* S
- *Terminal inventory level is* S

Specification Model

- Time begins at $t = 0$
- Review times are $t = 0, 1, 2, \ldots$
- l_{i-1} : inventory level at *beginning* of i^{th} interval
- o_{i-1} : amount ordered at time $t = i - 1$, $(o_{i-1} \geq 0)$
- d_i : demand quantity *during* i^{th} interval, $(d_i \geq 0)$
- Inventory at end of interval can be negative

Inventory Level Considerations

- Inventory level is reviewed at $t = i - 1$
- If at least s, no order is placed

  If less than s, inventory is replenished to S

$$o_{i-1} = \begin{cases} 0 & l_{i-1} \geq s \\ S - l_{i-1} & l_{i-1} < s \end{cases}$$

- Items are delivered immediately
- At end of i^{th} interval, inventory diminished by d_i

$$l_i = l_{i-1} + o_{i-1} - d_i$$

Time Evolution of Inventory Level

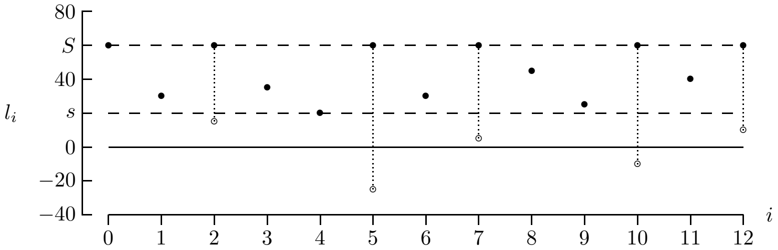
Algorithm 1.3.1

```
l_0 = S;      /* the initial inventory level is S */
i = 0;
while (more demand to process ) {
    i++;
    if (l_{i-1} < s)
        o_{i-1} = S - l_{i-1};
    else
        o_{i-1} = 0;
    d_i = GetDemand();
    l_i = l_{i-1} + o_{i-1} - d_i;
}
n = i;
o_n = S - l_n;
l_n = S;      /* the terminal inventory level is S */
return l_1, l_2, ..., l_n and o_1, o_2, ..., o_n;
```

Example 1.3.1: SIS with Sample Demands

Let $(s, S) = (20, 60)$ and consider $n = 12$ time intervals

i	:	1	2	3	4	5	6	7	8	9	10	11	12
d_i	:	30	15	25	15	45	30	25	15	20	35	20	30



Output Statistics

- What statistics to compute?
- *Average demand* and *average order*

$$\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i \qquad \bar{o} = \frac{1}{n} \sum_{i=1}^{n} o_i.$$

- For Example 1.3.1 data

$$\bar{d} = \bar{o} = 305/12 \simeq 25.42 \text{ items per time interval}$$

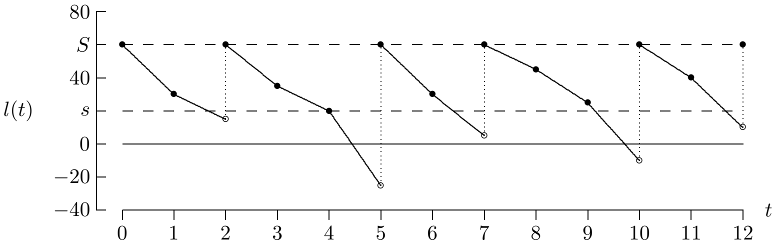
Flow Balance

- Average demand and order *must* be equal
- Ending inventory level is S
- Over the simulated period, all demand is satisfied
- Average "flow" of items in equals average "flow" of items out

- The inventory system is *flow balanced*

Constant Demand Rate

- Holding and shortage costs are proportional to time-averaged inventory levels
- Must know inventory level for all t
- *Assume* the demand rate is constant between review times

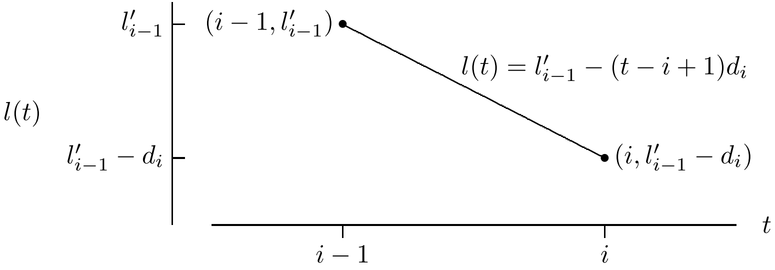


Inventory Level as a Function of Time

- The inventory level at any time t in i^{th} interval is

$$l(t) = l'_{i-1} - (t - i + 1)d_i$$

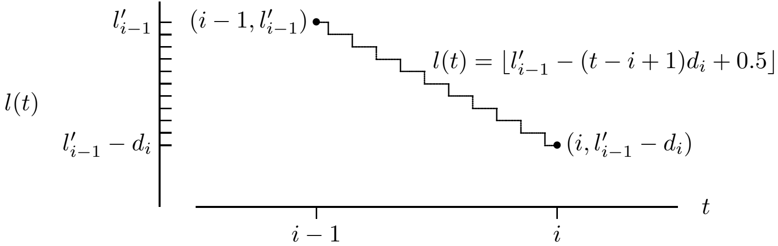
if demand rate is constant between review times



- $l'_{i-1} = l_{i-1} + o_{i-1}$ represents inventory level *after* review

Inventory Decrease Is Not Linear

- Inventory level at any time t is an integer
- $l(t)$ should be rounded to an integer value
- $l(t)$ is a stair-step, rather than linear, function



Time-Averaged Inventory Level

- $l(t)$ is the basis for computing the time-averaged inventory level
- Case 1: If $l(t)$ remains non-negative over i^{th} interval

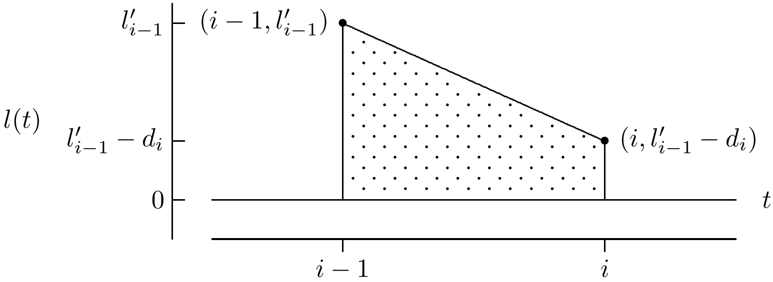
$$\bar{l}_i^+ = \int_{i-1}^{i} l(t)dt$$

- Case 2: If $l(t)$ becomes negative at some time τ

$$\bar{l}_i^+ = \int_{i-1}^{\tau} l(t)dt \qquad \bar{l}_i^- = -\int_{\tau}^{i} l(t)dt$$

- $\bar{l}_i^+$ is the time-averaged *holding level*
  $\bar{l}_i^-$ is the time-averaged *shortage level*

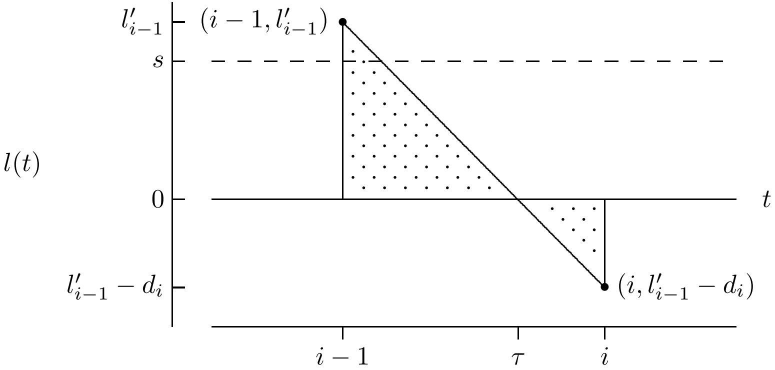
- No shortage during i^{th} time interval iff. $d_i \leq l'_{i-1}$



- Time-averaged holding level: area of a trapezoid

$$\bar{l}_i^+ = \int_{i-1}^{i} l(t)\,dt = \frac{l'_{i-1} + (l'_{i-1} - d_i)}{2} = l'_{i-1} - \frac{1}{2} d_i$$

- Inventory becomes negative iff. $d_i > l'_{i-1}$



Case 2: Back Ordering (Cont.)

- $l(t)$ becomes negative at time $t = \tau = i - 1 + (l'_{i-1}/d_i)$
- Time-averaged holding and shortage levels for i^{th} interval computed as the areas of triangles

$$\bar{l}_i^{+} = \int_{i-1}^{\tau} l(t)\, dt = \cdots = \frac{(l'_{i-1})^2}{2d_i}$$

$$\bar{l}_i^{-} = -\int_{\tau}^{i} l(t)\, dt = \cdots = \frac{(d_i - l'_{i-1})^2}{2d_i}$$

Time-Averaged Inventory Level

- *Time-averaged holding level* and *time-averaged shortage level*

$$\bar{l}^+ = \frac{1}{n}\sum_{i=1}^{n} \bar{l}_i^+ \qquad\qquad \bar{l}^- = \frac{1}{n}\sum_{i=1}^{n} \bar{l}_i^-$$

- Note that time-averaged shortage level is positive
- The *time-averaged inventory level* is

$$\bar{l} = \frac{1}{n}\int_0^n l(t)\,dt = \bar{l}^+ - \bar{l}^-$$

Computational Model

- `sis1` is a trace-driven computational model of the SIS
- Computes the statistics

$$\bar{d},\ \bar{o},\ \bar{l}^{+},\ \bar{l}^{-}$$

  and the order frequency $\bar{u}$

$$\bar{u} = \frac{\text{number of orders}}{n}$$

- Consistency check: compute $\bar{o}$ and $\bar{d}$ separately, then compare

Example 1.3.4: Executing sis1

- Trace file sis1.dat contains data for $n = 100$ time intervals
- With $(s, S) = (20, 80)$

$$\bar{o} = \bar{d} = 29.29 \qquad \bar{u} = 0.39 \qquad \bar{l}^{+} = 42.40 \qquad \bar{l}^{-} = 0.25$$

- After Chapter 2, we will generate data randomly (no trace file)

Operating Costs

A facility's cost of operation is determined by:

- c_{item} : *unit cost of new item*
- c_{setup} : *fixed cost for placing an order*
- c_{hold} : *cost to hold one item for one time interval*
- c_{short} : *cost of being short one item for one time interval*

Case Study

- Automobile dealership that uses weekly periodic inventory review
- The facility is the showroom and surrounding areas
- The items are new cars
- The supplier is the car manufacturer
- "...customers are people convinced by clever advertising that their lives will be improved significantly if they purchase a new car from this dealer." (S. Park)
- Simple (one type of car) inventory system

Example 1.3.5: Case Study Materialized

- Limited to a maximum of $S = 80$ cars
- Inventory reviewed every Monday
- If inventory falls below $s = 20$, order cars sufficient to restore to S
- For now, ignore delivery lag
- Costs:

• Item cost is	c_{item}	=	\$8000	per item
• Setup cost is	c_{setup}	=	\$1000	
• Holding cost is	c_{hold}	=	\$25	per week
• Shortage cost is	c_{hold}	=	\$700	per week

Per-Interval Average Operating Costs

- The average operating costs *per time interval* are

  - *item cost :* $c_{\text{item}} \cdot \bar{o}$
  - *setup cost :* $c_{\text{setup}} \cdot \bar{u}$
  - *holding cost :* $c_{\text{hold}} \cdot \bar{l}^{+}$
  - *shortage cost :* $c_{\text{short}} \cdot \bar{l}^{-}$

- The average *total* operating cost *per time interval* is their sum
- For the stats and costs of the hypothetical dealership:

  - *item cost :* $\$8000 \cdot 29.29 = \$234,320$ *per week*
  - *setup cost :* $\$1000 \cdot 0.39 = \$390$ *per week*
  - *holding cost :* $\$25 \cdot 42.40 = \$1,060$ *per week*
  - *shortage cost :* $\$700 \cdot 0.25 = \$175$ *per week*

Cost Minimization

- By varying s (and possibly S), an optimal policy can be determined
- Optimal $\iff$ minimum average cost
- Note that $\bar{o} = \bar{d}$, and $\bar{d}$ depends only on the demands
- Hence, item cost is independent of (s, S)
- Average *dependent* cost is

$$\text{avg setup cost} + \text{avg holding cost} + \text{avg shortage cost}$$

Experimentation

- Let S be fixed, and let the demand sequence be fixed
- If s is systematically increased, we expect:
  - average setup cost and holding cost will increase as s increases
  - average shortage cost will decrease as s increases
  - average dependent cost will have 'U' shape, yielding an optimum
- From results (next slide), minimum cost is \$1550 at $s = 22$

Example 1.3.7: Simulation Results