Basic Concept of Digital Images

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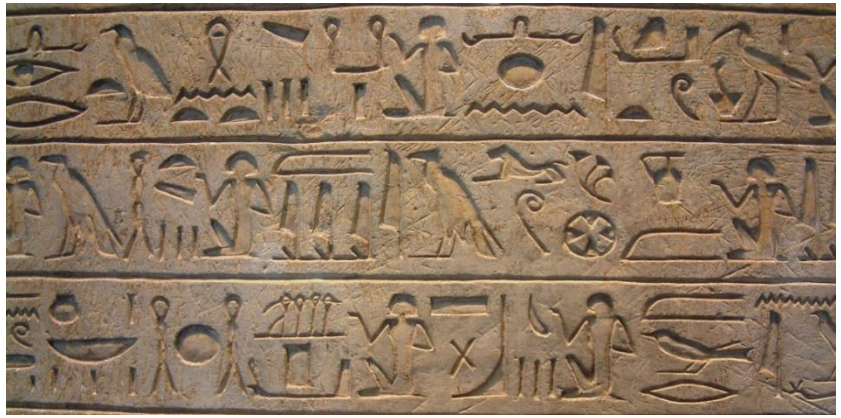
The images …

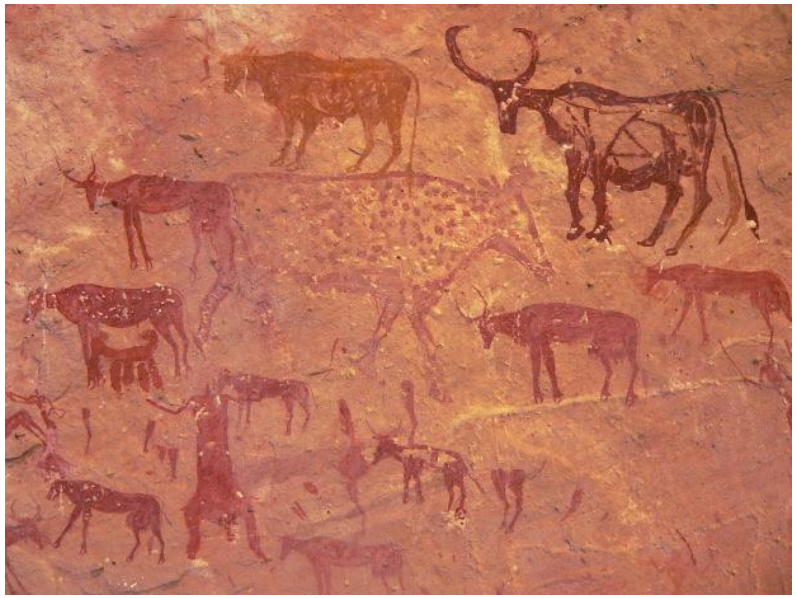
- We are surrounded by images…
- The eye interprets the image by making **color**, **motion**, and **depth** become real additional dimensions to the initial information.
- Digital images are sampled to be represented by a finite number of samples...

Cultural role of images

“A picture is worth a thousand words...”

“The language of images has belonged to man since his origins, written language has only belonged to him for a few millennia”





Visual communication is the most immediate and effective form of communication

Cultural role of images

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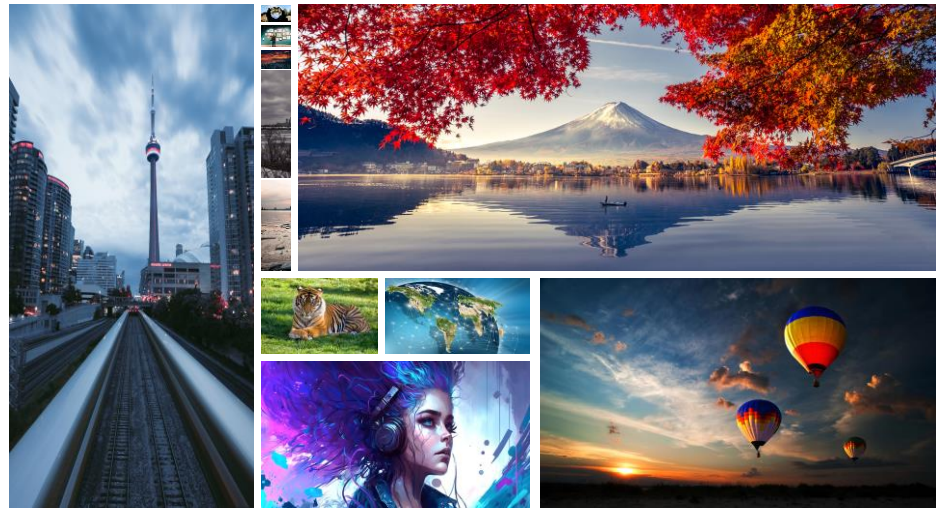


«... Except it takes up more space! »

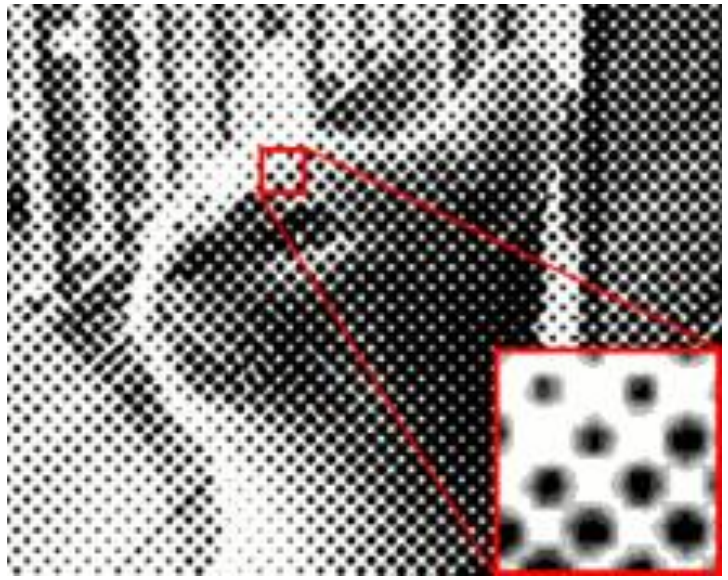
First non-digital photography

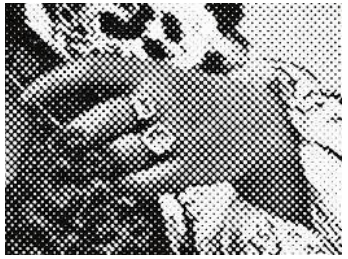
- Here is the first photographic document in history (1827).
- The subject is occasional: a roof visible from the window of the author, Joseph Nicéphore Niépce (1765-1833). It was titled «View from a window of the Gras at Saint-Loup-de-Varenne» (or more briefly «View from the window at Le Gras»).
- The heliographic plate he prepared was "exposed" for 8 hours.

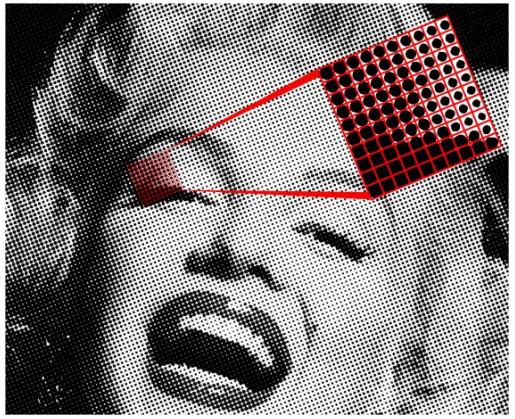
https://en.wikipedia.org/wiki/Heliography

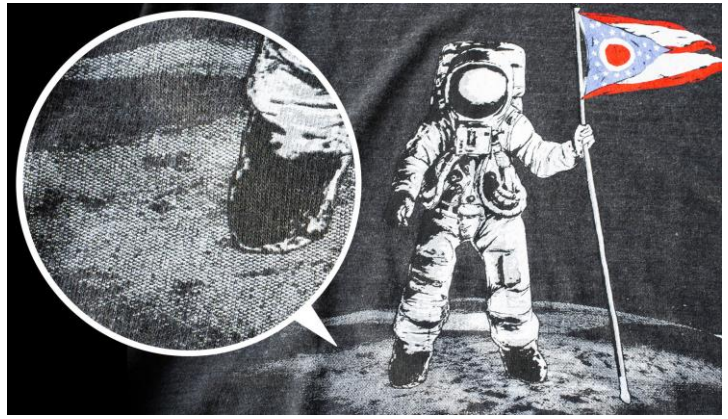
A little history of digital images

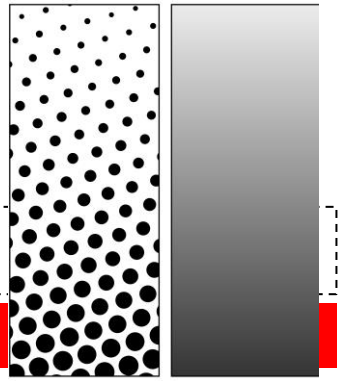
- The first application of digital images occurs in newspaper prints.
- In 1920 an image was transmitted via cable between New York and London in order to appear in a newspaper.
- The transmission protocol is specific to the image and the result is **printed in halftoning** by special printers.











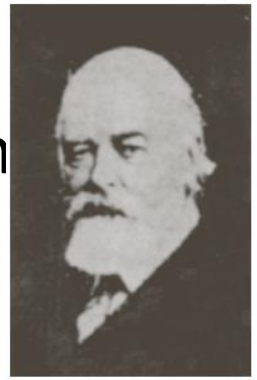
Left: Halftone dots. **Right**: Example of how the human eye would see the dots from a sufficient distance.

https://en.wikipedia.org/wiki/Halftone

A little history of digital images

- Halftoning printing has been used for many years.

- In 1922 the type of printing changed an up to 5 levels of gray could be obtained.



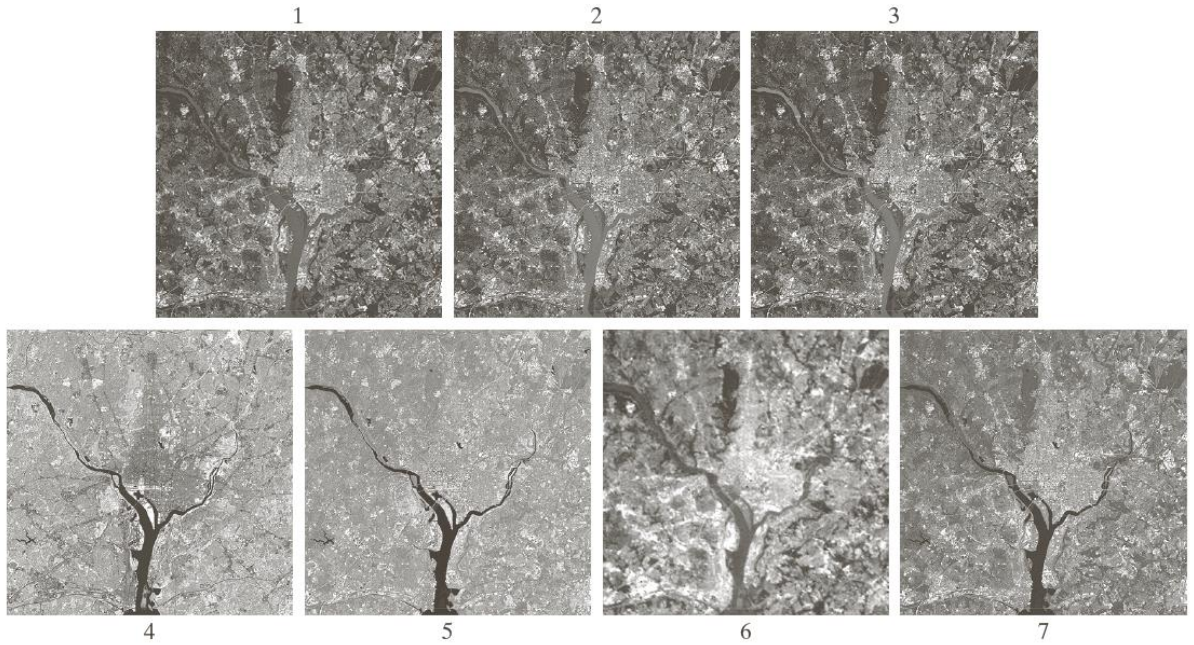
- In 1929 the levels of gray became 15

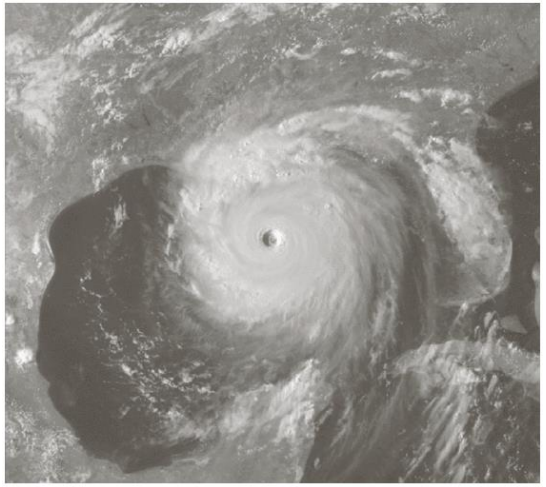
The first processing of a digital image

- The first time an image was processed by computer was in 1964, when a NASA computer received and processed an image of the moon and corrected some optical distortions.

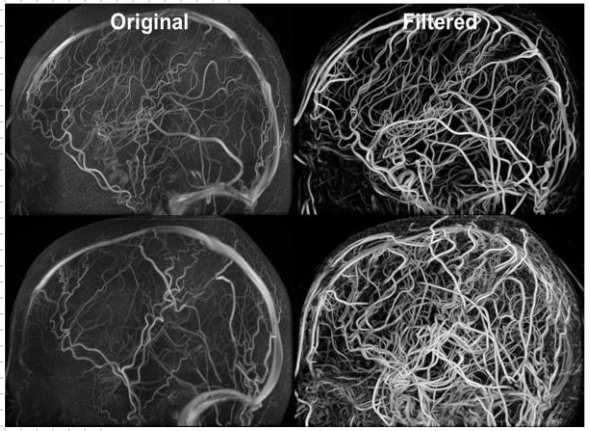
What is the purpose of processing a digital image?

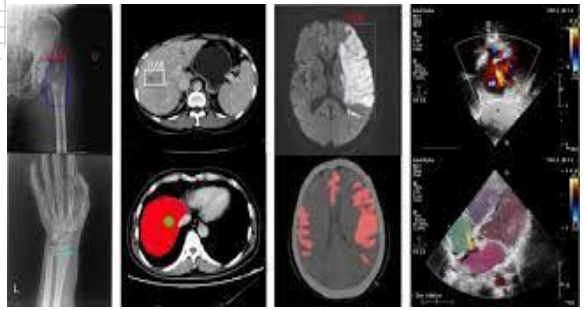
From the satellite

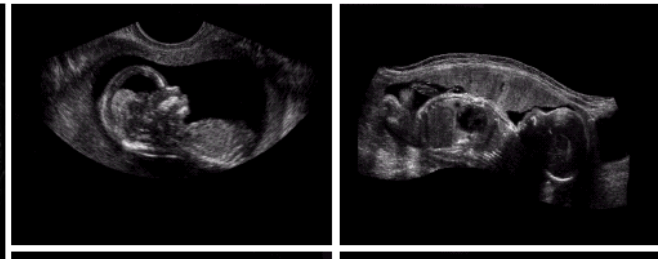


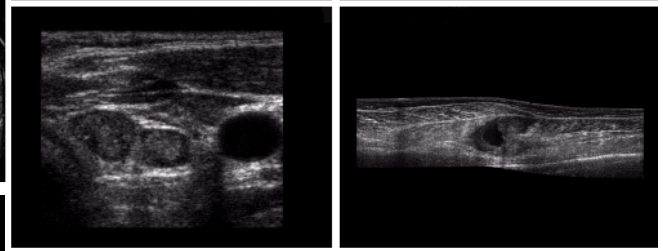


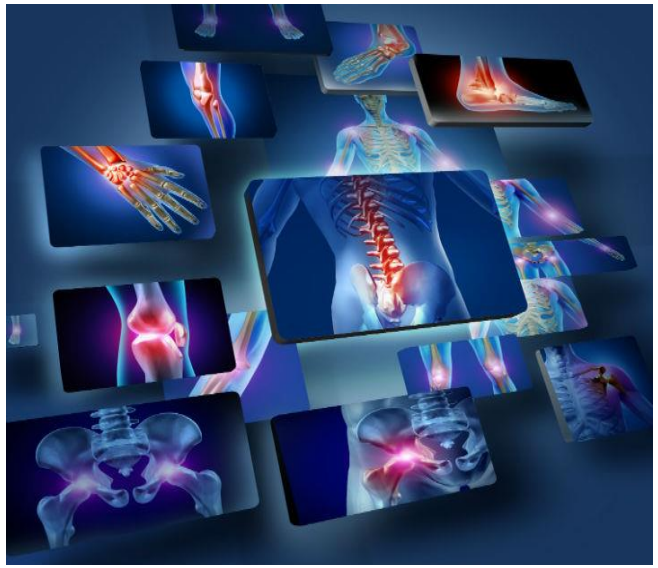
Medical

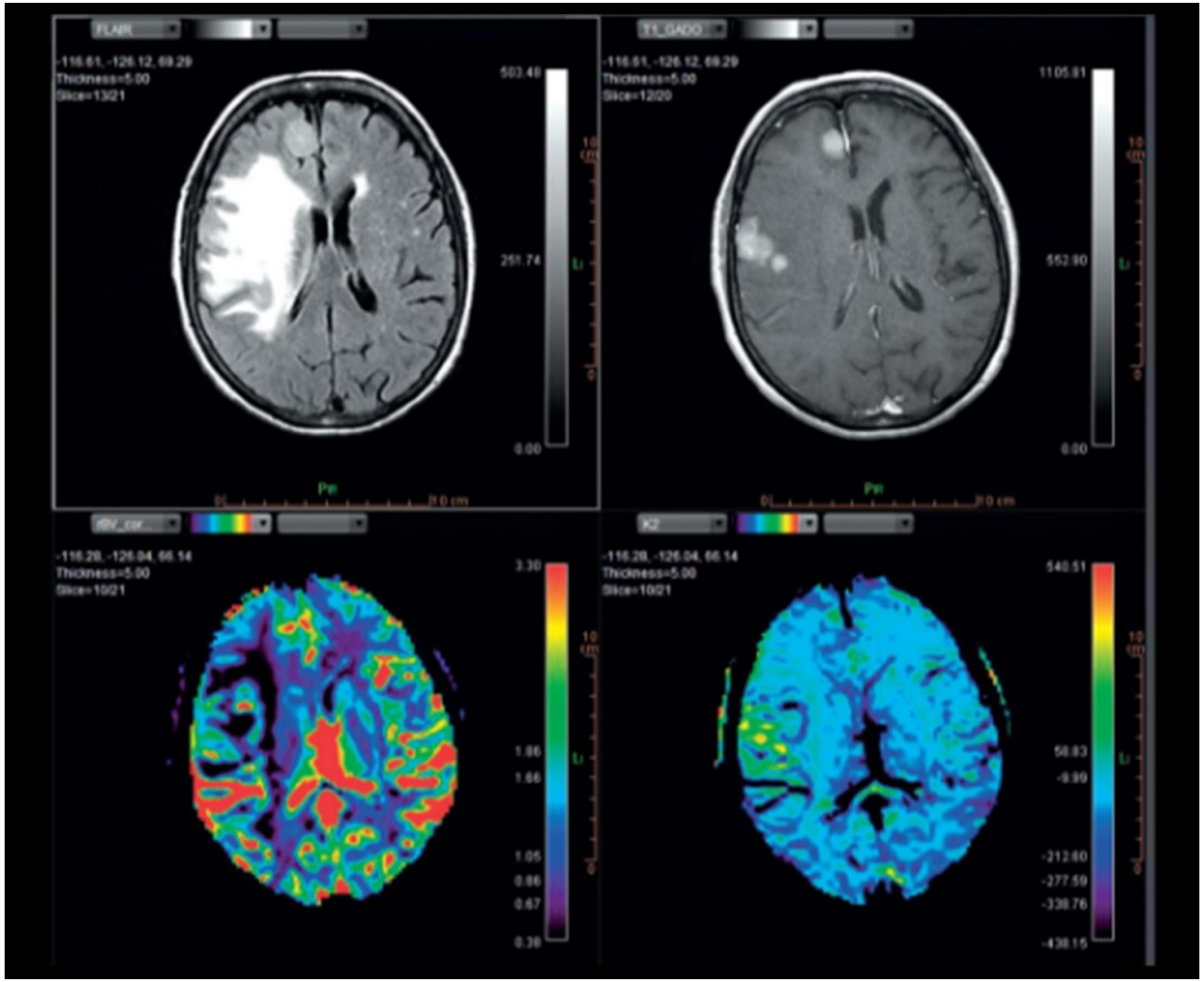




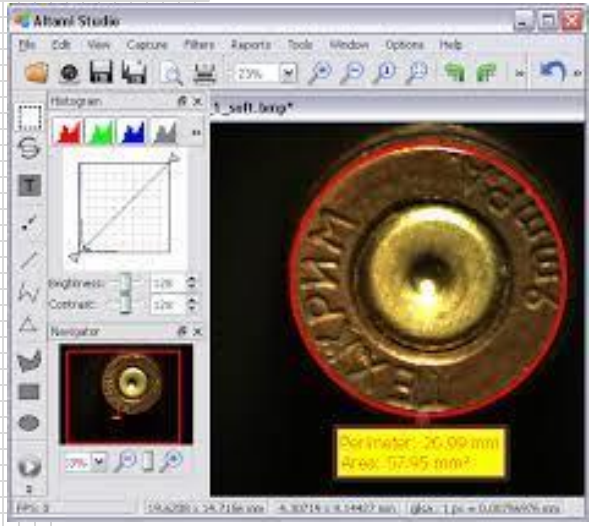








Forensics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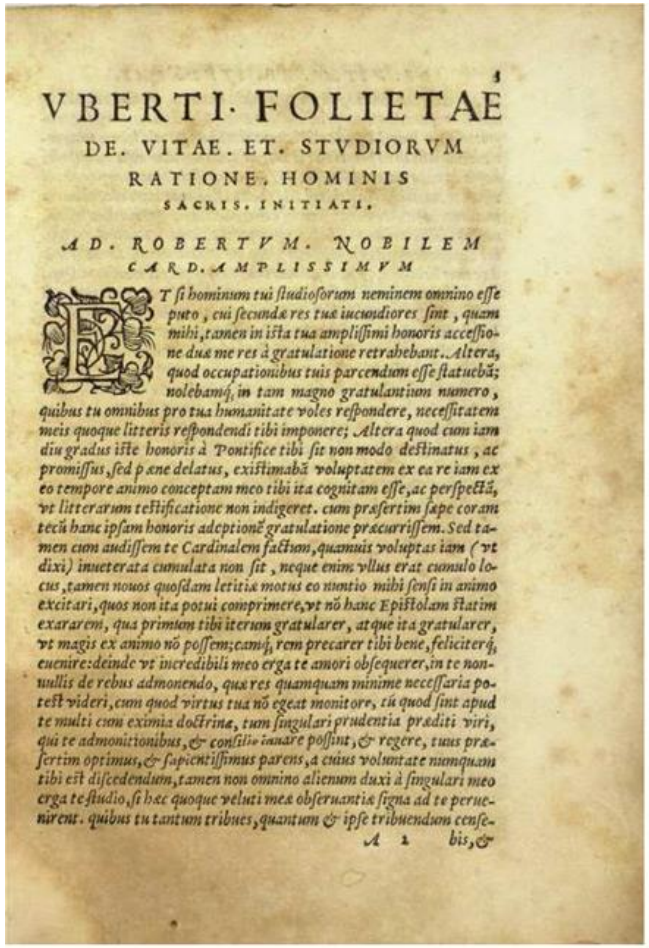
3

For cultural heritage



VBERTI· FOLIETAE

DE. VITAE. ET. STVDIORVM

RATIONE. HOMINIS

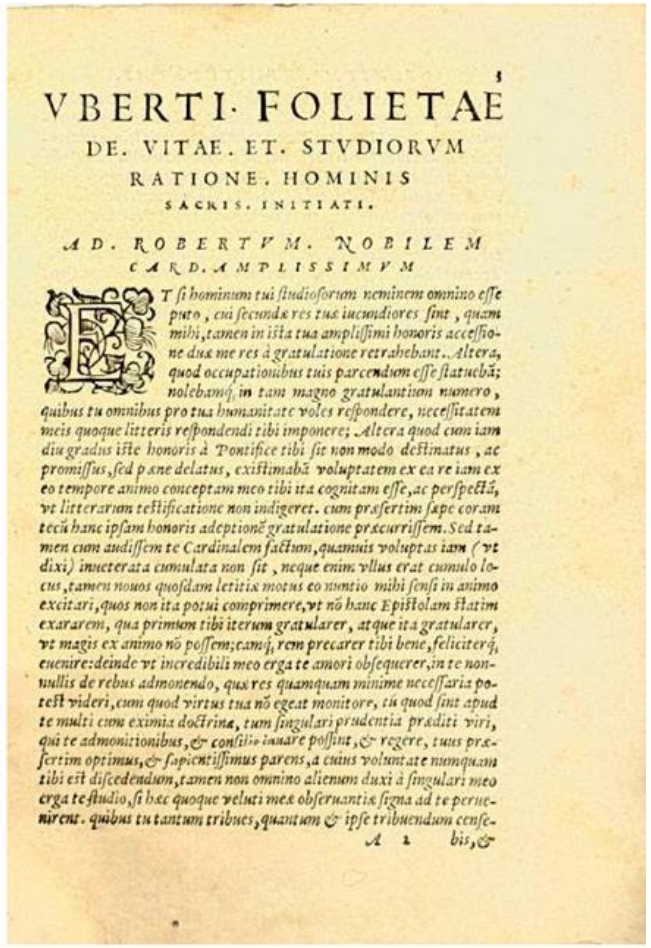
SACRIS. INITIATI.

AD. ROBERTVM. NOBILEM

CARD. AMPLISSIMVM

ET si hominum tui studiosorum neminem omnino esse puto, cui secundæ res tuæ iucundiores sint, quam mihi, tamen in ista tua amplissimi honoris accessione duæ me res à gratulatione retrahebant. Altera, quod occupationibus tuis parcendum esse statuebã; nolebamq; in tam magno gratulantium numero, quibus tu omnibus pro tua humanitate voles respondere, necessitatem meis quoque litteris respondendi tibi imponere; Altera quod cum iam diu gradus iste honoris à Pontifice tibi sit non modo destinatus, ac promissus, sed pene delatus, existimabã voluptatem ex ea re iam ex eo tempore animo conceptam meo tibi ita cognitam esse, ac perspectã, vt litterarum testificatione non indigeret. cum præsertim sæpe coram tecũ hanc ipsam honoris adeptionẽ gratulatione præcurrissem. Sed tamen cum audissem te Cardinalem factum, quamuis voluptas iam (vt dixi) inueterata cumulata non sit, neque enim vllus erat cumulo locus, tamen nouos quosdam letitiæ motus eo nuntio mihi sensi in animo excitari, quos non ita potui comprimere, vt nõ hanc Epistolam statim exararem, qua primum tibi iterum gratularer, atque ita gratularer, vt magis ex animo nõ possem; camq; rem precarer tibi bene, feliciterq; euenire: deinde vt incredibili meo erga te amori obsequerer, in te nonnullis de rebus admonendo, quæ res quamquam minime necessaria potest videri, cum quod virtus tua nõ egeat monitore, tũ quod sint apud te multi cum eximia doctrina, tum singulari prudentia præditi viri, qui te admonitionibus, & consilio iuuare possint, & regere, tuus præsertim optimus, & sapientissimus parens, a cuius voluntate numquam tibi est discedendum, tamen non omnino alienum duxi à singulari meo erga te studio, si hæc quoque veluti meæ obseruantiæ signa ad te perue-

A 2 bis, &



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Before we move on: Lena or Lenna (www.lenna.org)

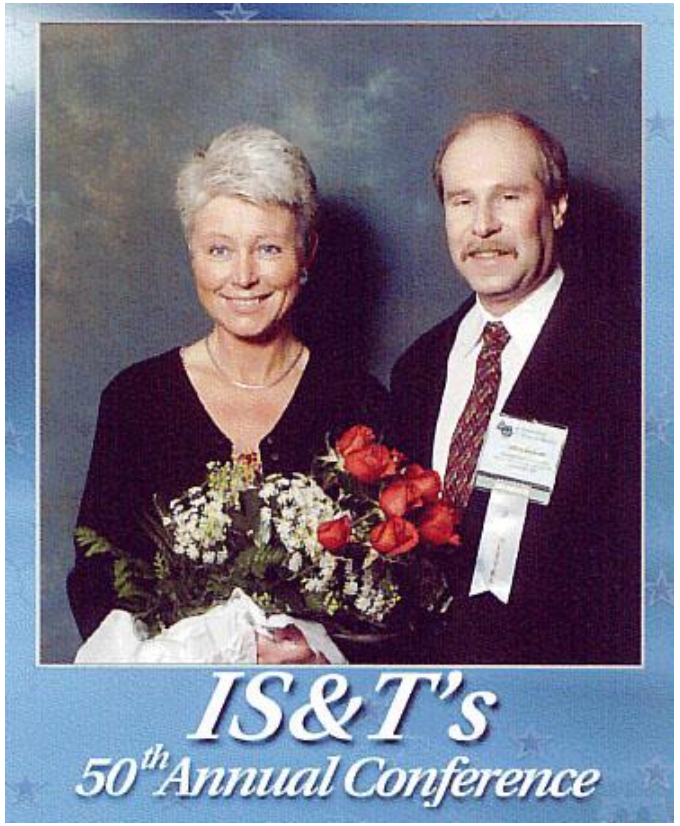


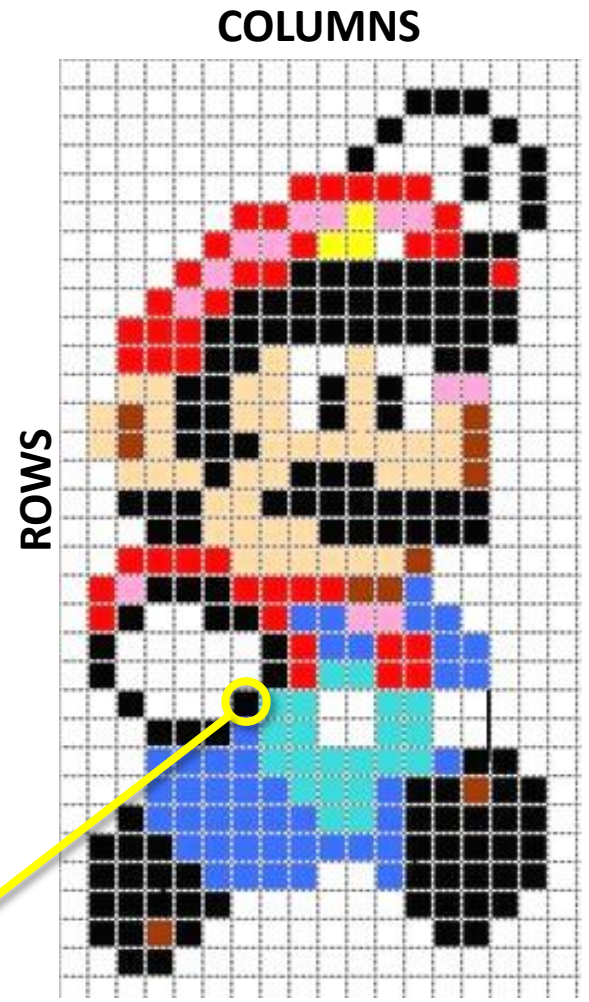
Image representation

What is a digital image?

Definizione

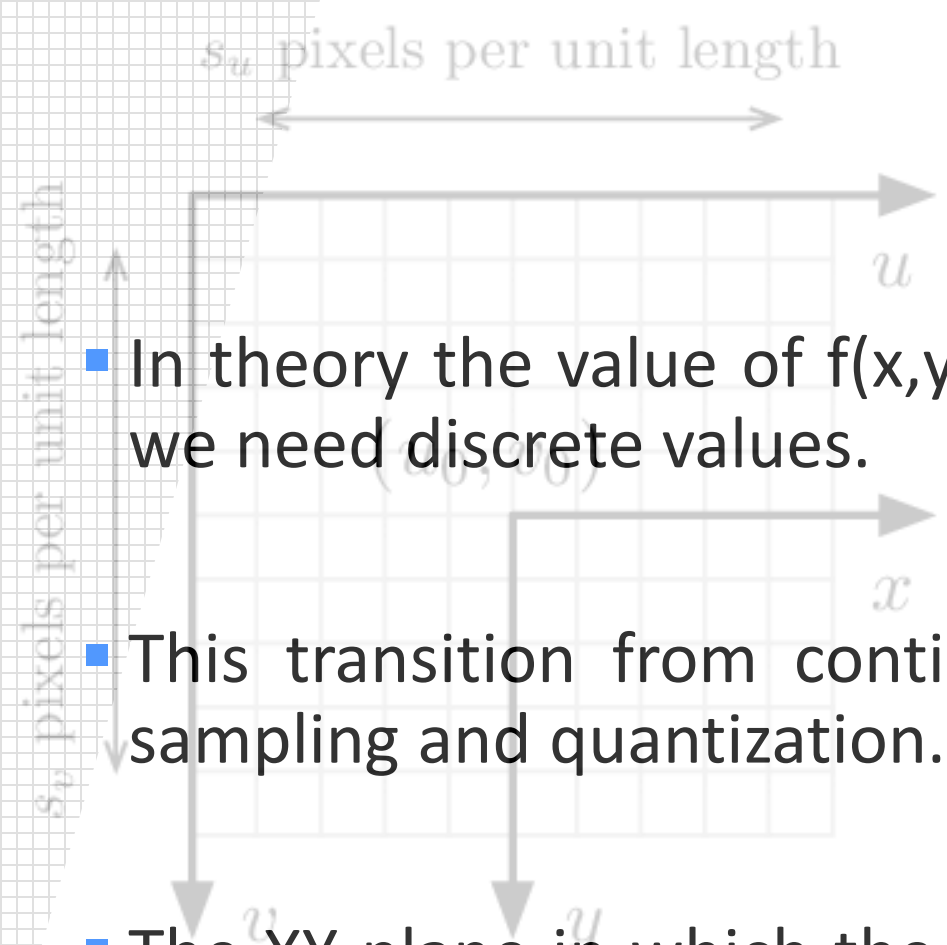
A monochromatic digital image is a matrix I = f(x,y) of discrete values of light intensity (gray levels), consisting of M*N pixels (picture elements, sometimes called pel), each of which has a value belonging to the interval [0, L-1] L being the possible levels of intensity (or gray levels).

All the operations that can be done on matrices can be done on an image



Pixel (PICTURE ELEMENT)

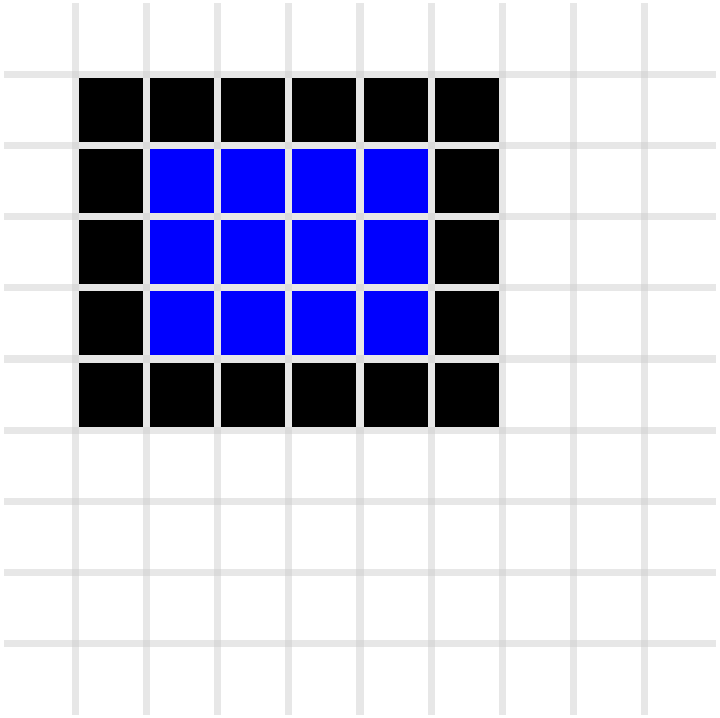
Discretization

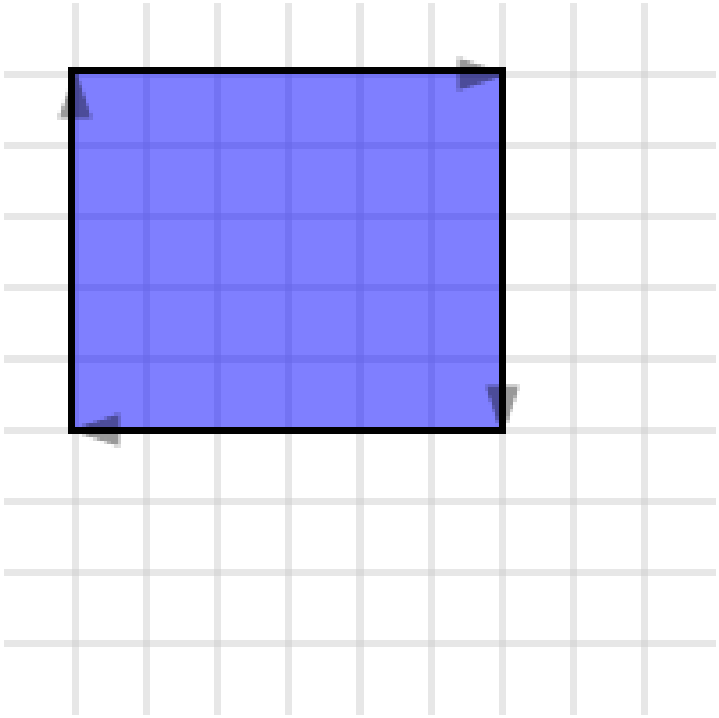


- In theory the value of f(x,y) is a real number, but to produce a digital image we need discrete values.
- This transition from continuous to discrete is done by the operations of sampling and quantization.
- The XY plane in which the coordinates of the image lie is called the **SPATIAL DOMAIN** and the **x,y variables** are called the spatial variables or **spatial coordinates**.

Vector and Raster Images

Vector Formats





Vector Graphics

- In vector graphics, an image is described as a series of geometric shapes.
- Rather than a "finite" series of pixels, a vector image viewer receives information on how to draw the image on the DISPLAY DEVICE in a specific reference system.
- Vector images can be printed with special tools (plotters)

Comparison

Raster

Pro

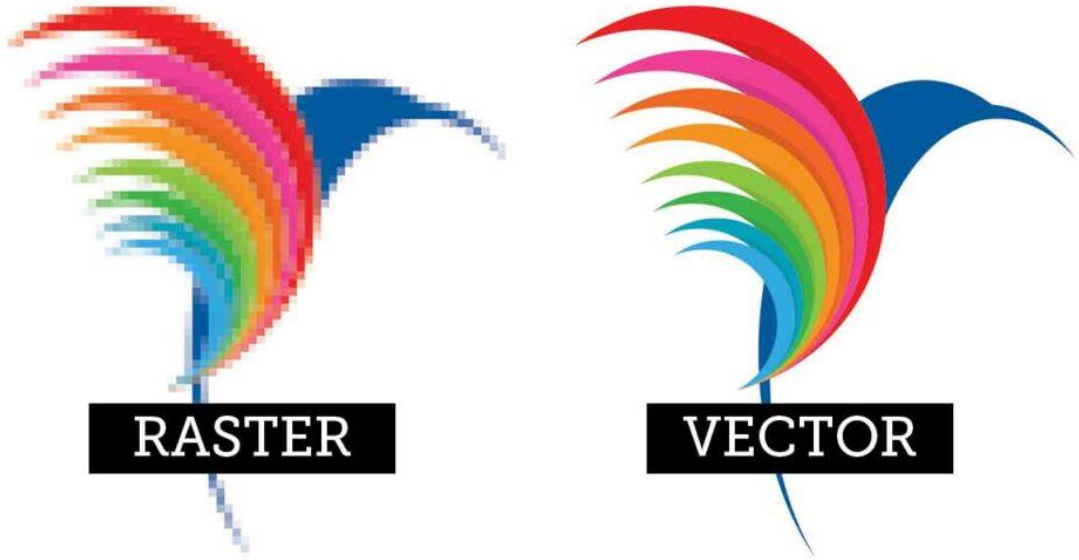
Photorealism

Web-based standards

Cons:

No semantic description.

Large size



Vector

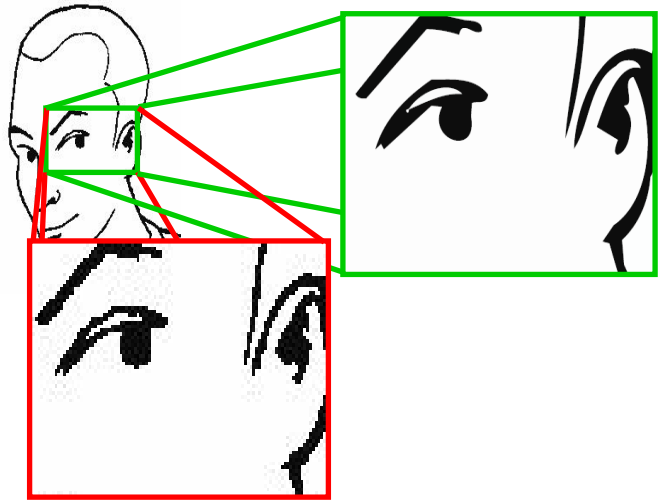
Pro

Transformations on the plane are simple (Zooming, Scaling, Rotating)

Cons:

Not photorealistic

Proprietary vector formats



Vettoriale

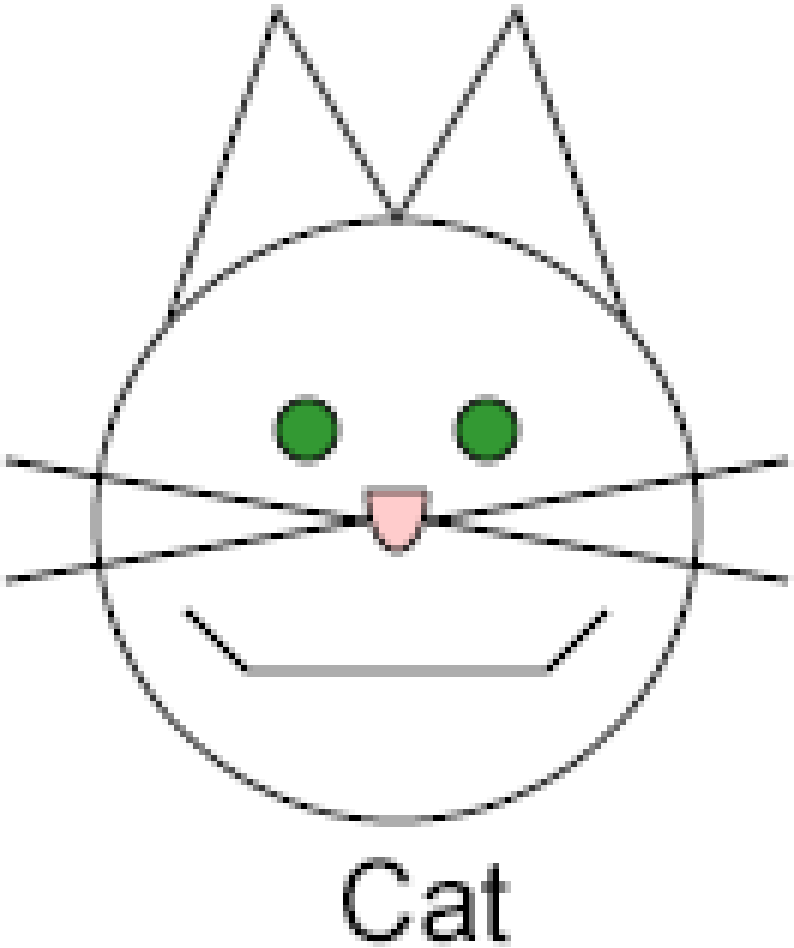
Raster

Un esempio concreto di scalabilità

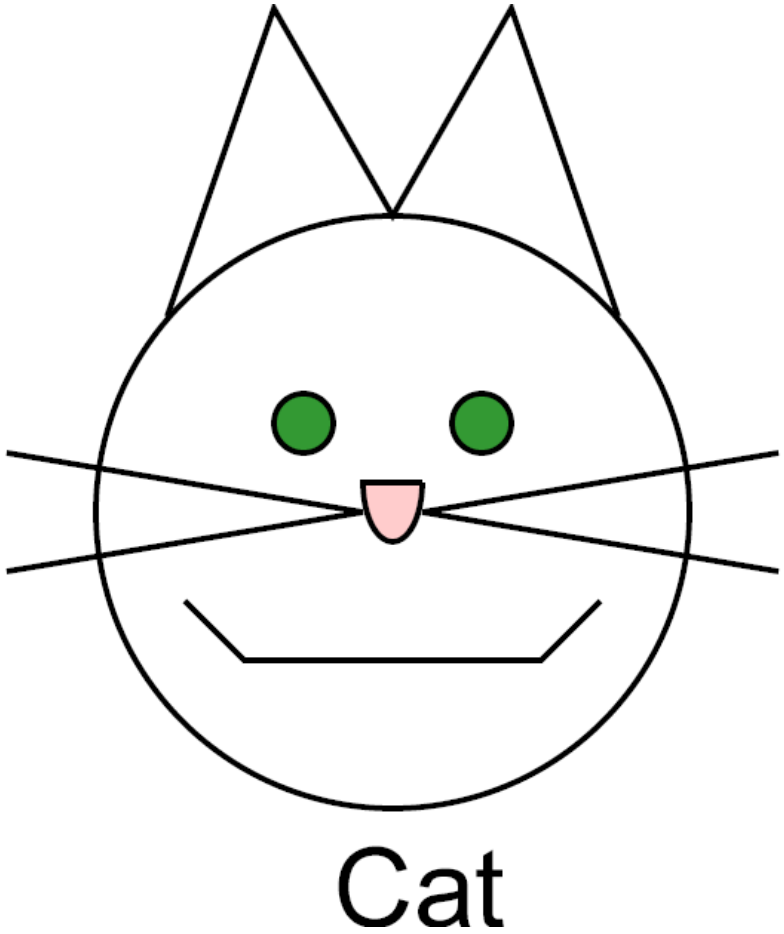
Raster
(140x170)

Vector
(140x170)

Raster 2x



Vector 2x



Vectorialisation

Q=1	19,330	0,976
Q=2	20,178	1,343
Q=4	21,016	1,753
Q=8	21,975	2,251
Q=16	22,670	3,097
Q=32	23,461	4,064
Q=64	24,326	5,693
Q=128	25,180	7,660
Q=256	26,058	10,430
Q=512	26,852	14,169
Q=1024	27,596	19,281
Q=2048	28,174	26,703
Q=4096	28,768	37,641
Q=8192	29,320	53,429

The visual perceived accuracy increases as the Q parameter increase.

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Q=128	25,180	7,660
Q=256	26,058	10,430
Q=512	26,852	14,169
Q=1024	27,596	19,281
Q=2048	28,174	26,703
Q=4096	28,768	37,641
Q=8192	29,320	53,429

The visual perceived accuracy increases as the Q parameter increase.

Vectorialisation

Q=1	19,330	0,976
Q=2	20,178	1,343
Q=4	21,016	1,753
Q=8	21,975	2,251
Q=16	22,670	3,097
Q=32	23,461	4,064
Q=64	24,326	5,693
Q=128	25,180	7,660
Q=256	26,058	10,430
Q=512	26,852	14,169
Q=1024	27,596	19,281
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Q=512	26,852	14,169
Q=1024	27,596	19,281
Q=2048	28,174	26,703
Q=4096	28,768	37,641
Q=8192	29,320	53,429

The visual perceived accuracy increases as the Q parameter increase.

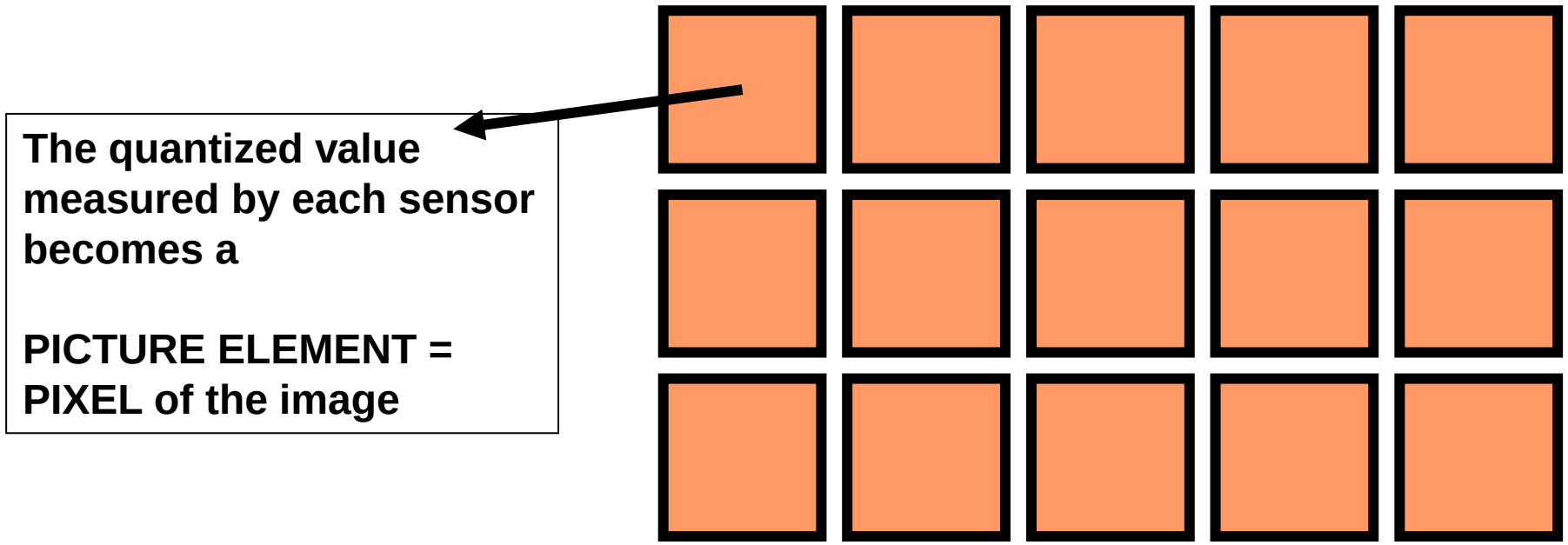
Vectorialisation

	PSNR	bpp
Q=1	19,330	0,976
Q=2	20,178	1,343
Q=4	21,016	1,753
Q=8	21,975	2,251
Q=16	22,670	3,097
Q=32	23,461	4,064
Q=64	24,326	5,693
Q=128	25,180	7,660
Q=256	26,058	10,430
Q=512	26,852	14,169
Q=1024	27,596	19,281
Q=2048	28,174	26,703
Q=4096	28,768	37,641
Q=8192	29,320	53,429

The visual perceived accuracy increases as the Q parameter increase.

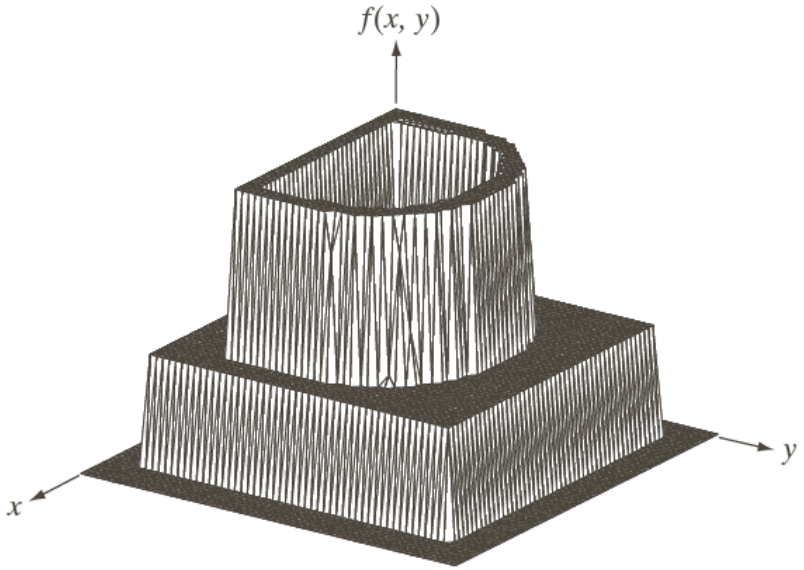
Representations and Types of Images

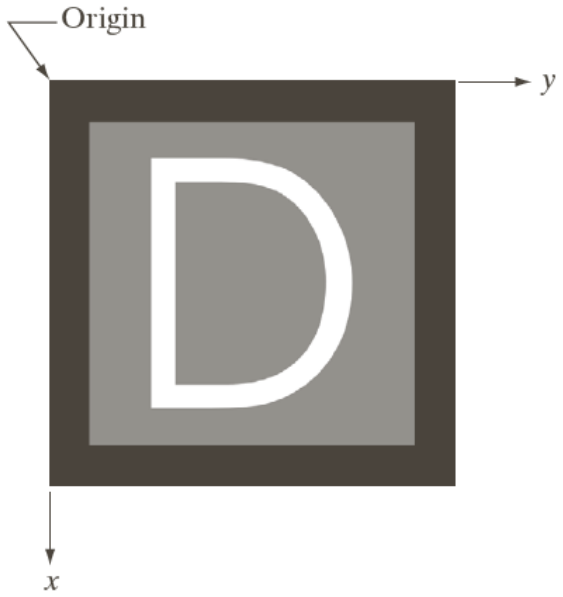
PIXEL

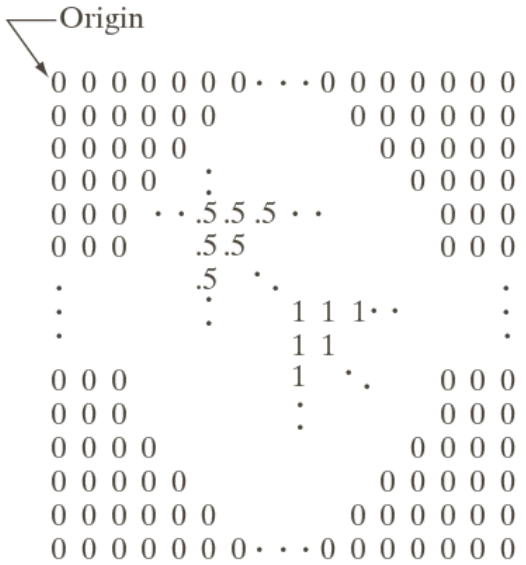


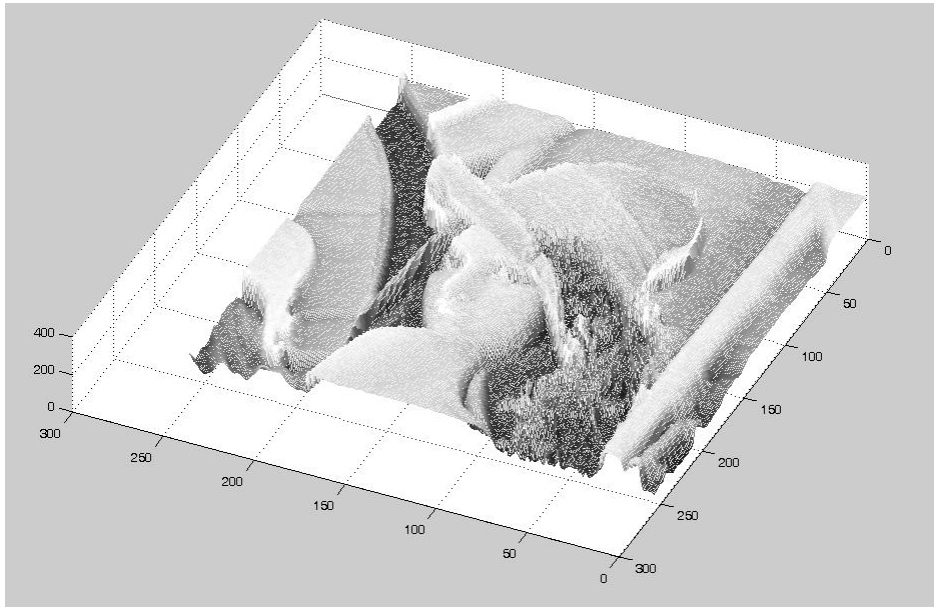
- Digital images are thus represented by matrix of pixels (called **bitmap**) stored in a specific format (Jpeg, Gif, Window Bitmap, PNG, Tiff) in compressed form.

XY Cartesian coordinate system is structured as shown in the figures









Command Window

File Edit Debug Desktop Window Help

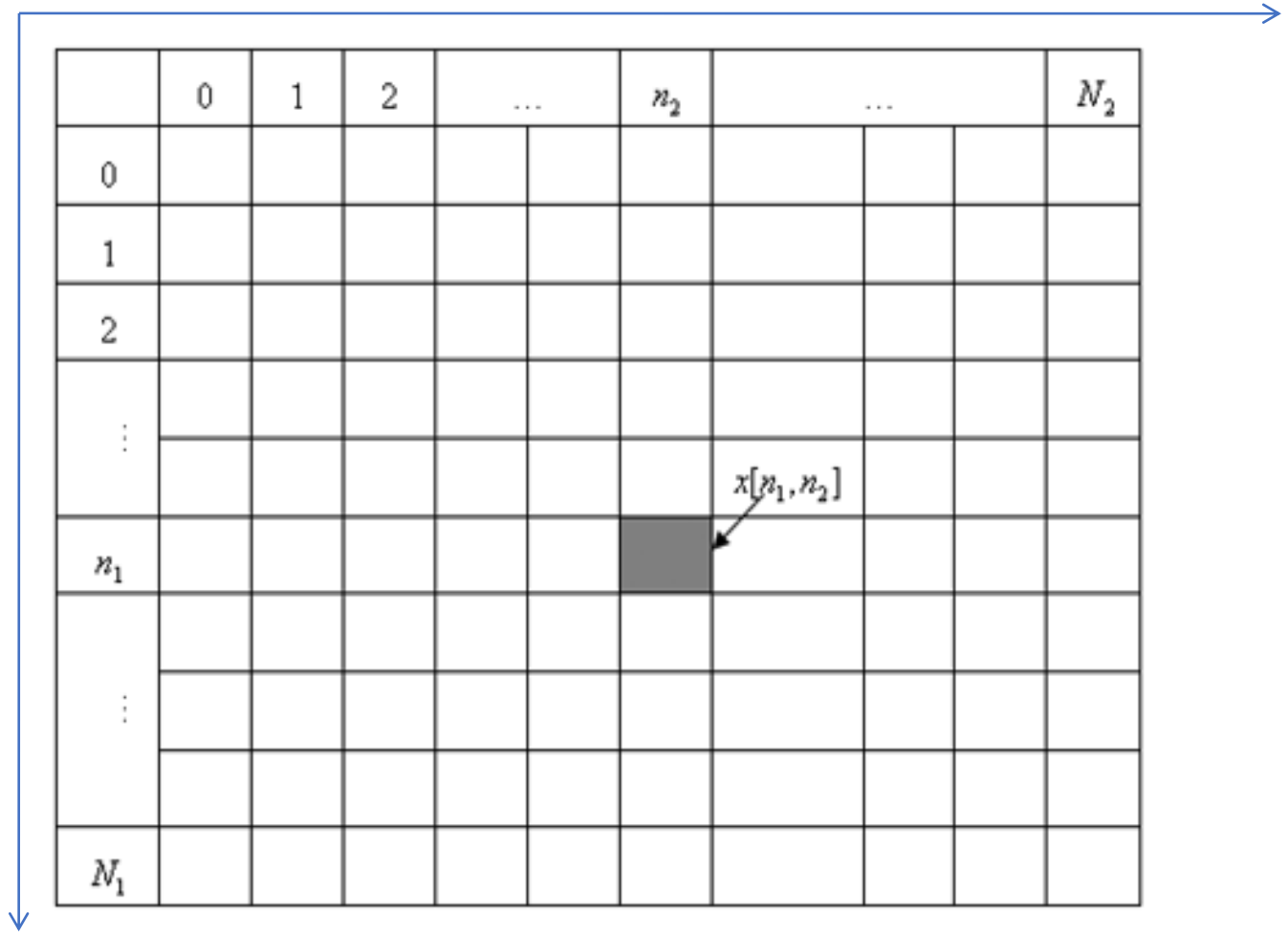
103	94	70	121	207	198
67	77	117	186	195	191
117	135	171	193	188	190
129	142	155	154	170	163
91	106	120	115	120	117
82	94	111	106	95	99
109	113	121	117	121	127
117	123	126	125	133	140
138	141	140	141	146	149
146	147	145	151	146	145
143	144	142	143	139	142
144	145	143	144	143	145
147	148	146	145	146	147
149	149	145	144	148	147
148	147	142	141	145	143
149	148	143	141	142	140
148	147	140	138	138	136
145	145	139	137	138	136
140	138	137	137	135	132
137	137	137	138	134	132

Note

- It is important to remember that conventionally the first element of the matrix is always the top left element.

- The right axis is oriented from top to bottom; the top axis is oriented from left to right.

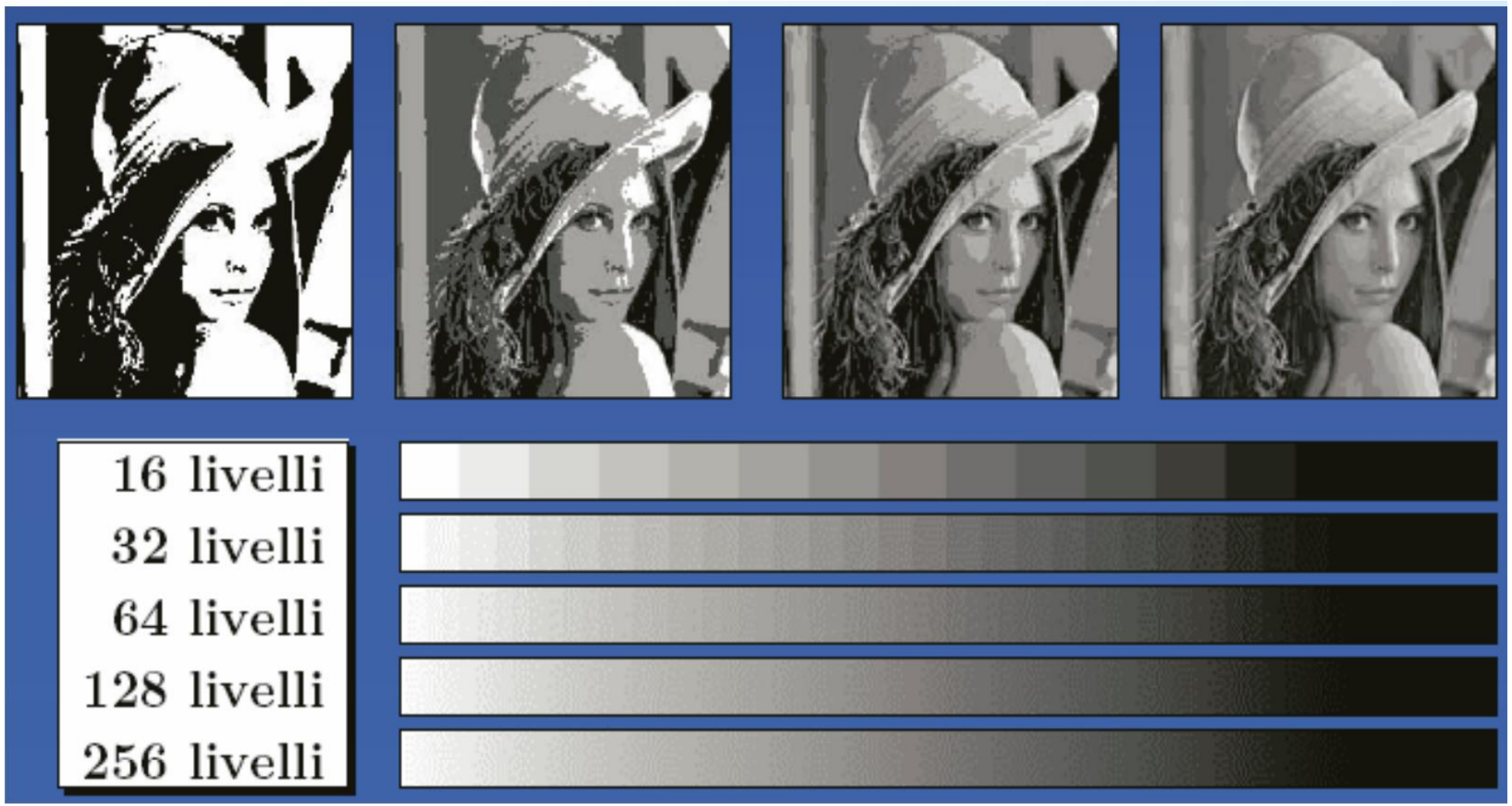
Image Representation



Types of images - **Black and white**

- **1 bit per pixel**
- At position **(i,j)** there will be either value **0** or value **1**

Gray levels



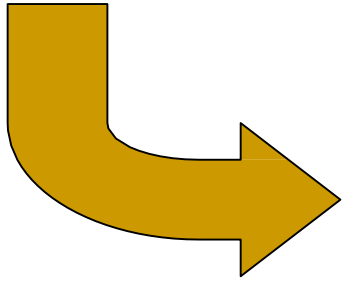
Types of images - Greyscale

- **8 bit per pixel**
- At position **(i,j)** there will be a value between **[0, 255]**

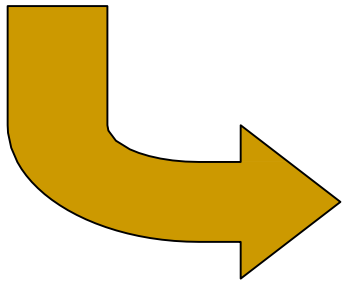
Types of images – Color (e.g. RGB)

- **8 bit per canale**. Since there are 3 channels I will have **24 bit**
- At position **(i,j)** there will be a triplet of the type **(x, y, z)** with x, y, z taking values between **[0, 255]**

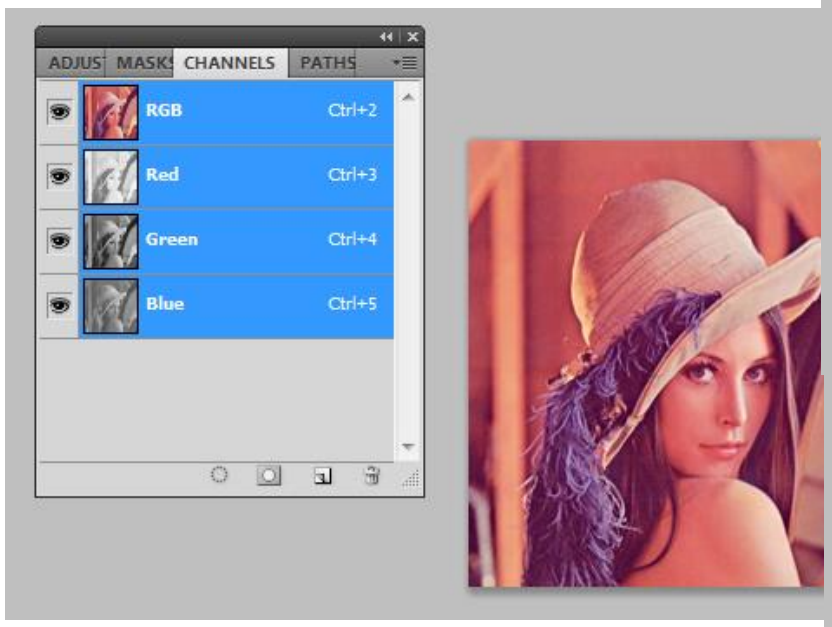
RGB image

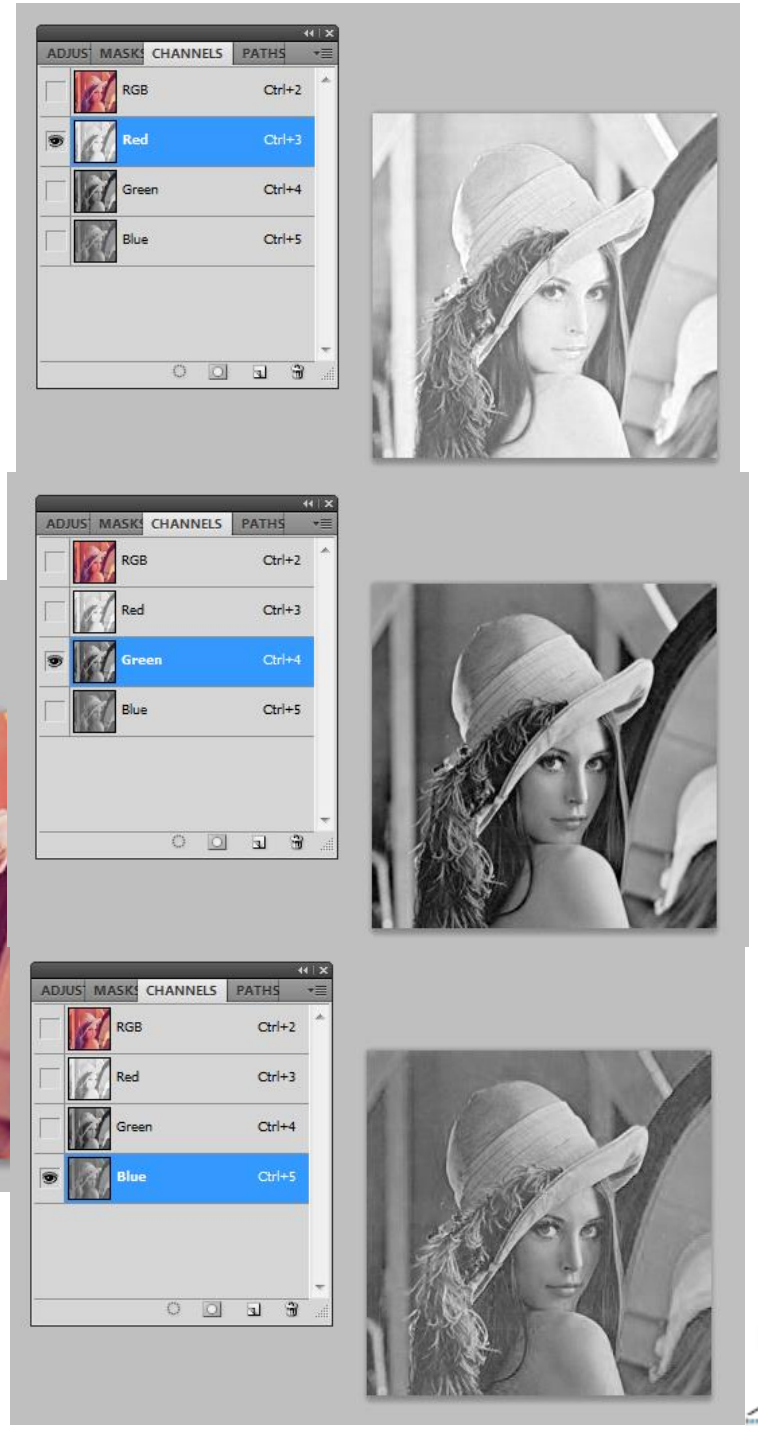


RGB images: actually ...



Lena


ADJUS
MASKS
CHANNELS
PATHS
RGB Ctrl+2
Red Ctrl+3
Green Ctrl+4
Blue Ctrl+5


ADJUS
MASKS
CHANNELS
PATHS
RGB Ctrl+2
Red Ctrl+3
Green Ctrl+4
Blue Ctrl+5

Operations on Images and Matrices

Operations on Images and Matrices

- A raster digital image can be represented by a matrix;
- All the operations that can be done on matrices can be done on an image.
- Such operations do not necessarily make logical sense. For example, what does it mean to multiply two images from a visual point of view?
- What is the range of values after such operations?

Product

- WARNING: The row-by-column product rule applies to matrices, whereas in image processing it is used to do the dot product between two matrices, that is, the **point-to-point product** of the **corresponding elements**.

row-by-column product rule

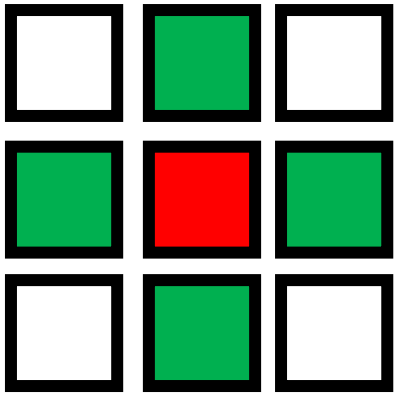
$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1\times 3+0\times 2+2\times 1) & (1\times 1+0\times 1+2\times 0) \\ (-1\times 3+3\times 2+1\times 1) & (-1\times 1+3\times 1+1\times 0) \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

Image product

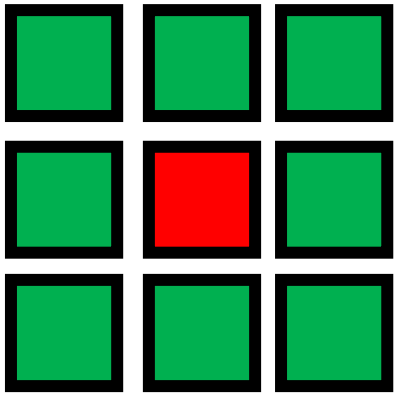
$$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 3 & -4 \end{bmatrix}$$

Neighborhood N_p

- The 4 connected neighbors of a given pixel are those to its left and right and those above and below it.



- Neighbors 8 connected are those 4 connected to which the 4 diagonal pixels are added.



Affine operations

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\sin\theta + w\cos\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Forward mapping

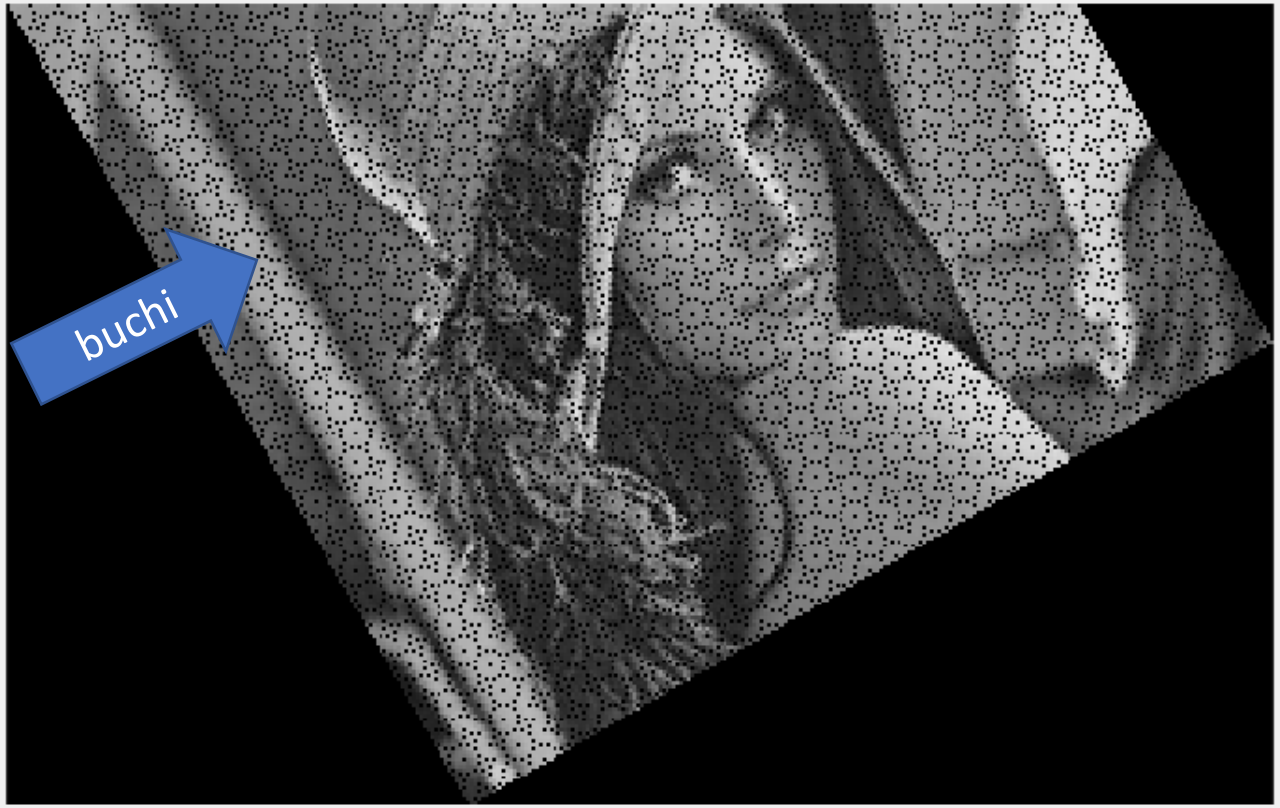
- Where (v,w) is the input pixel, (x,y) the output pixel and T the affine matrix.

- To obtain the value

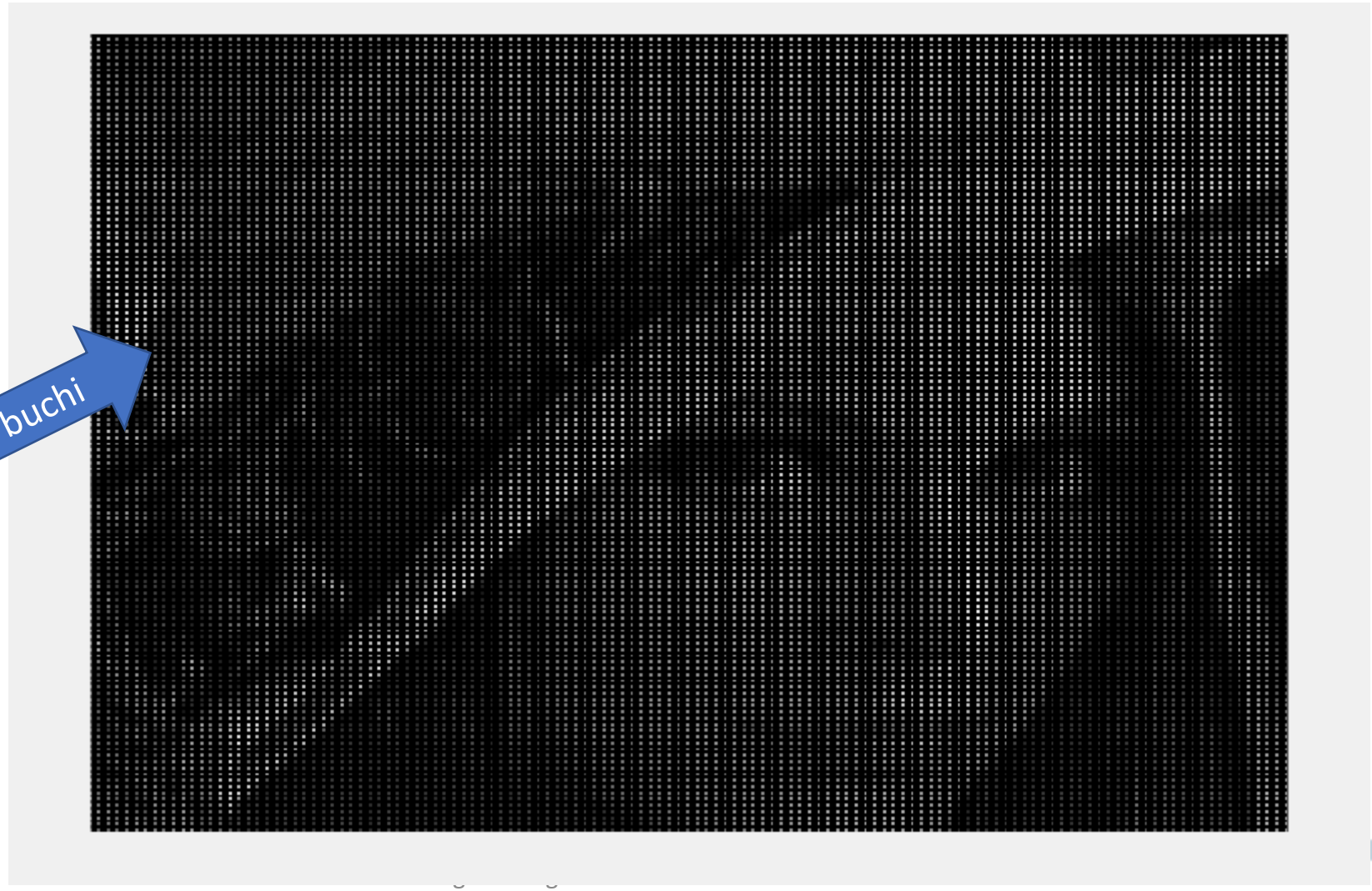
$$[x\ y\ 1] = [v\ w\ 1] * T$$

- In this case, the input image is scrolled and for each pixel (v,w) the position of the new image (x,y) is calculated

Forward mapping (rotation)



Forward mapping (scaling)



Forward mapping *Vs* Inverse mapping

- **Forward mapping**
  - In this case, the input image is scrolled and for each pixel (v,w) the position of the new image (x,y) is calculated

- **Inverse mapping**
  - Visits the spatial positions of the output pixels (x,y) and for each of them calculates the corresponding coordinates in the input image (an inverse formula occurs)

Inverse mapping

- Visits the spatial positions of the output pixels (x,y) and for each of them calculates the corresponding coordinates in the input image (an inverse formula occurs)

- Obviously

$$[v\ w\ 1] = [x\ y\ 1] * \text{inversa}(T)$$

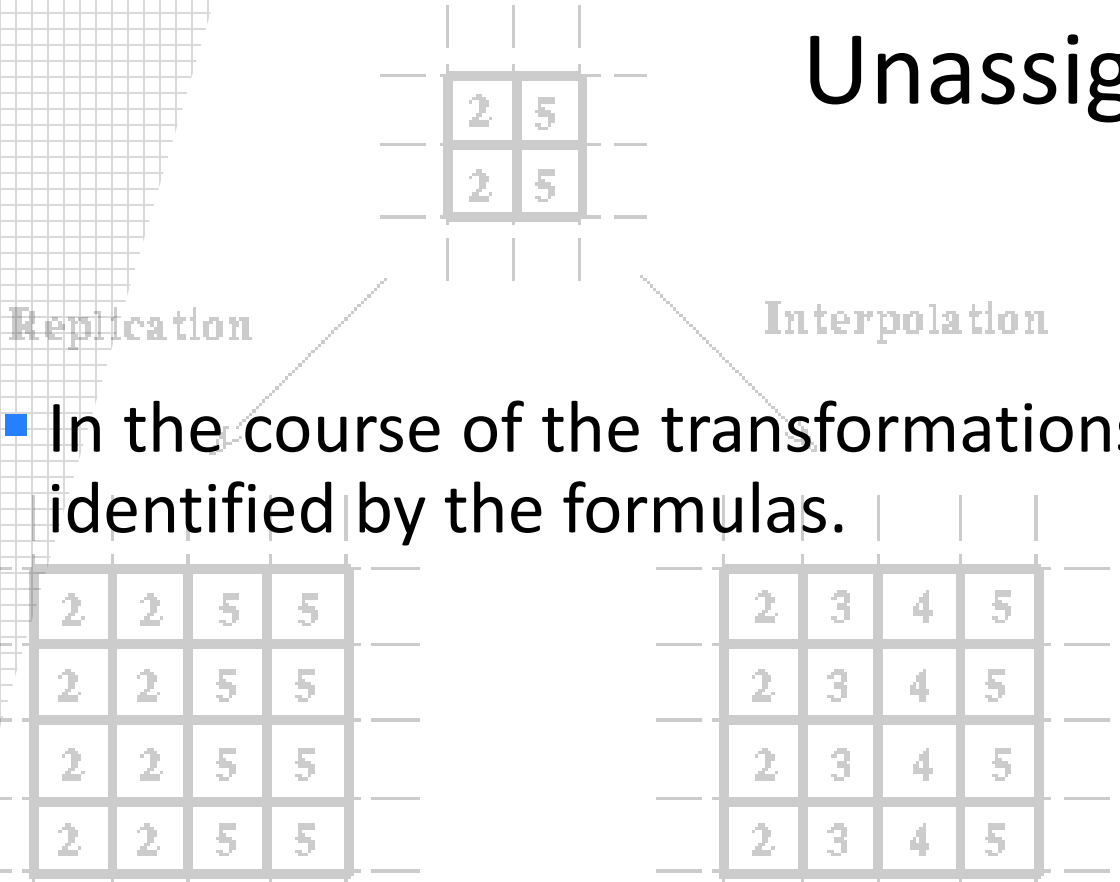
Combinations

- Moreover, affine transformations can be combined with each other simply by multiplying the corresponding T matrices.

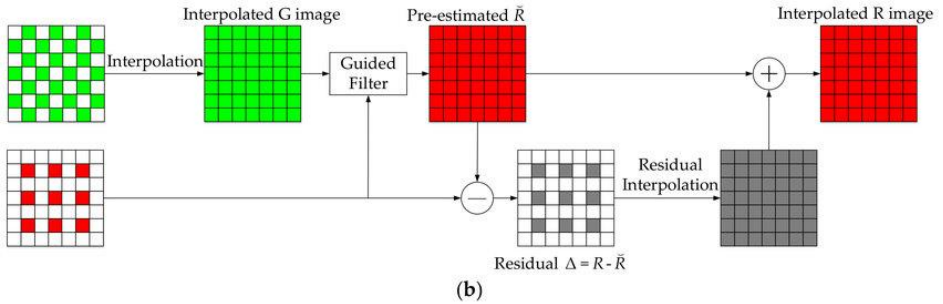
```
theta=38;
tx=40;
ty=23;
cx=2;
cy=2;

Tr=[cosd(theta) sind(theta) 0; -sind(theta) cosd(theta) 0; 0 0 1];
Tt=[1 0 0; 0 1 0; tx ty 1];
Ts=[cx 0 0 ; 0 cy 0; 0 0 1];
T=Tr*Tt*Ts;
```

Interpolation Operation

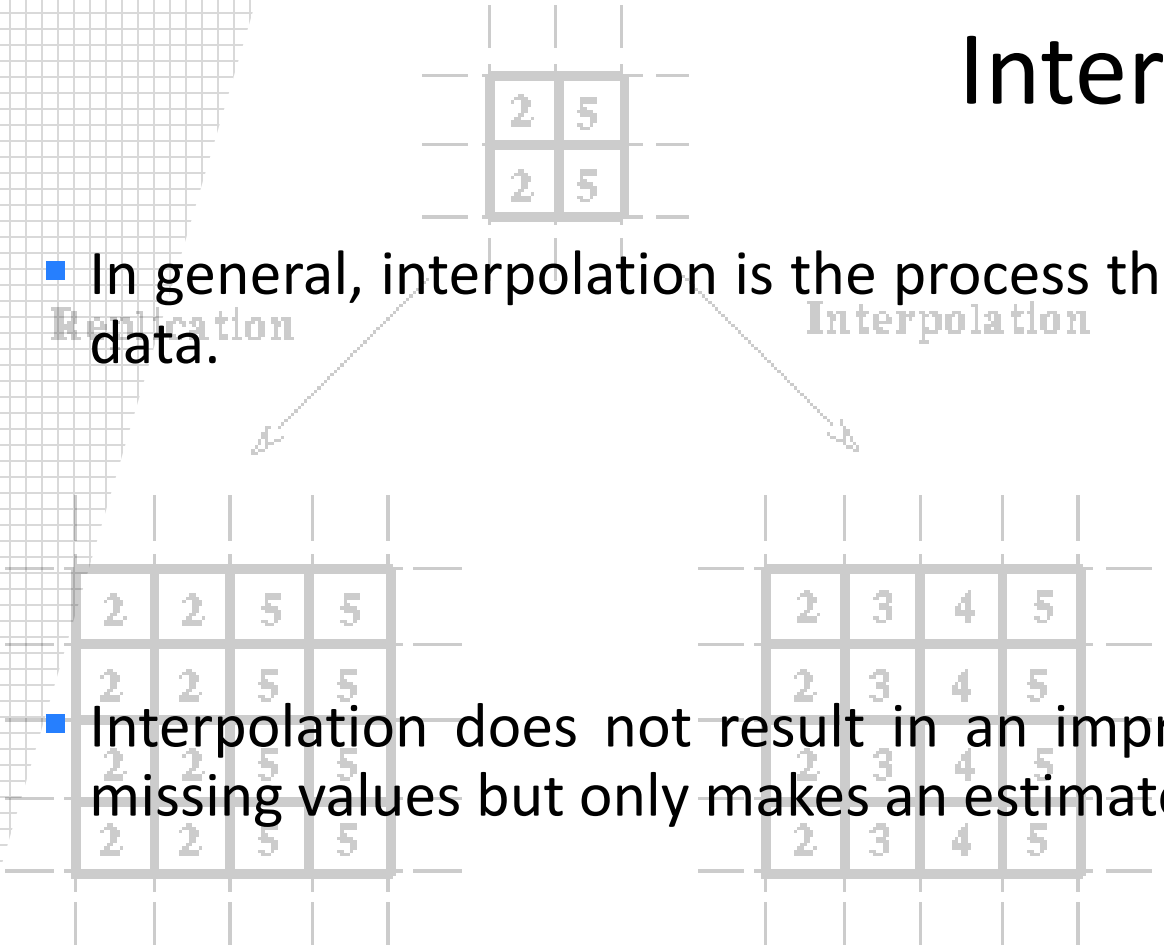
Unassigned values



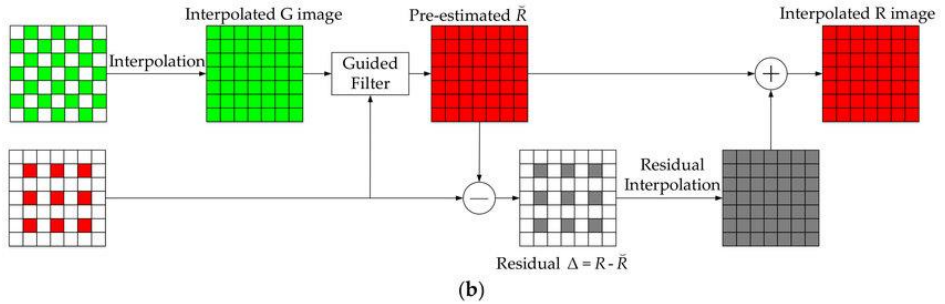
- In the course of the transformations, there may be pixel values that are never identified by the formulas.
- An interpolation process is applied for them.



Interpolation



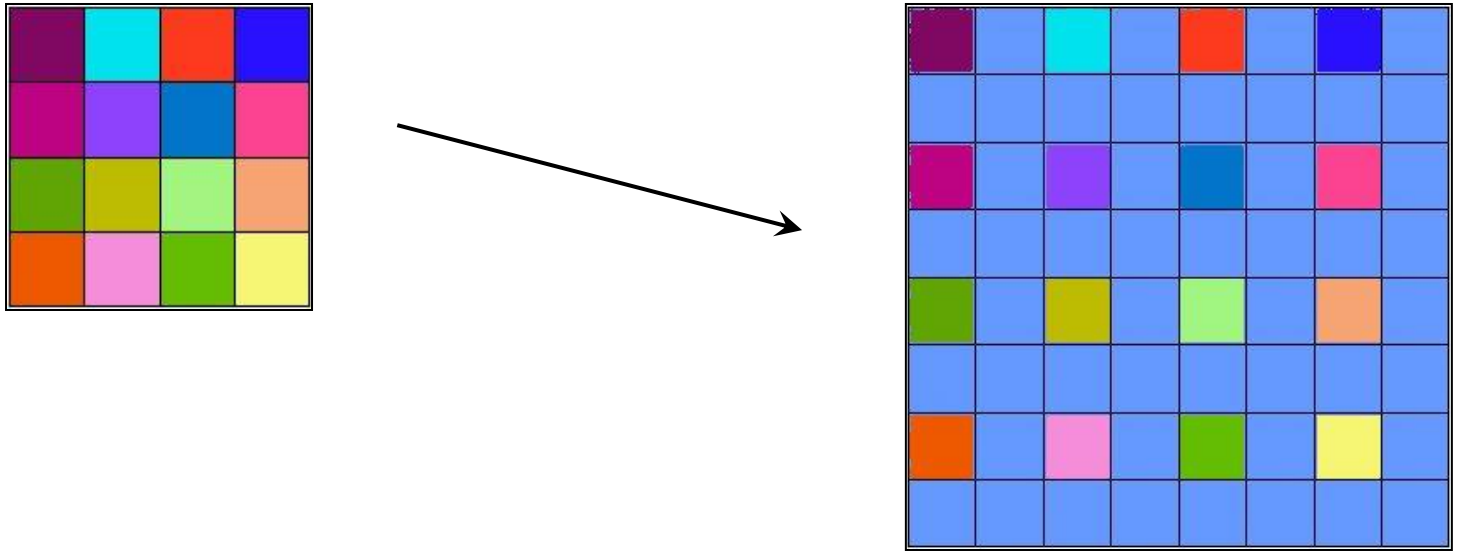
- In general, interpolation is the process that starting from real data estimates the unknown data.
- Interpolation does not result in an improvement in image quality as if it "regains" the missing values but only makes an estimate of the unknown values.



Zooming in

- Interpolation is also performed in zooming processes.
- A 2× zooming means that the size of the image is doubled. If I have an m × n image it will become 2m × 2n. Which means that the total number of pixels will be quadrupled!
- Zooming can be done differently for individual dimensions. For example, one could double the number of rows and triple the number of columns. In this case, our interpolation algorithms will be more complicated.

Zooming in (2x)



After placing the already known values, it is necessary to estimate the values in the empty areas.

Various types of interpolation

There are several types of interpolation:

- Nearest neighbor (or replication)
- Bilinear
- Bicubic
- Others ...

Replication Interpolation

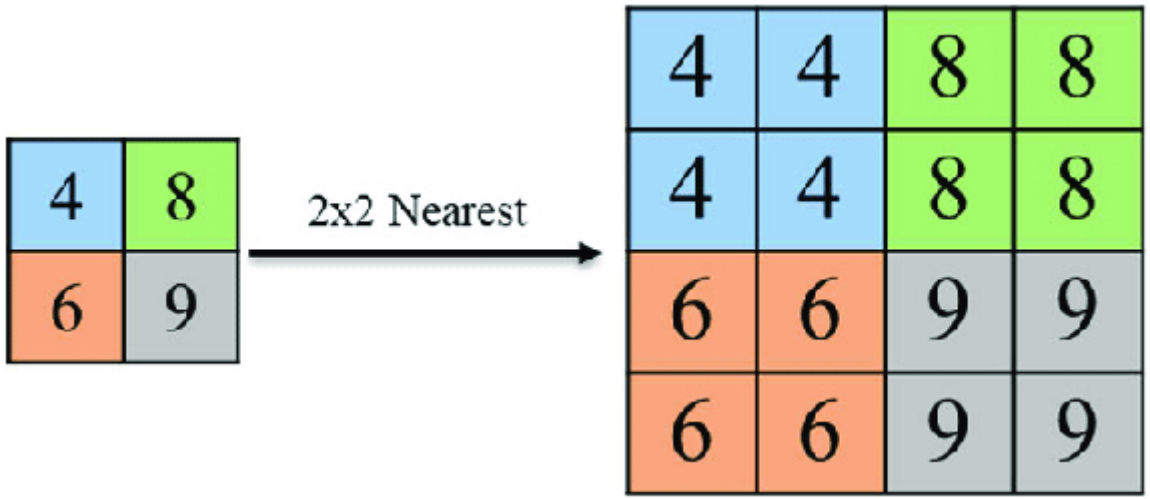
(a)

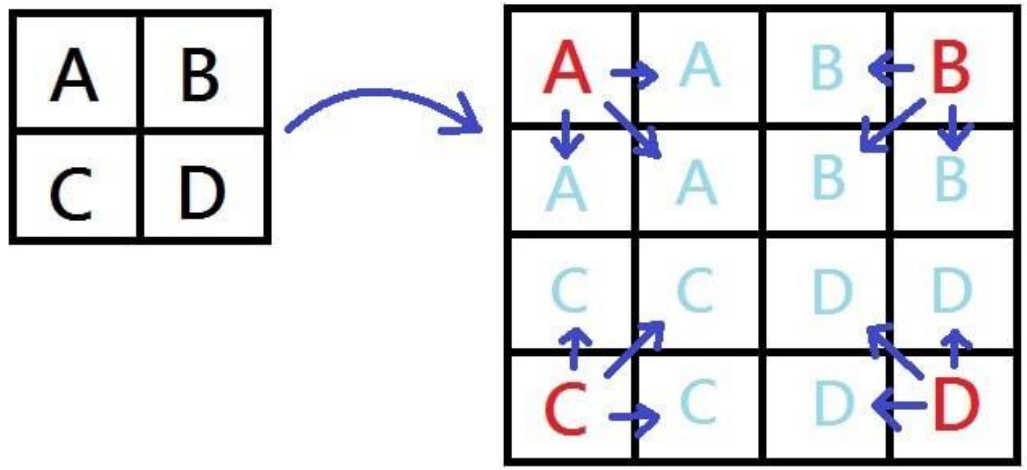
(b)

Replication o nearest neighbor

- This method assigns to each new position the intensity of the nearest pixel in the original image.

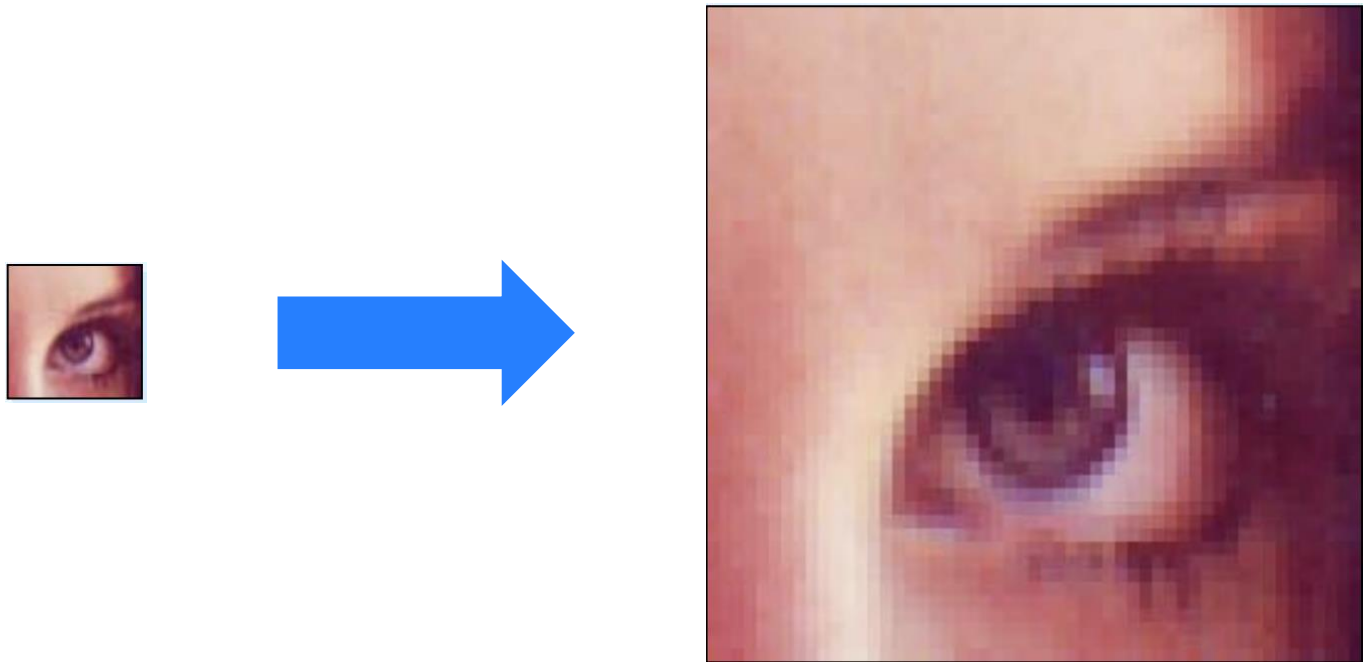
- This approach is very simple but introduces artifacts such as distortions along the sides of the objects represented in the image.





Replication o nearest neighbor

- The main problem with this approach (called "**Pixelization**") consists in the occurrence of **block artifacts**.



- To remove remove Pixelization, so-called "adaptive adaptive" operators can be used (e.g. **Bicubic Interpolation**)

Bilinear Interpolation

(a)

(b)

Bilinear Interpolation

- In bilinear interpolation, the four nearest pixels are used to estimate the intensity to be assigned to each new position. Suppose that (x, y) are the coordinates of the position to be assigned an intensity value and that v(x, y) equals the intensity value. For bilinear interpolation, the assigned value is obtained by the equation

$$v(x, y) = a_0 + a_1 x + a_2 y + a3xy$$

Where the four coefficients are determined from the four equations in the four unknowns obtainable using the four pixels closest to the point (x, y).

- Bilinear interpolation produces better results than replication with a modest increase in computational complexity.

Bilinear Interpolation

- The value assigned to the new spatial positions is given by a linear combination of the gray levels of the four nearest pixels.

- For example, suppose we need to determine the value of pixel x(1/2, 1/2), then we will need to interpolate the levels assigned to pixels x(0, 0), x(0, 1), x(1, 0) and x(1, 1) by solving the following system of equations:

- $$\begin{cases} v(0,0) = a0 \\ v(0,1) = a0 + a \\ v(1,0) = a0 + a1 \\ v(1,1) = a0 + a1 + a2 + a3 \end{cases} \Rightarrow \begin{cases} a0 = v(0,0) \\ a1 = v(1,0) - v(0,0) \\ a2 = v(0,1) - v(0,0) \\ a3 = v(0,0) - v(0,1) - v(1,0) + v(1,1) \end{cases}$$

- Then,

- $$v\left(\frac{1}{2},\frac{1}{2}\right) = a_0 + \frac{a_1}{2} + \frac{a_2}{2} + \frac{a_3}{2} = v(0,0) + \frac{v(1,0),-v(0,0)}{2} + \frac{v(0,1)-v(0,0)}{2} + \frac{v(0,0)-v(0,1)-v(1,0)+v(1,1)}{4} = \frac{v(0,0)+v(0,1)+v(1,0)+v(1,1)}{4}$$

- Note how the central value is obtained precisely as a weighted average of the four neighboring pixels, just as happens in linear interpolation (order 1 interpolation) in the one-dimensional case.

Bicubic Interpolation

(a)

(b)

Bicubic Interpolation

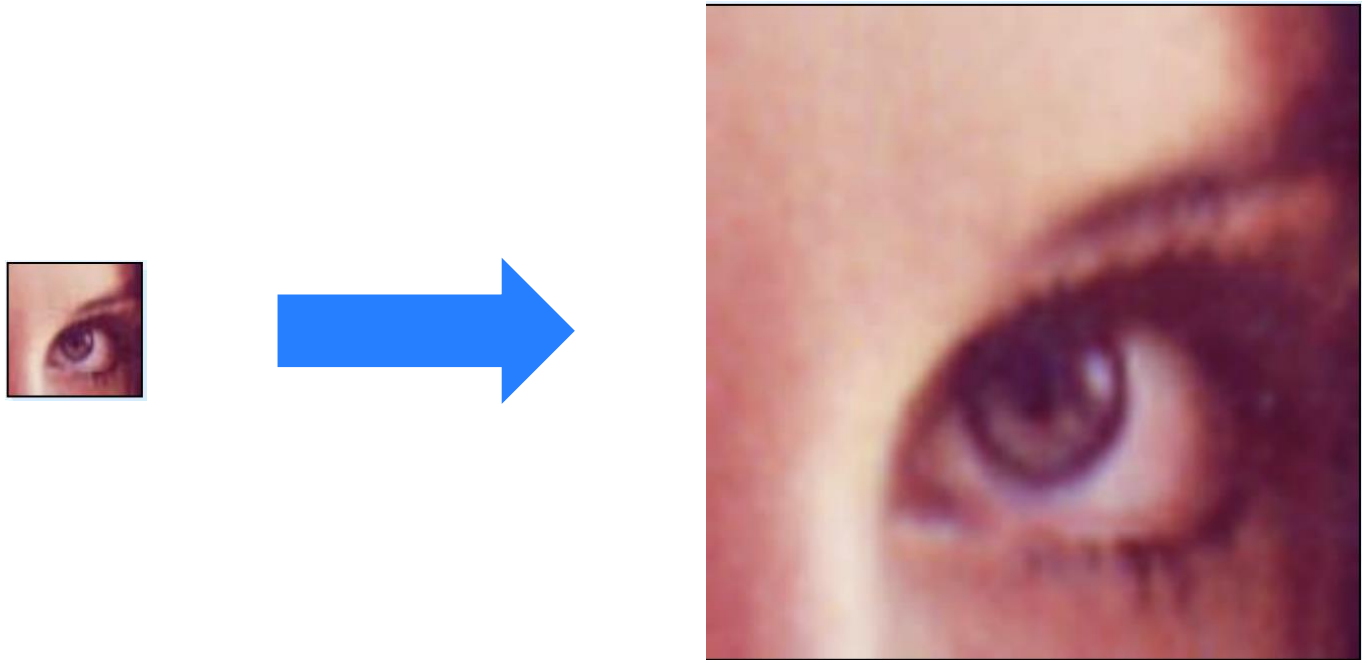
- Bicubic interpolation uses the sixteen pixels closest to the point. The intensity value assigned to the point (x, y) is obtained through the equation

$$v(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

- Where the sixteen coefficients are determined from sixteen equations in sixteen unknowns that can be written using the sixteen points closest to (x, y).
- Generally, bicubic interpolation preserves details better than bilinear interpolation. Bicubic interpolation is the standard technique used in commercial editing programs such as Adobe Photoshop and Corel Photopaint.

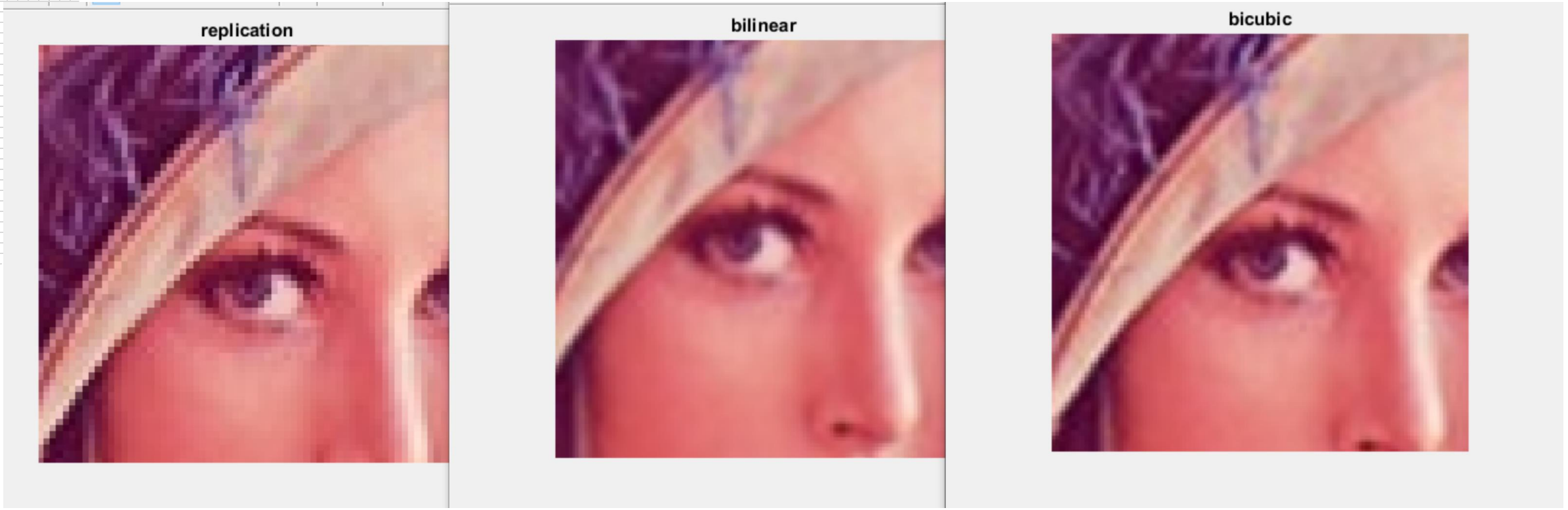
Bicubic Interpolation

- Missing pixels are calculated using a local average, making use of an approximation with one or more cubic functions.



- The main problem with this approach concerns the fact that **images appear blurred**

Replication, Bilinear and Bicubic - Example



What to do at the edges?

One problem is with edges: how to interpolate to edges?

POSSIBLE SOLUTIONS:

- Do nothing
- Interpolating with the values present even if they are fewer in number than those used for other pixels.

In the implementation …

One problem is with edges: how to interpolate to edges?

POSSIBLE SOLUTIONS:

a) Interpolating only the central areas of the image

b) Assume that all around the image there is 0

c) Add a row at the beginning equal to the previous rows, a row at the end equal to the last row, a column at the beginning equal to the starting column, and a column at the end equal to the ending column.

Consider only the central areas of the image.

- In this case, the values at the edges are not calculated

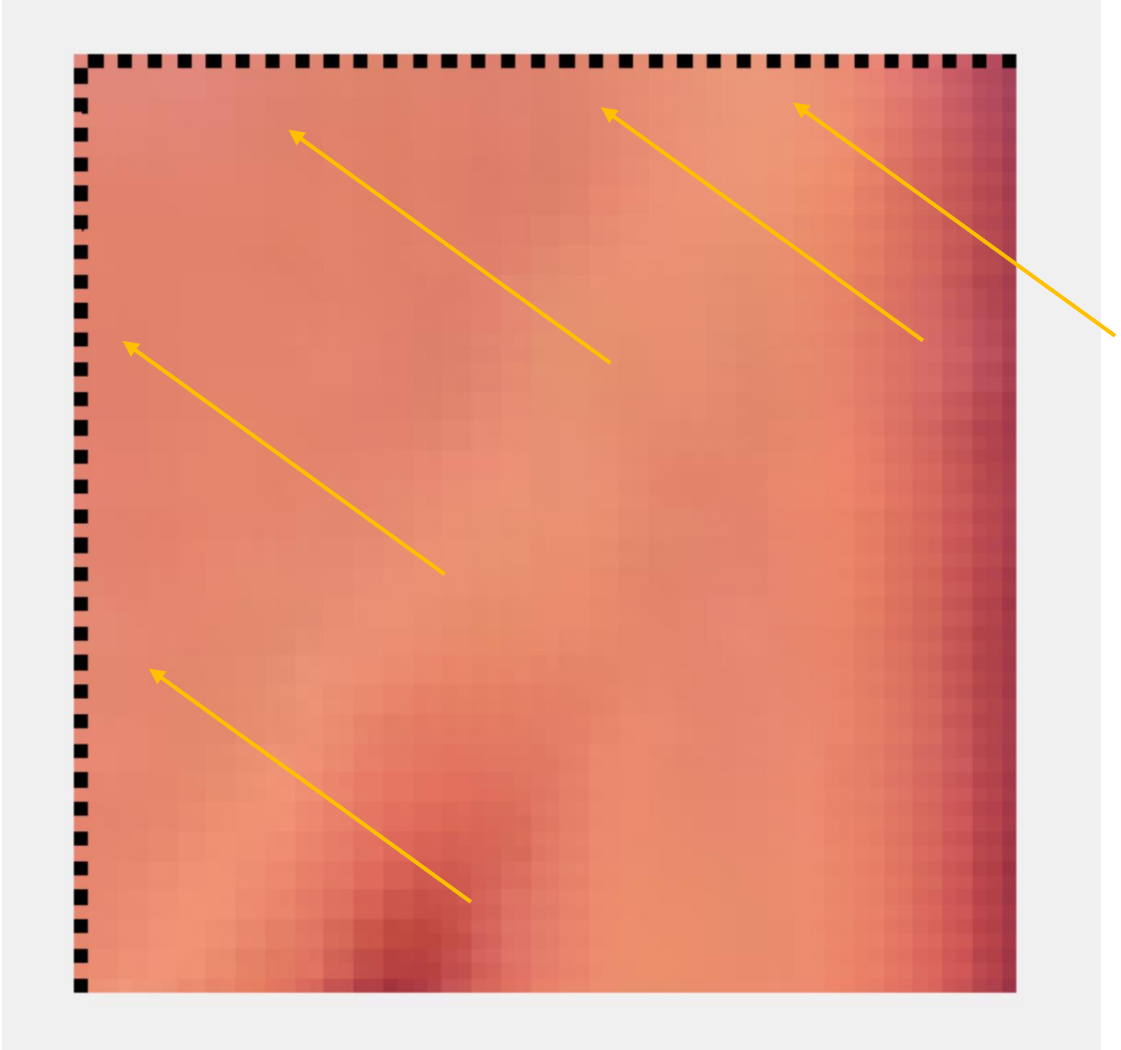
input

1	2	3
4	5	6
7	8	9

output

1	1.5	2	2.5	3	
2.5	3	3.5	4	4.5	
4	4.5	5	5.5	6	
5.5	6	6.5	7	7.5	
7	7.5	8	8.5	9	

The problem



After doing the calculations replicate the last rows and columns

- In this case to do the calculations we replicate the values in the "isolated" rows and columns

input

1	2	3
4	5	6
7	8	9

output

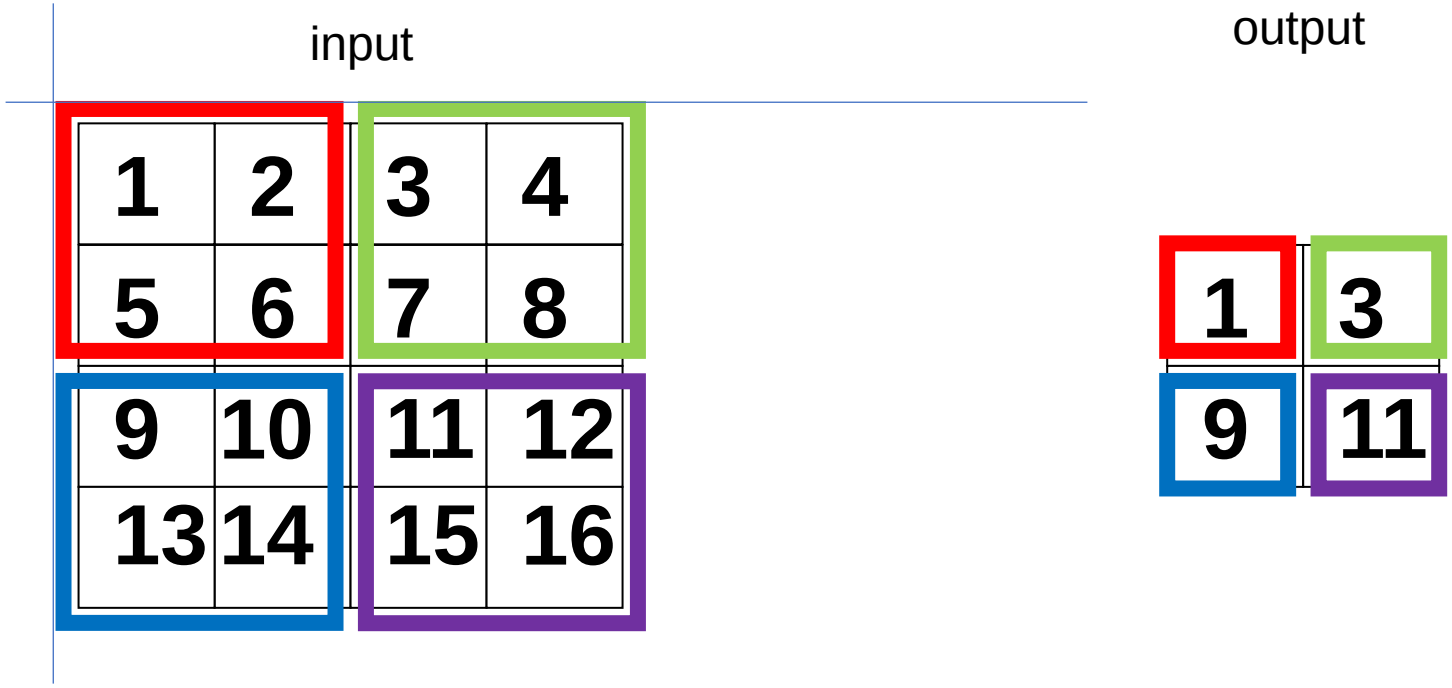
1	1.5	2	2.5	3	3
2.5	3	3.5	4	4.5	4.5
4	4.5	5	5.5	6	6
5.5	6	6.5	7	7.5	7.5
7	7.5	8	8.5	9	9
7	7.5	8	8.5	9	9

Zooming out

- If zooming is done with a number less than 1, you get an image smaller than the original.
- If I reduce an image, I have a process called "decimation."
- Given an mxn image with a zooming out of 0.5 you will get an m/2 × n/2 image.

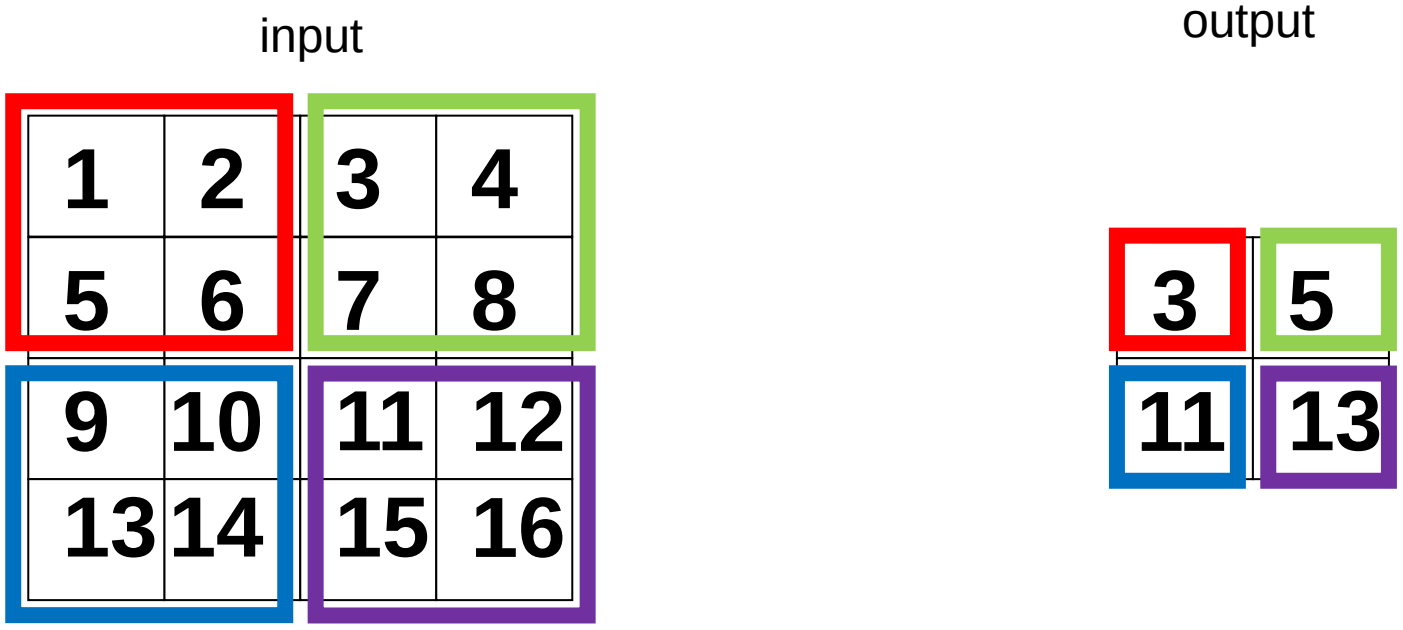
Decimation: method 1

- Every four pixels one is chosen.



Decimation: method 2

- Of four pixels, the average value is calculated.



Estimating the quality of an algorithm

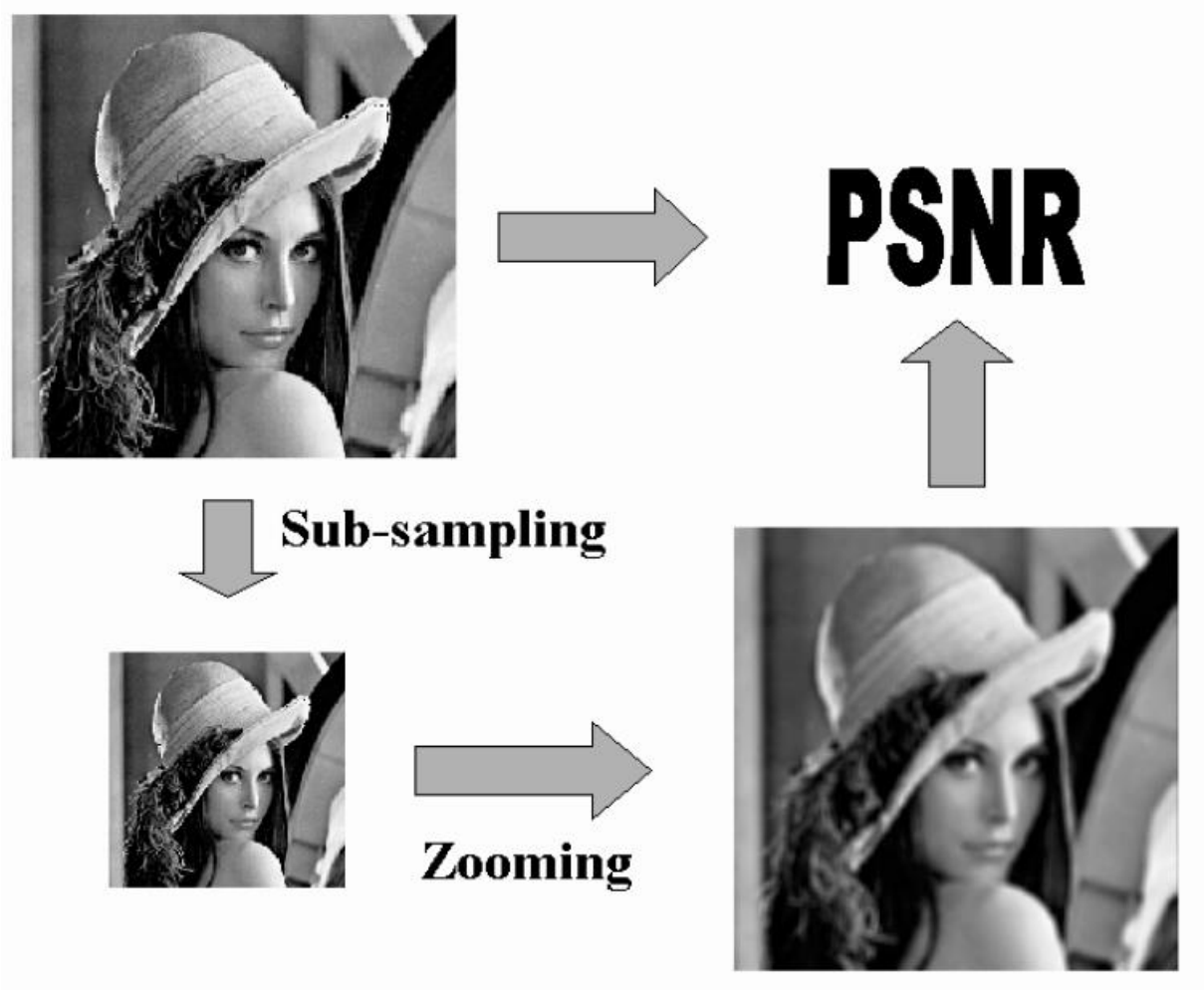
- ***MSE***: (**Mean Square Error**) this parameter is used to estimate the mean square error between two images; the lower this index is, the smaller the difference between the images.

- ***PSNR***: (**Peak Signal to NoiseRatio**) parameter to measure the quality of a compressed image compared to the original, it depends on the difference between the encoded image and the original image. The higher its value, the greater the "similarity" to the original.

PSNR (Peak Signal to Noise Ratio)

- It is a full reference technique.
- To calculate it, it is necessary to have both the image to be evaluated I' (MxN) and an optimal version of it I (MxN).
- PSNR **is not the best parameter** for evaluating the quality of an interpolation algorithm, but **it is the most widely used**.

Scheme for the calculation of PSNR in the case of zooming



PSNR: formule

- To calculate the PSNR we need the MSE (Mean Square Error):

$$MSE = \frac{1}{MN}\sum_{x=1}^{M}\sum_{y=1}^{N}[I'(x,y) - I(x,y)]^2$$

- The PSNR is calculated by one of the following formulas (they are equivalent):

$$PSNR = -10\log_{10}\frac{MSE}{S^2} \qquad PSNR = 20\log_{10}\left(\frac{S}{\sqrt{MSE}}\right), \; PSNR = 10\log_{10}\left(\frac{S^2}{MSE}\right)$$

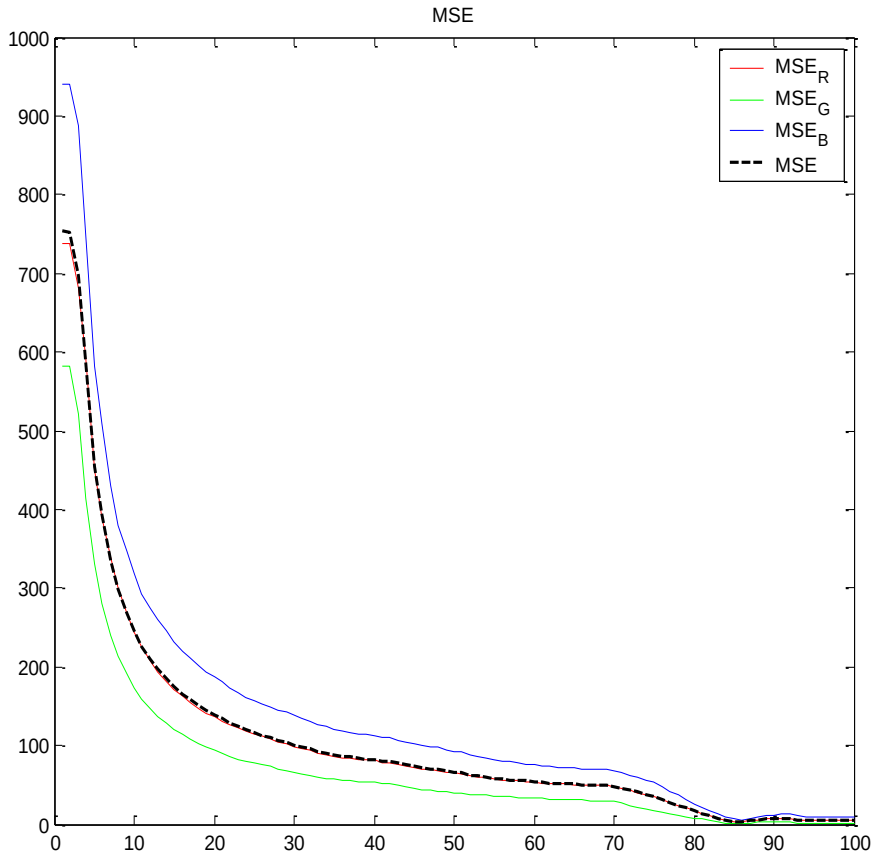
S is the maximum pixel value (usually 255),

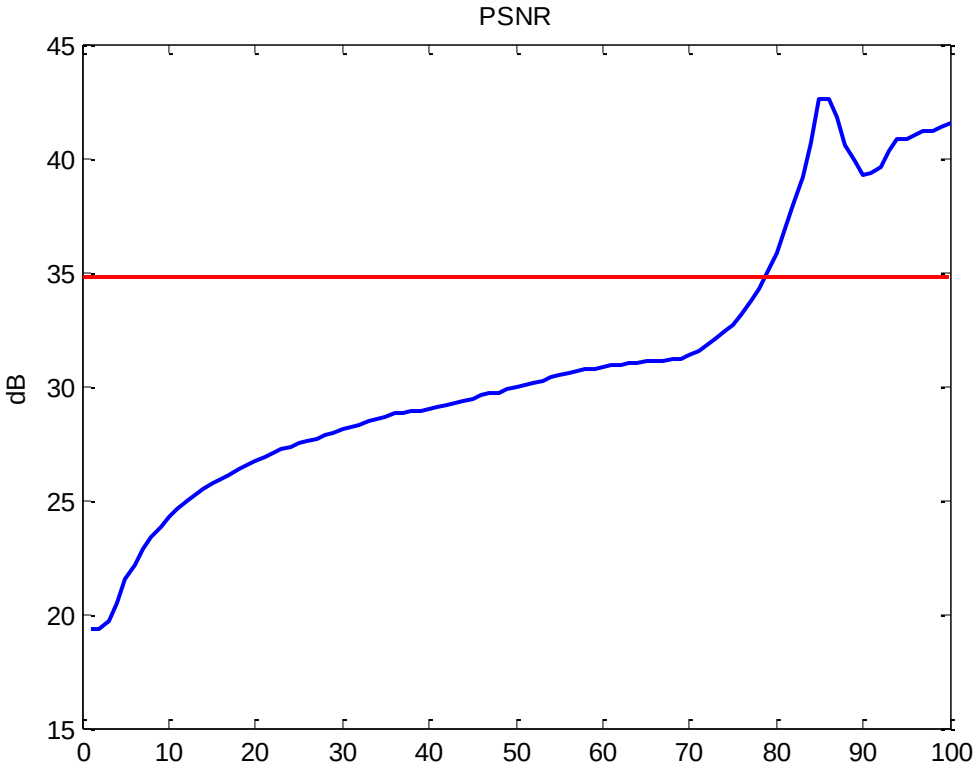
MSE e PSNR

MSE and PSNR are widely used because they are simple to calculate, however, they do not always give a result faithful to that given by the human visual system. Indeed:

- The sensitivity of the HVS system to errors may be different for different types of errors, and may also vary according to the visual context. This difference cannot be adequately captured by the MSE.
- Two distorted images can have very different types of errors while having the same MSE.
- Both metrics are also strongly affected by "imperceptible" spatial movements (translations, rotations, flipping of rows and/columns)

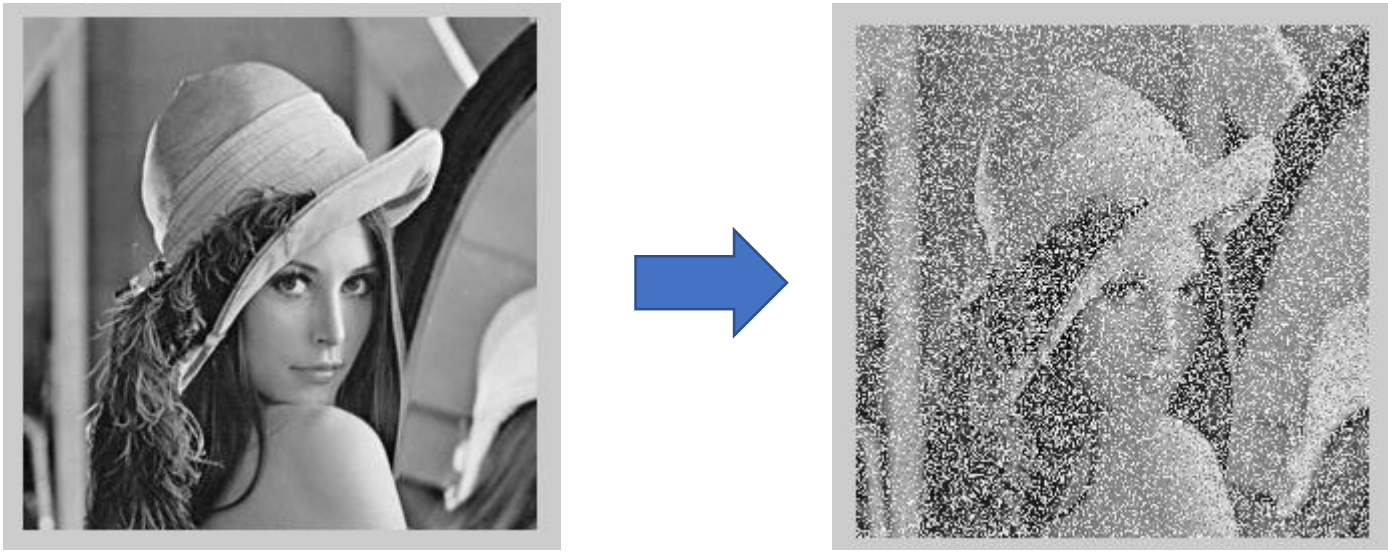
PSNR vs MSE


MSE
MSE_R
MSE_G
MSE_B
MSE
0
100
200
300
400
500
600
700
800
900
1000
0
10
20
30
40
50
60
70
80
90
100

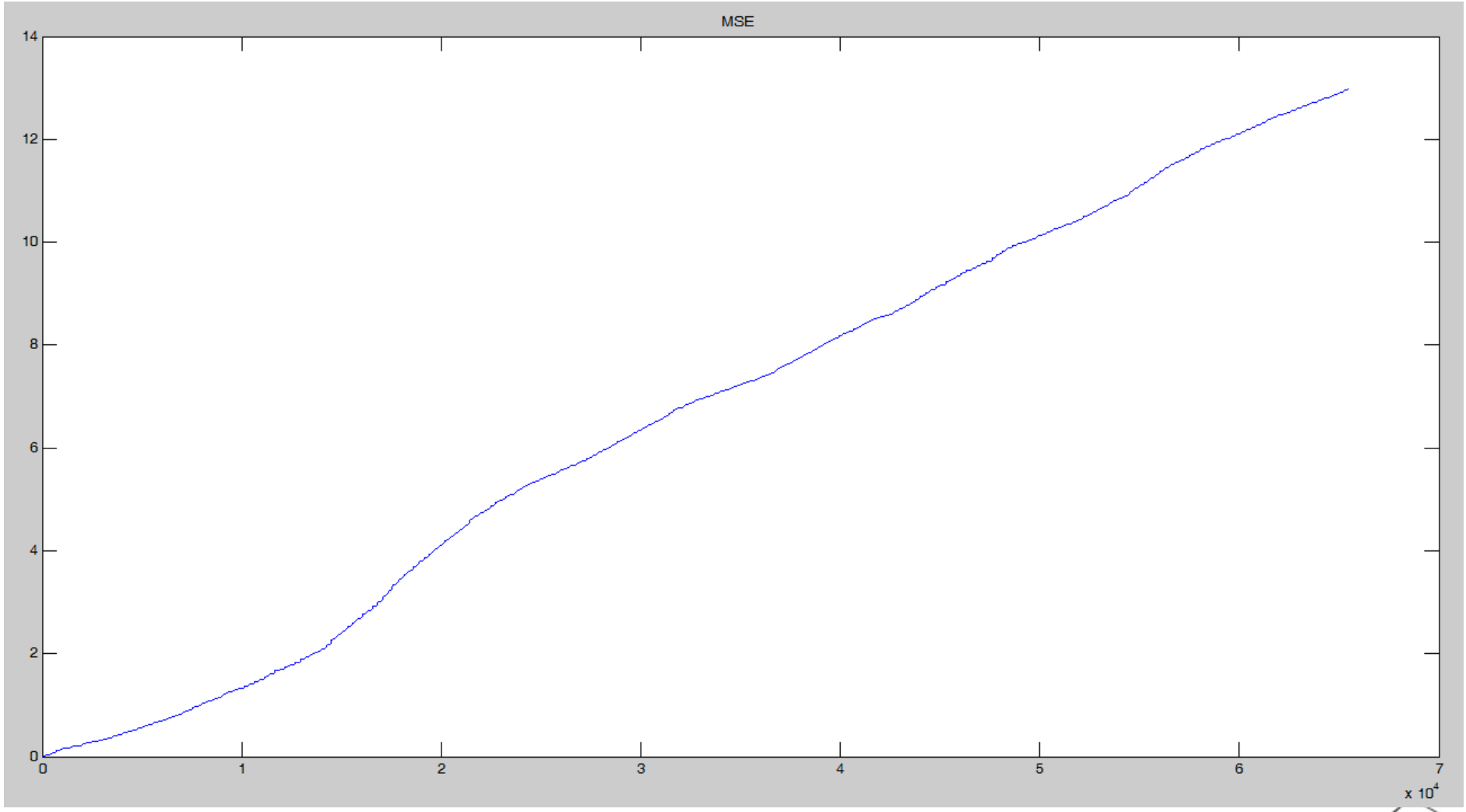

PSNR
dB
15
20
25
30
35
40
45
0
10
20
30
40
50
60
70
80
90
100

Graphical trend

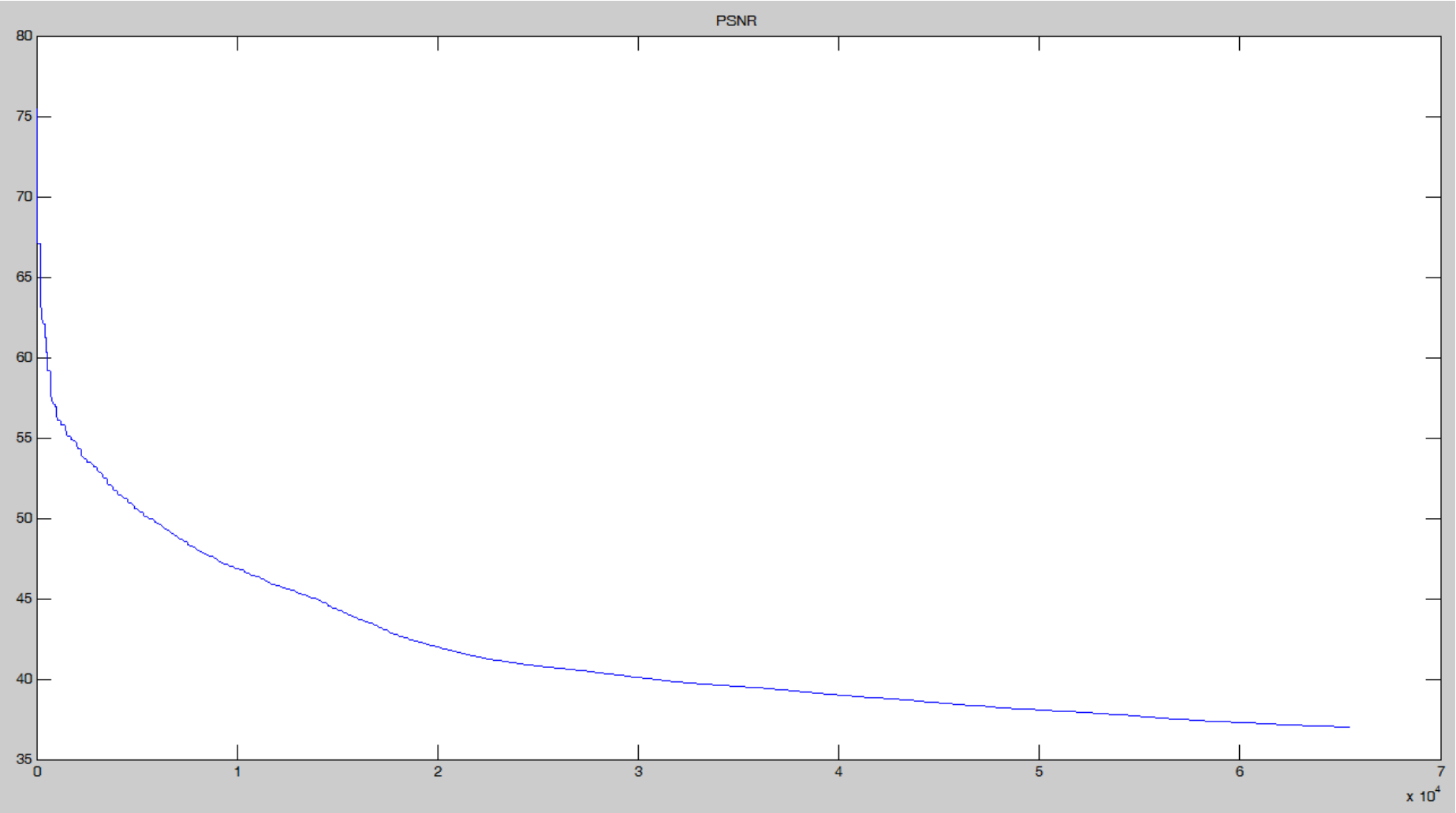
- Here is how the PSNR and MSE varies if we randomly vary the pixels of an image.
- Each time a pixel varies I calculate the PSNR and MSE.



MSE



PSNR



MSE and PSNR for RGB images

- Usually one of the following solutions is used.
  - The simple average of the MSE (or PSNR) values over the 3 channels.
  - A linear combination that weighs the green component more heavily
  - ...

Basic Concept of Digital Images

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