DEFINIZIONE PER CASI

- SIANO $M_1(\vec{x}), \dots, M_k(\vec{x})$ PREDICATI **DECIDIBILI** TALI CHE
  - $M_1(\vec{x}) \vee \dots \vee M_k(\vec{x}) \equiv \underline{true}, \quad \forall \vec{x}$
  - $M_i(\vec{x}) \wedge M_j(\vec{x}) \equiv \underline{false}, \quad \forall \vec{x} \quad \forall i \neq j$

  CIOE', PER OGNI $\vec{x}$, UNO ED UNO SOLO TRA $M_1(\vec{x}), \dots, M_k(\vec{x})$ E' VERO.

- SIANO $f_1(\vec{x}), \dots, f_k(\vec{x})$ FUNZIONI CALCOLABILI (ANCHE PARZIALI)

  [CON $\vec{x} = (x_1, x_2, \dots, x_m)$]

NELLE IPOTESI PRECEDENTI, LA FUNZIONE

$$g(\vec{x}) =_{df} \begin{cases} f_1(\vec{x}) & \text{SE} \quad M_1(\vec{x}) \\ f_2(\vec{x}) & \text{SE} \quad M_2(\vec{x}) \\ \vdots & \vdots \quad\quad \vdots \\ f_k(\vec{x}) & \text{SE} \quad M_k(\vec{x}) \end{cases}$$

DEFINITA PER CASI E' CALCOLABILE

DIM. SIANO $e_1, e_2, \dots, e_k$ TALI CHE

$$f_1 = \phi^{(m)}_{e_1}$$

$$f_2 = \phi^{(m)}_{e_2}$$

$$\vdots$$

$$f_k = \phi^{(m)}_{e_k}$$

SUPPONIAMO CHE $g(\vec{c})\downarrow$, CON $\vec{c}=(c_1,c_2,\dots,c_m)$.

ALLORA PER QUALCHE i

- $M_i(\vec{c})=true$ (E QUINDI $M_j(\vec{c})=false,\ \forall j\neq i$)
- $f_i(\vec{c})\downarrow$ (E QUINDI $(\exists y)(\exists t)\, S_m(e_i,\vec{c},y,t)$)
- $g(\vec{c})=f_i(\vec{x})$

SI HA: $(\exists y)(\exists t)\, S_m(e_i,\vec{c},y,t)$

$$\leftrightarrow (\exists w)\, S_m(e_i,\vec{c},(w)_1,(w)_2)$$

$$\leftrightarrow (\exists w)\,(S_m(e_i,\vec{c},(w)_1,(w)_2)$$

$$\leftrightarrow (\mu w)\,(S_m(e_i,\vec{c},(w)_1,(w)_2))\downarrow$$

POSTO $\overline{w} = (\mu w)\left(S_m(e_i, \vec{c}, (w)_1, (w)_2)\right)$, SI HA

$$S_m(e_i, \vec{c}, (\overline{w})_1, (\overline{w})_2) = \underline{true}$$

DA CUI $g(\vec{c}) = f_i(\vec{c}) = \phi^{(m)}_{e_i}(\vec{c}) = (\overline{w})_1$

MA POICHÉ $- \; M_i(\vec{c}) = \underline{true}$, E

$- \; M_j(\vec{c}) = \underline{false}$, $\forall j \neq i$

SI HA, PER OGNI $w \in \mathbb{N}$,

$$S_m(e_i, \vec{c}, (w)_1, (w)_2)$$
$$\Leftrightarrow \left(S_m(e_1, \vec{c}, (w)_1, (w)_2) \wedge M_1(\vec{c})\right) \vee \ldots$$
$$\ldots \vee \left(S_m(e_k, \vec{c}, (w)_1, (w)_2) \wedge M_k(\vec{c})\right)$$

E QUINDI

$$g(\vec{c}) = (\overline{w})_1 = \left((\mu w)\left(S_m(e_i, \vec{c}, (w)_1, (w)_2)\right)\right)_1$$
$$= \left((\mu w)\left(\left(S_m(e_1, \vec{c}, (w)_1, (w)_2) \wedge M_1(\vec{c})\right) \vee \ldots\right.\right.$$
$$\left.\left.\ldots \vee \left(S_m(e_k, \vec{c}, (w)_1, (w)_2) \wedge M_k(\vec{c})\right)\right)\right)_1$$

D'ALTRA PARTE, SE $g(\vec{c})\uparrow$, ALLORA PER QUALCHE i

- $M_i(\vec{c}) = true$ (E QUINDI $M_j(\vec{c}) = false$, $\forall j \neq i$)
- $f_i(\vec{c})\uparrow$ (E QUINDI $(\exists y)(\exists t)\, S_m(e_i, \vec{c}, y, t) = \underline{false}$)

MA ALLORA

$$\underline{false} = (\exists y)(\exists t)\, S_m(e_i, \vec{c}, y, t)$$

$$\leftrightarrow (\exists w)\, S_m(e_i, \vec{c}, (w)_1, (w)_2)$$

$$\leftrightarrow (\exists w)\, \big(S_m(e_1, \vec{c}, (w)_1, (w)_2) \land M_1(\vec{c})\big) \lor \ldots$$
$$\ldots \lor \big(S_m(e_k, \vec{c}, (w)_1, (w)_2) \land M_k(\vec{c})\big)$$

E QUINDI $\Big((\mu w)\Big(\big(S_m(e_1, \vec{c}, (w)_1, (w)_2) \land M_1(\vec{c})\big) \lor \ldots$
$\ldots \lor \big(S_m(e_k, \vec{c}, (w)_1, (w)_2) \land M_k(\vec{c})\big)\Big)\Big)_1 \uparrow$

PERTANTO SI HA, PER OGNI $\vec{x}$,

$$g(\vec{x}) = \Big((\mu w)\big((S_m(e_1, \vec{x}, (w)_1, (w)_2) \wedge M_1(\vec{x})) \vee \dots$$

$$\dots \vee (S_m(e_k, \vec{x}, (w)_1, (w)_2) \wedge M_k(\vec{x}))\big)\Big)_1$$

DA CUI SEGUE LA CALCOLABILITÀ DELLA FUNZIONE $g(\vec{x})$. $\blacksquare$