UNIVERSITY OF CATANIA – Department of Mathematics and Computer Sciences

Master degree in Mathematics - April $18^{th}$ 2018

**Exercise 1.** Let $\Omega$ be a bounded domain in $\mathbb{R}^n$ and let $\{f_j\}$ be harmonic in $\Omega$ for any $j$. Assume $\{f_j\}$ converges uniformly to $f$ in any ball $B \subset\subset \Omega$. Show that $f$ is harmonic in $\Omega$. What about unbounded domains ?

**Exercise 2.** Let $\Omega$ be a bounded domain in $\mathbb{R}^n$ and let $\{f_j\}$ be harmonic in $\Omega$ for any $j$. Assume that

(H1) $f_j(x) \leq f_{j+1}(x) \qquad \forall j \in \mathbb{N} \quad \forall x \in \Omega\,.$

(H2) $\{f_j\}$ converges to $f$ in $\Omega$.

Show $f$ is harmonic in $\Omega$. Is it possible to replace (H2) by

(H3) There exists $x_0 \in \Omega \ : \ f_j(x_0)$ converges?

**Exercise 3.** Evaluate the limit of the sequence $f_n(x) = (1 - n|x|)^+$ in $\mathcal{D}(\mathbb{R})$.

**Exercise 4.** Evaluate the derivative of $(-1)^{[x]}$ in $\mathbb{R}$.

**Exercise 5.** Let $\psi$ a $C^\infty(\mathbb{R})$ function and let $T$ be a distribution in $\mathbb{R}$. Evaluate the derivative of $\psi T$.

**Exercise 6.** Evaluate the limit of

$$f_j(x) = \begin{cases} \dfrac{1}{x} & \text{se } |x| > 1/j \\ j & \text{se } 0 < x < 1/j \\ -j & \text{se } -1/j < x < 0 \end{cases}$$

in $\mathcal{D}(\mathbb{R})$.