# UNIVERSITY OF CATANIA - Department of Mathematics and Computer Sciences 

Master degree in Mathematics - May $23^{\text {rd }} 2022$

Exercise 1. Prove that

$$
\int_{\Omega}|D u|^{2} d x \leq c\left(\int_{\Omega}|u|^{2} d x\right)^{1 / 2}\left(\int_{\Omega}\left|D^{2} u\right|^{2} d x\right)^{1 / 2} \quad \forall u \in W^{2,2}(\Omega) \cap W_{0}^{1,2}(\Omega)
$$

where $c$ is a constant that does not depend on $u$.
Exercise 2. Write down Euler-Lagrange equation for

$$
\int_{\Omega}\left[\left(\frac{\partial u}{\partial x}\right)^{2}+x^{2}\left(\frac{\partial u}{\partial y}\right)^{2}\right] d x d y
$$

Exercise 3. Give an example of a domain where Harnack inequality for equation

$$
u_{x x}+x^{2} u_{y y}=0 \quad \text { in } \mathbb{R}^{2} .
$$

is true.

Exercise 4. Find all real $\alpha$ such that

$$
u(x)=|x|^{\alpha}
$$

belongs to $W^{1,2}\left(B_{1}(0)\right)$ where $B_{1}(0)$ is the unit ball in $\mathbb{R}^{n}$.

