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ORTHOGONALLY ADDITIVE HOLOMORPHIC FUNCTIONS ON C*–ALGEBRAS

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Abstract. Let A be a C*-algebra. We prove that a holomorphic function of bounded type $f : A \to \mathbb{C}$ is orthogonally additive on A_{sa} if, and only if, it is additive on elements having zero-product if, and only if, there exist a positive functional φ in A^*, a sequence (ψ_n) in $L_1(A^{**},\varphi)$ and a power series holomorphic function h in $\mathscr{H}_b(A,A^*)$ such that

$$h(a) = \sum_{k=1}^{\infty} \psi_k \cdot a^k \text{ and } f(a) = \langle 1_{A^{**}}, h(a)\rangle = \int h(a)\, d\varphi,$$

for every a in A, where $1_{A^{**}}$ denotes the unit element in A^{**} and $L_1(A^{**},\varphi)$ is a non-commutative L_1-space.

1. Introduction

Let A be a C*-algebra whose self-adjoint part is denoted by A_{sa}. Two elements a and b in A are said to be *orthogonal* (denoted by $a \perp b$) if $ab^* = b^*a = 0$. When $ab = 0 = ba$ we shall say that a and b *have zero-product*.

Let A be a C*-algebra and let X be a complex Banach space. A mapping $f : A \to X$ is said to be *orthogonally additive* (respectively, *orthogonally additive on* A_{sa}) if for every a,b in A (respectively, a,b in A_{sa}) with $a \perp b$ we have $f(a+b) = f(a) + f(b)$. We shall say that f is *additive on elements having zero-product* if for every a,b in A with $ab = 0 = ba$ we have $f(a+b) = f(a) + f(b)$.

Let X and Y be Banach spaces. A (continuous) m-homogeneous polynomial P from X to Y is a mapping $P : X \longrightarrow Y$ for which there is a (continuous) multilinear symmetric operator $A : X \times \ldots \times X \to Y$ such that $P(x) = A(x,\ldots,x)$, for every $x \in X$. All the polynomials considered in this paper are assumed to be continuous.

Orthogonally additive n-homogeneous polynomials over $C(K)$-spaces and Banach lattices have been independently studied by Y. Benyamini, S. Lassalle and J.G. Llavona (cf. [1]) and D. Pérez and I. Villanueva (cf. [9]); a short proof was published by

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D. Carando, S. Lassalle and I. Zalduendo [3]. Orthogonally additive n-homogeneous polynomials over general C^*-algebras were described by C. Palazuelos, I. Villanueva and the first author of this note in [8] (see also [2, §3]). These results extend the characterization given by K. Sundaresan for L^p-spaces [12]. In the setting of C^*-algebras we have:

THEOREM 1. [8, Theorem 2.8 and Corollary 3.1] *Let A be a C*-algebra, X a Banach space, $n \in \mathbb{N}$, and P an n-homogeneous polynomial from A to X. The following are equivalent.*

(a) *There exists a bounded linear operator $T : A \to X$ satisfying*

$$P(x) = T(x^n),$$

for every $x \in A$, and $\|P\| \leqslant \|T\| \leqslant 2\|P\|$.

(b) *P is additive on elements having zero-products.*

(c) *P is orthogonally additive on A_{sa}.*

A mapping f from a Banach space X to a Banach space Y is said to be *holomorphic* if for each $x \in X$ there exists a sequence of polynomials $P_k(x) : X \to Y$, and a neighbourhood V_x of x such that the series

$$\sum_{k=0}^{\infty} P_k(x)(y-x)$$

converges uniformly to $f(y)$ for every $y \in V_x$. A holomorphic function $f : X \longrightarrow Y$ is said to be of *bounded type* if it is bounded on all bounded subsets of X, in this case its Taylor series at zero, $f = \sum_{k=0}^{\infty} P_k$, has infinite radius of uniform convergence, i.e. $\limsup_{k\to\infty} \|P_k\|^{\frac{1}{k}} = 0$ (compare [5, §6.2]). We refer to [5] for the basic facts and definitions used in this paper.

Homogeneous polynomials on a C^*-algebra A are the simplest examples of holomorphic functions on A.

In a recent paper, D. Carando, S. Lassalle and I. Zalduendo considered orthogonally additive holomorphic functions of bounded type from $C(K)$ to $\mathbb{C}$ (cf. [4]). These authors noticed that the characterizations obtained for orthogonally additive n-homogeneous polynomial can not be expected for orthogonally additive holomorphic functions of bounded type from $C(K)$ to $\mathbb{C}$. They actually show that there is no entire function $\Phi : \mathbb{C} \to \mathbb{C}$ such that the mapping $h : C(K) \to C(K)$, $h(f) = \Phi \circ f$ factors all degree-2 orthogonally additive scalar polynomials over $C(K)$. In the same paper, the authors quoted above gave an alternative characterization of all orthogonally additive scalar holomorphic functions of bounded type over $C(K)$-spaces.

We recall that, given a Borel regular measure μ on a compact Hausdorff space K, a holomorphic function h in $\mathscr{H}_b(C(K), L_1(\mu))$ is a *power series function* if there exists a sequence $(g_k)_k \subseteq L_1(\mu)$ such that

$$h(x) = \sum_{k=0}^{\infty} g_k\, x^k, \ (x \in C(K)).$$

The following theorem is due to D. Carando, S. Lassalle and I. Zalduendo.

THEOREM 2. [4, Theorem 3.3] *Let K be a compact Hausdorff space. A holomorphic function of bounded type $f : C(K) \to \mathbb{C}$ is orthogonally additive if and only if there exist a Borel regular measure μ on K and a power series function $h \in \mathcal{H}_b(C(K), L_1(\mu))$ such that*

$$f(x) = \int_K h(x)\, d\mu,$$

for every $x \in C(K)$.

The above theorem can be read as follows: A holomorphic function of bounded type $f : C(K) \longrightarrow \mathbb{C}$ is orthogonally additive if and only if there exist a Borel regular measure μ on K and a power series function $h \in \mathcal{H}_b(C(K), L_1(\mu))$ such that

$$f(x) = \langle 1_{C(K)}, h(x) \rangle, \qquad x \in C(K).$$

Recently, J.A. Jaramillo, A. Prieto and I. Zalduendo introduced and studied "locally" orthogonally additive holomorphic functions defined on an open subset of $C(K)$ (see [6]).

In this note we study orthogonally additive scalar holomorphic functions of bounded type on a general C*-algebra. In our main result (see Theorem 5) we prove that a scalar holomorphic function of bounded type f from a C*-algebra A is orthogonally additive on A_{sa} if and only if there exist a positive functional φ in A^*, a sequence (ψ_n) in $L_1(A^{**}, \varphi)$ and a power series holomorphic function h in $\mathcal{H}_b(A, A^*)$ such that

$$h(a) = \sum_{k=1}^{\infty} \psi_k \cdot a^k \text{ and } f(a) = \langle 1_{A^{**}}, h(a) \rangle = \int h(a)\, d\varphi,$$

for every a in A, where $1_{A^{**}}$ denotes the unit element in A^{**} and $L_1(A^{**}, \varphi)$ is one of the non-commutative L_1-spaces studied, for example, by H. Kosaki in [7].

2. Holomorphic mappings on general C*-algebras

In this section we shall study orthogonally additive complex-valued holomorphic functions of bounded type on a general C*-algebra.

The following lemma was essentially obtained in [4].

LEMMA 3. *Let $f : A \longrightarrow X$ be a holomorphic function of bounded type from a C^*-algebra to a complex Banach space and let $f = \sum_{k=0}^{\infty} P_k$ be its Taylor series at zero. Then, f is orthogonally additive (respectively, orthogonally additive on A_{sa} or additive on elements having zero-product) if, and only if, all the P_k's satisfy the same property. In such a case, $P_0 = 0$.*

Proof. The proof of [4, Lemma 1.1] remains valid here. □

Before dealing with the main theorem of this section, we shall recall some technical results on non-commutative $L_p(\varphi)$ spaces which are needed later. The construction presented here is inspired by those results established by H. Kosaki and G. Pisier in [7, §3] and [10, §3], respectively.

For each element x in a C*-algebra, A, the *(Jordan) modulus* of x is defined by

$$|x| := \left(\frac{x^*x + xx^*}{2}\right)^{\frac{1}{2}}.$$

We shall denoted by A_+ the set of all positive elements in A.

Let φ be a positive functional in A^*. We may equip A with a scalar product defined by

$$(x,y)_\varphi = \varphi\left(\frac{x^*y + yx^*}{2}\right), (x,y \in A).$$

Let $N = N_\varphi := \{x \in A \ : \ \varphi(|x|^2) = 0\}$. Then, N is a norm-closed subspace of A. Let A^0 denote the Banach space A/N equipped with the quotient norm. The space $(A/N, (.,.)_\varphi)$ is a prehilbert space. The completion of $(A/N, (.,.)_\varphi)$ is a Hilbert space which shall be denoted by $L_2(A,\varphi)$ or simply by $L_2(\varphi)$.

Let $J_\varphi : A^0 \hookrightarrow L_2(\varphi)$ denote the natural embedding. It is clear that J_φ is a continuous operator which has norm dense range and $\|J_\varphi\| \leqslant \|\varphi\|^{\frac{1}{2}}$. Considering the injection $\iota = J_\varphi^* \circ J_\varphi : A^0 \longrightarrow (A^0)^* \subseteq A^*$, the space $L_1(A,\varphi) = L_1(\varphi)$ is defined as the norm closure of $\iota(A^0)$ in A^*.

It should be also mentioned here that, for each $x + N \in A^0$

$$\iota(x+N) = \frac{x\cdot\varphi + \varphi\cdot x}{2}, \tag{1}$$

where $x\cdot\varphi$, $\varphi\cdot x$ defined as elements in A^* are given by

$$x\cdot\varphi(y) = \varphi(yx) \quad \text{and} \quad \varphi\cdot x(y) = \varphi(xy),$$

(compare [10, page 124]).

When A is a von Neumann algebra and φ is a positive element in the predual of A, for each element $a+N$ in A^0, we have $\iota(a+N) = \frac{a\cdot\varphi+\varphi\cdot a}{2} \in A_*$. Since A_* is a norm closed subspace of A^*, we have $L_1(A,\varphi) \subseteq A_*$.

$L_1(\varphi)$ is a closed subspace of A^*, therefore given g in $L_1(\varphi)$, we can compute $g(1_{A^{**}}) = \langle g, 1_{A^{**}}\rangle$, where $1_{A^{**}}$ denotes the unit element in A^{**}. According to the notation employed in the abelian case, we shall write

$$\int g \, d\varphi = \langle g, 1_{A^{**}}\rangle.$$

In order to be consistent with the terminology used in the commutative setting, we shall say that a mapping $h : A \to A^*$ is a *power series* function if $h(a) = \sum_{k=0}^\infty g_k \cdot a^k$ or

$h(a)=\sum_{k=0}^{\infty} a^k \cdot g_k$, for every $a\in A$, where (g_k) is a sequence in A^*. Clearly, every power series function h is a holomorphic function of bounded type.

The following generalisation of Radon-Nikodym theorem, due to S. Sakai, will be needed later.

THEOREM 4. [11, Proposition 1.24.4] *Let $\mathscr{M}$ be a von Neumann algebra, and let φ,ψ be two normal positive linear functionals on $\mathscr{M}$ such that $\psi\leqslant\varphi$. Then, there exists a positive element t_0 in $\mathscr{M}$, with $0\leqslant t_0\leqslant 1$, such that*

$$\psi(x)=\frac{t_0\cdot\varphi+\varphi\cdot t_0}{2}\,(x)=\frac{1}{2}\,\varphi(t_0\,x+x\,t_0),\ \textit{for every}\ x\in\mathscr{M}.$$

Now, we are ready to state the main result of this section.

THEOREM 5. *Let $f:A\longrightarrow\mathbb{C}$ be a holomorphic function of bounded type defined on a C^*-algebra. Then the following are equivalent:*

(a) *f is orthogonally additive on A_{sa}.*

(b) *f is additive on elements having zero-product.*

(c) *There exist a positive functional φ in A^* and a power series holomorphic function h in $\mathscr{H}_b(A,A^*)$ such that*

$$f(a)=\langle 1_{A^{**}},h(a)\rangle=\int h(a)\,d\varphi,$$

for every a in A, where $1_{A^{**}}$ denotes the unit element in A^{**}.

Proof. The implications $(c)\Rightarrow(b)\Rightarrow(a)$ are clear. We shall prove $(a)\Rightarrow(c)$.

Let $f=\sum_{k=1}^{\infty}P_k$ be the Taylor series expression of f at zero. Since f is orthogonally additive on A_{sa}, by Lemma 3, each k-homogeneous polynomial P_k is orthogonally additive on A_{sa}. Thus, by Theorem 1 (c.f. [8]), for each natural k, there exists $\varphi_k\in A^*$ such that

$$P_k(a)=\varphi_k(a^k)\quad(a\in A),$$

with

$$\|P_k\|\leqslant\|\varphi_k\|\leqslant 2\|P_k\|,\quad\forall k\in\mathbb{N}.$$

Let us write each φ_k in the form

$$\varphi_k=(\varphi_{k,1}-\varphi_{k,3})+i(\varphi_{k,2}-\varphi_{k,4})$$

where $\varphi_{k,j}\in(A^*)^+$, for $j=1,\ldots,4$, $k\in\mathbb{N}$.

Since $\|\varphi_{k,j}\|\leqslant\|\varphi_k\|$ for all $j=1,\ldots,4$ and $k\in\mathbb{N}$, and the series $\sum_{k=0}^{\infty}\|P_k\|\,\lambda^k$ has infinite radius of convergence, the expression

$$\varphi=\sum_{k=1}^{\infty}\sum_{j=1}^{4}\varphi_{k,j}$$

defines a positive functional in A^*.

Since, trivially, $\varphi_{k,j} \leqslant \varphi$, for each $j = 1,...,4$ and $k \in \mathbb{N}$, by Theorem 4 (c.f. [11, Theorem 1.24.4]) applied to the von Neumann algebra A^{**}, for each $j = 1,...,4$ and $k \in \mathbb{N}$, there exist $0 \leqslant t_{k,j} \leqslant 1$ in A^{**} such that

$$\varphi_{k,j}(a) = \varphi\left(\frac{t_{k,j}\, a + a\, t_{k,j}}{2}\right), \ (a \in A^{**}).$$

Let us consider the space $L_1(A^{**}, \varphi)$ and the natural embedding

$$A/N \hookrightarrow A^{**}/\overline{N}^{w^*} \stackrel{\iota}{\longrightarrow} L_1(A^{**}, \varphi),$$

where $N = \{x \in A \ : \ \varphi(|x|^2) = 0\}$ and $\overline{N}^{w^*} = \{x \in A^{**} \ : \ \varphi(|x|^2) = 0\}$. When (1) is applied to φ, considered as a functional on A^{**}, gives

$$\iota(a + \overline{N}^{w^*}) = \frac{a \cdot \varphi + \varphi \cdot a}{2} \ \ (a \in A^{**}).$$

For each $(j,k) \in \{1,\ldots,4\} \times \mathbb{N}$, $\iota(t_{k,j} + \overline{N}^{w^*})$ is positive in A^*, and we have

$$\begin{aligned} \|\iota(t_{k,j} + \overline{N}^{w^*})\|_{L_1(\varphi)} &= \langle \iota(t_{k,j} + \overline{N}^{w^*}), 1_{A^{**}} \rangle = \iota(t_{k,j} + \overline{N}^{w^*})(1_{A^{**}}) \\ &= \varphi(t_{k,j}) = \|\varphi_{k,j}\|_{A^*}. \end{aligned}$$

Let us define $h : A \longrightarrow L_1(A^{**}, \varphi) \cdot A \subseteq A^*$ by

$$h(a) = \sum_{k=1}^{\infty} \iota(t_k + \overline{N}^{w^*}) \cdot a^k,$$

where, for each natural k, $t_k = (t_{k,1} - t_{k,3}) + i(t_{k,2} - t_{k,4})$ and for each element $x \in A$ and a functional $\psi \in A^*$, $\psi \cdot x$ denotes the element in A^* defined as $\psi \cdot x(y) = \psi(xy)$. It should be noticed here that $L_1(A^{**}, \varphi) \cdot A$ need not be, in general, a subset of $L_1(A^{**}, \varphi)$; we can only guarantee that $L_1(A^{**}, \varphi) \cdot A \subseteq A^*$.

In order to see that h is well defined, let us estimate

$$\begin{aligned} \sum_{k=1}^{\infty} \left\| \iota(t_k + \overline{N}^{w^*}) \cdot a^k \right\|_{A^*} &\leqslant \sum_{k=1}^{\infty} \sum_{j=1}^{4} \left\| \iota(t_{k,j} + \overline{N}^{w^*}) \right\|_{L_1(\varphi)} \ \|a\|_A^k \\ &= \sum_{k=1}^{\infty} \sum_{j=1}^{4} \|\varphi_{k,j}\|_{A^*} \ \|a\|^k \\ &\leqslant 4 \sum_{k=1}^{\infty} \|\varphi_k\|_{A^*} \ \|a\|^k \leqslant 8 \sum_{k=1}^{\infty} \|P_k\| \ \|a\|^k < \infty \end{aligned}$$

for every a in A, where in the last inequality we applied that $\lim_{k\to\infty} \|P_k\|^{\frac{1}{k}} = 0$.

This tell us that h is a well defined power series holomorphic function of bounded type. It follows from the construction that

$$\begin{aligned} f(a) &= \sum_{k=1}^{\infty} P_k(a) = \sum_{k=1}^{\infty} \varphi_k(a^k) = \sum_{k=1}^{\infty} [(\varphi_{k,1} - \varphi_{k,3}) + i(\varphi_{k,2} - \varphi_{k,4})]\ (a^k) \\ &= \sum_{k=1}^{\infty}\sum_{j=1}^{4} i^{(j-1)}\ \varphi\left(\frac{t_{k,j}\ a^k + a^k\ t_{k,j}}{2}\right) = \sum_{k=1}^{\infty}\sum_{j=1}^{4} i^{(j-1)}\ \frac{t_{k,j}\cdot\varphi + \varphi\cdot t_{k,j}}{2}\ (a^k) \\ &= \sum_{k=1}^{\infty}\sum_{j=1}^{4} i^{(j-1)}\ \imath(t_{k,j} + \overline{N}^{w^*})\ (a^k) = \sum_{k=1}^{\infty} \imath(t_k + \overline{N}^{w^*})\ (a^k) \\ &= \langle 1_{A^{**}}, \sum_{k=1}^{\infty} \imath(t_k + \overline{N}^{w^*}) \cdot a^k\rangle = \langle 1_{A^{**}}, h(a)\rangle = \int h(a)\, d\varphi. \quad \square \end{aligned}$$

Let A be a C*-algebra. We have proved that every scalar holomorphic function of bounded type on A which is orthogonally additive on A_{sa} factors through the norm-closure of $L_1(A^{**},\varphi)\cdot A$ in A^*, where φ is a suitable positive functional in A^*. Under some additional hypothesis, we shall prove that we can get a factorization through a non-commutative L_1 space.

Let $\mathscr{M}$ be a semi-finite von Neumann algebra with a faithful semi-finite normal trace τ (cf. [13, Theorem 2.15]). Let

$$\mathfrak{m}_\tau = \{xy\ :\ \tau(|x|^2), \tau(|y|^2) < \infty\},$$

then $\mathfrak{m}_\tau$ is a two-sided ideal of $\mathscr{M}$, called the *definition ideal* of the trace τ. The assignment $x \mapsto \|x\|_1 := \tau(|x|)$ defines a norm on $\mathfrak{m}_\tau$. Actually, the space $(\mathfrak{m}_\tau, \|x\|_1)$ can be identified with a subspace of $\mathscr{M}_*$ via the following norm-one bilinear form

$$\mathscr{M} \times \mathfrak{m}_\tau \to \mathbb{C} \tag{2}$$

$$(a,x) \mapsto \tau(ax).$$

For each $x \in \mathfrak{m}_\tau$, the symbol ω_x will denote the functional defined by $\omega_x(y) := \tau(xy)$. $L_1(\mathscr{M},\tau)$ is defined as the completion of $(\mathfrak{m}_\tau, \|\cdot\|_1)$. It is also known that, for each $x \in \mathfrak{m}_\tau$, $\|x\|_1 = \sup\{|\tau(yx)| : y \in \mathscr{M}, \|y\| \leqslant 1\}$. The bilinear form defined in (2) extends to a norm-one bilinear form on $\mathscr{M} \times L_1(\mathscr{M},\tau)$, and $L_1(\mathscr{M},\tau)$ is isometrically isomorphic to the predual $\mathscr{M}_*$ (cf. [13, Pages 319-321]). In the literature, the von Neumann algebra $\mathscr{M}$ is usually denoted by $L_\infty(\mathscr{M},\tau)$.

This notation is coherent with the one used before. When τ is a normal faithful finite trace on $\mathscr{M}$ then there exists a central (i.e. $\varphi(xy) = \varphi(yx)$ for every $x,y \in M$), positive and faithful functional φ in $\mathscr{M}_*$ such that $\tau = \varphi|_{M^+}$. In this case, the spaces $L_1(\mathscr{M},\tau)$ and $L_1(\mathscr{M},\varphi)$ coincide.

THEOREM 6. *Let A be a C^*-algebra such that A^{**} is a semi-finite von Neumann algebra and let $f : A \longrightarrow \mathbb{C}$ be a holomorphic function of bounded type. Suppose that τ is a faithful semi-finite normal trace on A^{**}. Then f is orthogonally additive on A_{sa}*

if, and only if, there exists a power series holomorphic function $h : A \to L_1(A^{**}, \tau)$ *such that*

$$f(a) = \langle h(a), 1_{A^{**}} \rangle = \int h(a)\, d\tau$$

for every a *in* A*, where* $1_{A^{**}}$ *denotes the unit element in* A^{**}.

Proof. Let $f = \sum_{k=1}^{\infty} P_k$ be the Taylor series of f at zero. Since f is orthogonally additive on A_{sa}, by Lemma 3, each k-homogeneous polynomial P_k is orthogonally additive on A_{sa}. Thus, by Theorem 1 (c.f. [8]), for each natural k, there exists $\varphi_k \in A^*$ such that

$$P_k(a) = \varphi_k(a^k) \quad (a \in A),$$

with

$$\|P_k\| \leqslant \|\varphi_k\| \leqslant 2\|P_k\|, \quad \forall k \in \mathbb{N}.$$

We have already mentioned that $(A^{**})_* = A^* = L_1(A^{**}, \tau)$. Since for each $k \in \mathbb{N}$, $\varphi_k \in A^*$, by construction, $\varphi_k = g_k \in L_1(A^{**}, \tau)$, with $\|g_k\|_1 = \|\varphi_k\|_{A^*}$.

Given $a \in A$ and $g \in L_1(A^{**}, \tau)$, there exists a sequence $(y_n)_n$ in $\mathfrak{m}_\tau$ such that $\|\omega_{y_n} - g\|_1 \to 0$, then the sequence $(\omega_{y_n a})_n$ is Cauchy in $L_1(A^{**}, \tau)$. The limit of $(\omega_{y_n a})_n$ is denoted by $g \cdot a$. Further, it is not hard to see that $g \cdot a(x) = g(ax)$, for all $x \in A$. In particular $\|g \cdot a\|_1 \leqslant \|g\|_1\, \|a\|_A$.

Let us define

$$h : A \longrightarrow L_1(A^{**}, \tau)$$

the mapping given by

$$h(a) = \sum_{k=1}^{\infty} g_k \cdot a^k.$$

Since

$$\left\| \sum_{k=1}^{\infty} g_k \cdot a^k \right\|_1 \leqslant \sum_{k=1}^{\infty} \|g_k\|_1\, \|a\|^k = \sum_{k=1}^{\infty} \|\varphi_k\|_{A^*}\, \|a\|^k < \infty,$$

we deduce that h is well defined and $f(a) = \langle h(a), 1_{A^{**}} \rangle = \int h(a)\, d\tau$, for all $a \in A$, where $1_{A^{**}}$ denotes the unit element in A^{**}. □

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REFERENCES

[1] Y. BENYAMINI, S. LASSALLE, AND J. G. LLAVONA, *Homogeneous orthogonally additive polynomials on Banach lattices*, Bull. London Math. Soc. **38**, 3 (2006), 459–469.

[2] M. BURGOS, F. J. FERNÁNDEZ-POLO, J. J. GARCÉ, AND A. M. PERALTA, *Orthogonality preservers Revisited*, Asian-European Journal of Mathematics **2**, 3 (2009), 387–405.

[3] D. CARANDO, S. LASSALLE, AND I. ZALDUENDO, *Orthogonally additive polynomials over $C(K)$ are measures – a short proof*, Integr. equ. oper. theory **56** (2006), 597–602.

[4] D. CARANDO, S. LASSALLE, AND I. ZALDUENDO, *Orthogonally Additive Holomorphic functions of Bounded Type over $C(K)$*, Proc. of the Edinburgh Math. Soc. **53** (2010), 609–618.
[5] S. DINEEN, *Complex Analysis on infinite dimensional Spaces*, Springer, 1999.
[6] J. A. JARAMILLO, A. PRIETO, I. ZALDUENDO, *Orthogonally additive holomorphic functions on open subsets of $C(K)$*, Rev. Mat. Complut. (2010), Pages 1–11. DOI: 10.1007/s13163-010-0055-2. Article in press.
[7] H. KOSAKI, *Applications of the complex interpolation method to a von Neumann algebra: noncommutative L^p -spaces*, J. Funct. Anal. **56**, 1 (1984), 29–78.
[8] C. PALAZUELOS, A. M. PERALTA, AND I. VILLANUEVA, *Orthogonally Additive Polynomials on C^* -Algebras*, Quart. J. Math. **59** (2008), 363–374.
[9] D. PÉREZ, AND I. VILLANUEVA, *Orthogonally additive polynomials on spaces of continuous functions*, J. Math. Anal. Appl. **306** (2005), 97–105.
[10] G. PISIER, *Factorization of operators through $L_{p\infty}$ or L_{p1} and noncommutative generalizations*, Math. Ann. **276**, 1 (1986), 105–136.
[11] S. SAKAI, *C^* -algebras and W^* -algebras*, Springer-Verlag, Berlin, 1971.
[12] K. SUNDARESAN, *Geometry of spaces of homogeneous polynomials on Banach lattices, Ap plied geometry and discrete mathematics*, 571–586, DIMACS Ser. Discrete Math. Theoret. Comput. Sci., 4, Amer. Math. Soc., Prov., RI, 1991.
[13] M. TAKESAKI, *Theory of operator algebras, I*, Springer-Verlag, New York-Heidelberg, 1979.

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