UNIVERSITY OF CATANIA – Department of Mathematics and Computer Sciences

Master degree in Mathematics - May 6^{th} 2020

Exercise 1. Let $u \in C^\alpha(\Omega)$ and $v \in C^\beta(\Omega)$. Prove that $w = \min(u,v) \in C^\gamma(\Omega)$ where $\gamma = \min(\alpha, \beta)$.

Exercise 2. Let $\{g_\varepsilon\}$, $\{h_\varepsilon\}$ for any $\varepsilon > 0$, defined by

$$g_\varepsilon(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < -\varepsilon \text{ or } x > 2\varepsilon \\ 0 & \text{otherwise} \end{cases} \qquad h_\varepsilon(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < -\varepsilon \text{ or } x > \varepsilon^2 \\ 0 & \text{otherwise} \end{cases}$$

Discuss the limits in $\mathcal{D}'(\mathbb{R})$ as $\varepsilon \to 0$.

Exercise 3. Let Ω be a bounded domain in $\mathbb{R}^n$ and let $c(x)$ be a continuous function in $\bar{\Omega}$ such that $c(x) \leq 0$ in Ω. Prove that any solution u to equation

$$-\Delta u + cu = 0 \qquad \forall x \in \Omega$$

satisfy Maximum principle. What if $c(x) \geq 0$?