Example

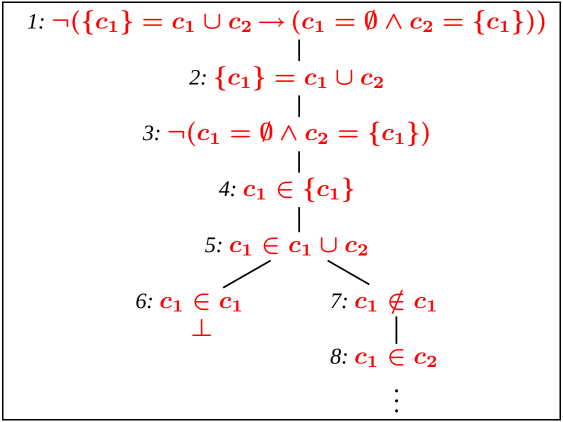
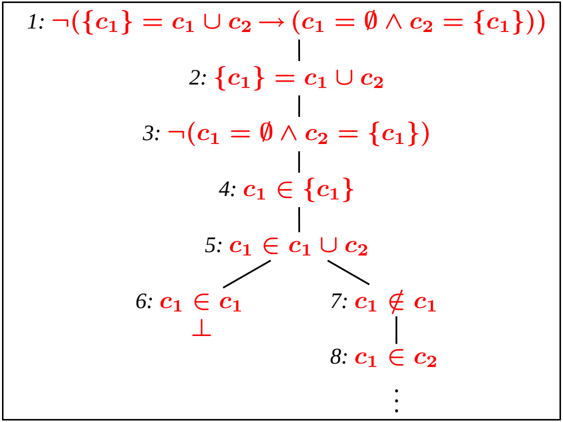


Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of
$\{c_1\} = c_1 \cup c_2 \longrightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$

Example



1: $\neg(\{c_1\} = c_1 \cup c_2 \longrightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\}))$

2: $\{c_1\} = c_1 \cup c_2$

3: $\neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$

4: $c_1 \in \{c_1\}$

5: $c_1 \in c_1 \cup c_2$

6: $c_1 \in c_1$

$\perp$

7: $c_1 \notin c_1$

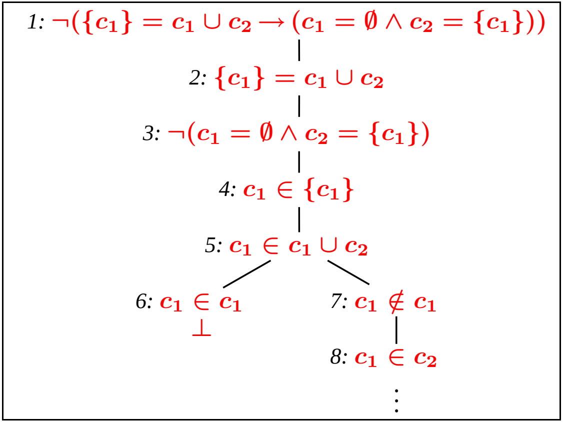
8: $c_1 \in c_2$

⋮

φ_2 and φ_3 are obtained by an application of the α-rule to φ_1

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \longrightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$

Example



1: $\neg(\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\}))$

2: $\{c_1\} = c_1 \cup c_2$

3: $\neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$

4: $c_1 \in \{c_1\}$

5: $c_1 \in c_1 \cup c_2$

6: $c_1 \in c_1$ $\qquad$ 7: $c_1 \notin c_1$

$\perp$ $\qquad$ 8: $c_1 \in c_2$

$\vdots$

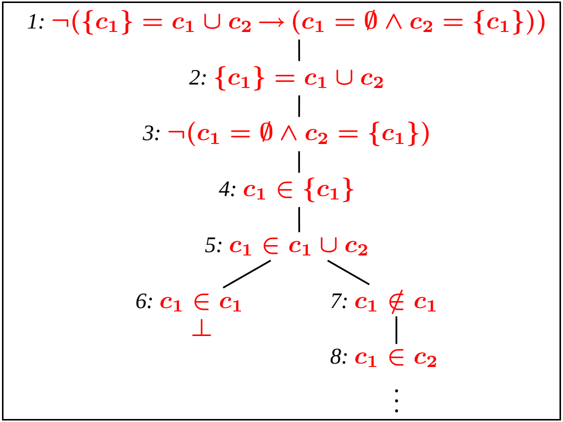
Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$

φ_4 is obtained by an application of the $\{\_\}$-rule

$$\overline{t_1 \in \{t_1\}}$$

under restriction **R1**

Example



1: $\neg(\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\}))$

2: $\{c_1\} = c_1 \cup c_2$

3: $\neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$

4: $c_1 \in \{c_1\}$

5: $c_1 \in c_1 \cup c_2$

6: $c_1 \in c_1$

$\perp$

7: $c_1 \notin c_1$

8: $c_1 \in c_2$

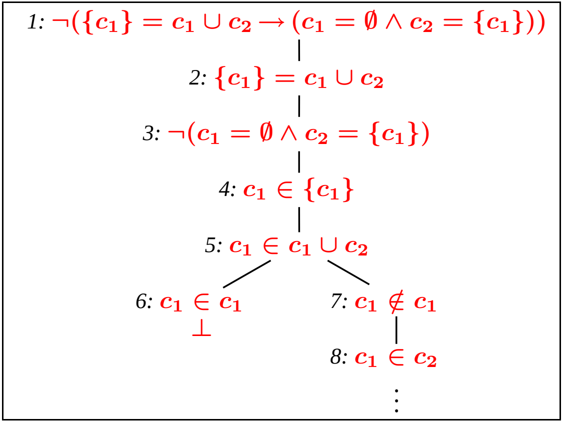
$\vdots$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$

φ_5 is obtained from φ_2 and φ_4 by an application of the $=$-rule

$$\frac{\begin{array}{c} t_1 = t_2 \\ \ell \end{array}}{\ell^{t_1}_{t_2}}$$

Example



1: $\neg(\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\}))$

2: $\{c_1\} = c_1 \cup c_2$

3: $\neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$

4: $c_1 \in \{c_1\}$

5: $c_1 \in c_1 \cup c_2$

6: $c_1 \in c_1$

$\perp$

7: $c_1 \notin c_1$

8: $c_1 \in c_2$

$\vdots$

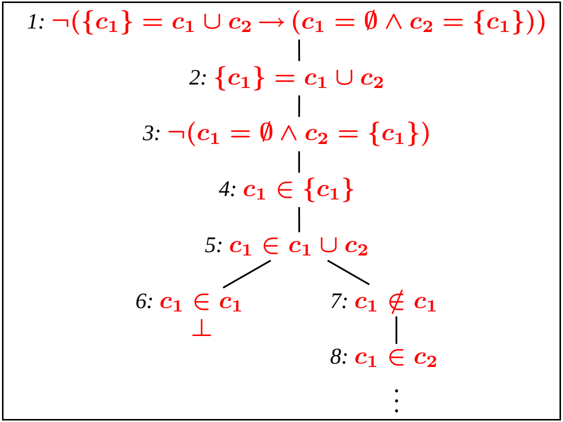
Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$

φ_6 and φ_7 are obtained from the ($\in$)-branching rule

$$\frac{}{s \in t \mid s \notin t}$$

applied to the pair (c_1, c_1), under restriction **R2**

Example



1: $\neg(\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\}))$

2: $\{c_1\} = c_1 \cup c_2$

3: $\neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$

4: $c_1 \in \{c_1\}$

5: $c_1 \in c_1 \cup c_2$

6: $c_1 \in c_1$

$\perp$

7: $c_1 \notin c_1$

8: $c_1 \in c_2$

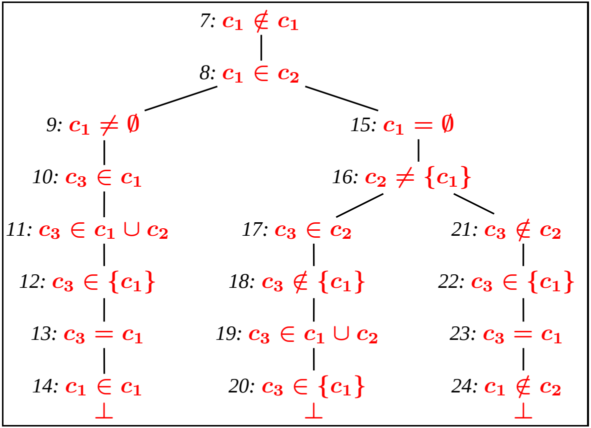
$\vdots$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$

φ_8 is obtained from φ_5 and φ_7 by an application of the $\cup$-rule

$$\frac{\begin{array}{c} s \in t_1 \cup t_2 \\ s \notin t_i \end{array}}{s \in t_{3-i}}$$

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

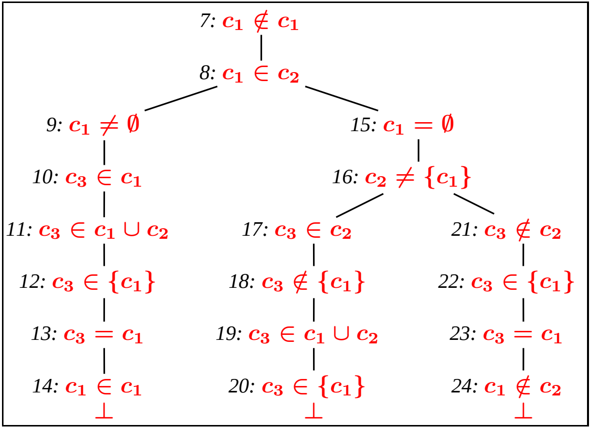
23: $c_3 = c_1$

24: $c_1 \notin c_2$

$\perp$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of
$\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \land c_2 = \{c_1\})$ (cntd)

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

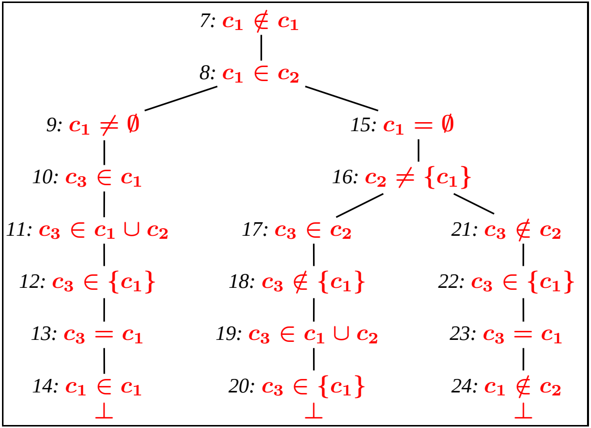
$\perp$

φ_9 and φ_{15} are obtained from φ_3 : $\neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$ by an application of the $\beta 1$-rule

$$\frac{\beta}{\beta_1 \mid \beta_1^c}$$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

$\perp$

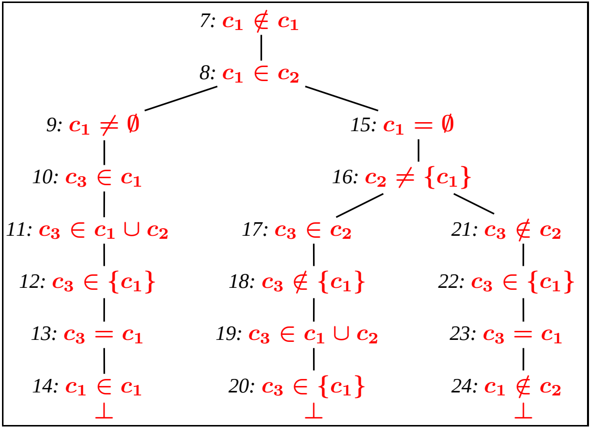
Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{10} is obtained from φ_9 by an application of the following derived rule

$$\frac{y \neq \emptyset}{c \in y} \qquad (\emptyset)$$

where c denotes a *new* set constant.

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

$\perp$

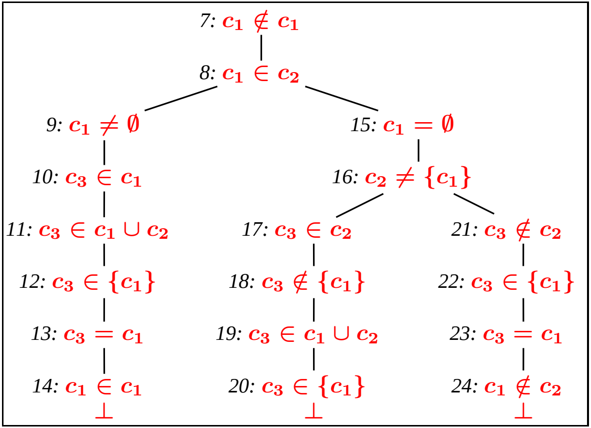
Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{11} is obtained from φ_{10} by an application of the $\cup$-rule

$$\frac{s \in t_i}{s \in t_1 \cup t_2}$$

under restriction **R1**

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$
10: $c_3 \in c_1$
11: $c_3 \in c_1 \cup c_2$
12: $c_3 \in \{c_1\}$
13: $c_3 = c_1$
14: $c_1 \in c_1$
$\bot$

15: $c_1 = \emptyset$
16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$
18: $c_3 \notin \{c_1\}$
19: $c_3 \in c_1 \cup c_2$
20: $c_3 \in \{c_1\}$
$\bot$

21: $c_3 \notin c_2$
22: $c_3 \in \{c_1\}$
23: $c_3 = c_1$
24: $c_1 \notin c_2$
$\bot$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{12} is obtained from $\varphi_2 : \ \{c_1\} = c_1 \cup c_2$ and φ_{11} by an application of the $=$-rule

$$\frac{\begin{array}{c} t_1 = t_2 \\ \ell \end{array}}{\ell^{t_2}_{t_1}}$$

Example (cntd)

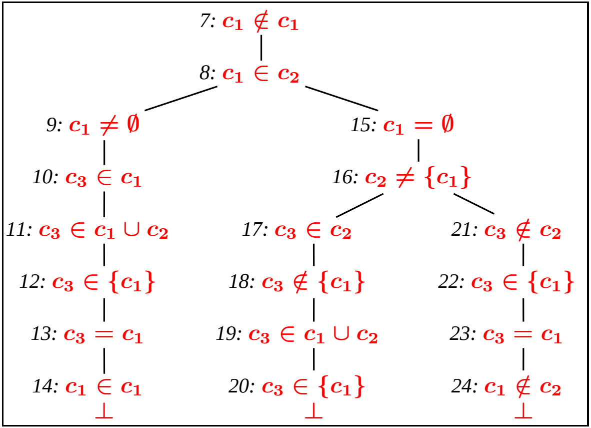
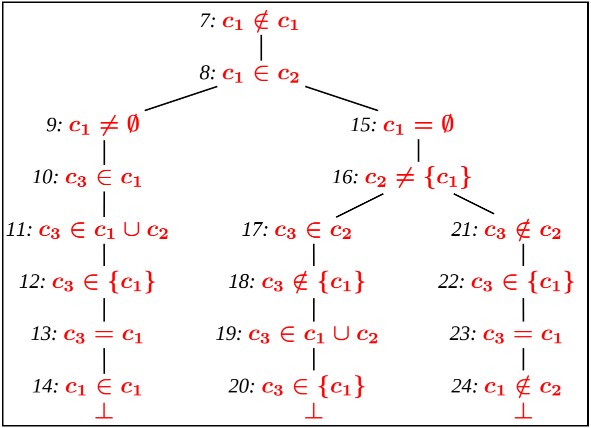


Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{13} is obtained from φ_{12} by an application of the $\{\_\}$-rule

$$\frac{s \in \{t_1\}}{s = t_1}$$

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

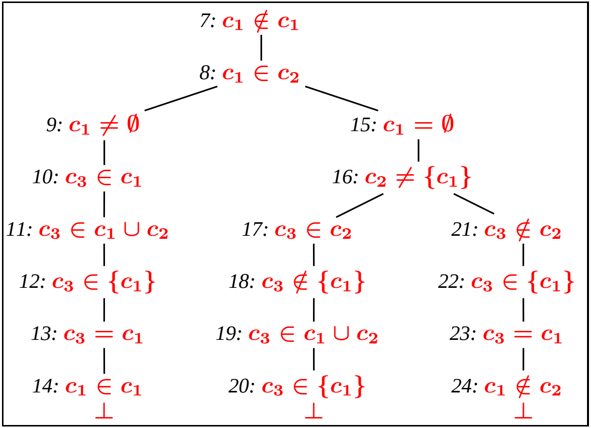
$\perp$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of
$\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{14} is obtained from φ_{13} and φ_{10} by an application of the $=$-rule

$$\frac{\begin{array}{c} t_1 = t_2 \\ \ell \end{array}}{\ell^{t_1}_{t_2}}$$

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

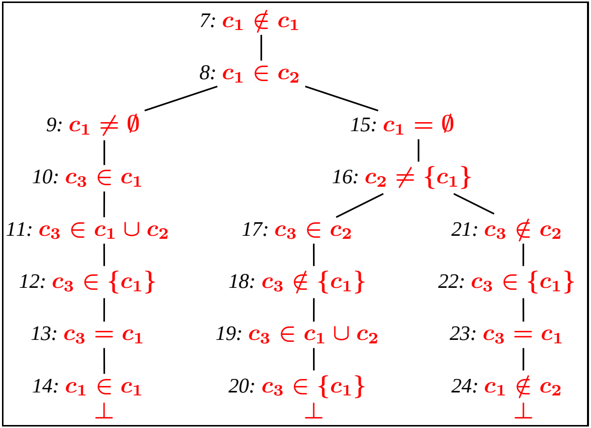
$\perp$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{16} is obtained from $\varphi_3 : \neg(c_1 = \emptyset \wedge c_2 = \{c_1\})$ and φ_{15} by an application of the β-rule

$$\frac{\begin{array}{c}\beta \\ \beta_1^c\end{array}}{\beta_2}$$

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

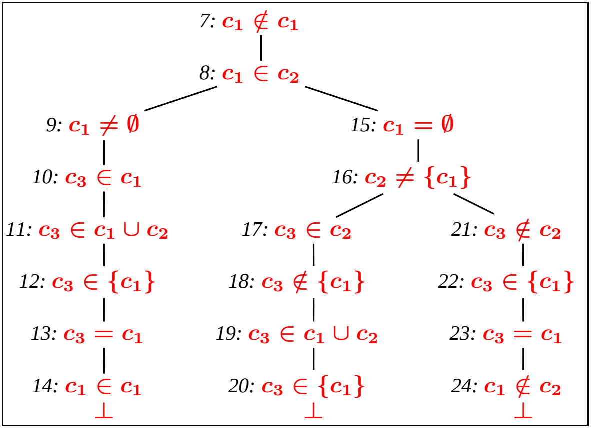
$\perp$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{17}, φ_{18}, φ_{21}, and φ_{22} are obtained from φ_{16} by an application of the **(*ext*)**-branching rule

$$\frac{t_1 \neq t_2}{\begin{array}{c|c} c \in t_1 & c \notin t_1 \\ c \notin t_2 & c \in t_2 \end{array}}$$

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

$\perp$

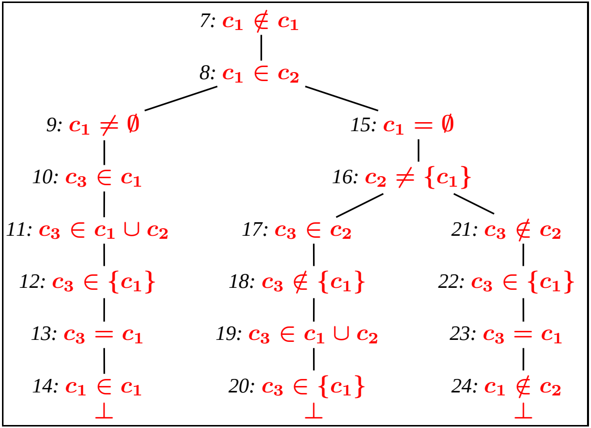
Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{19} is obtained from φ_{17} by an application of the $\cup$-rule

$$\frac{s \in t_i}{s \in t_1 \cup t_2}$$

under restriction **R1**

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

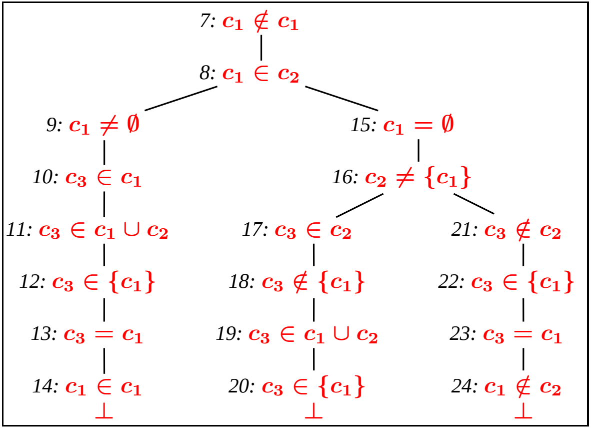
$\perp$

φ_{20} is obtained from $\varphi_2 : \{c_1\} = c_1 \cup c_2$ and φ_{19} by an application of the $=$-rule

$$\frac{\begin{array}{c} t_1 = t_2 \\ \ell \end{array}}{\ell^{t_2}_{t_1}}$$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

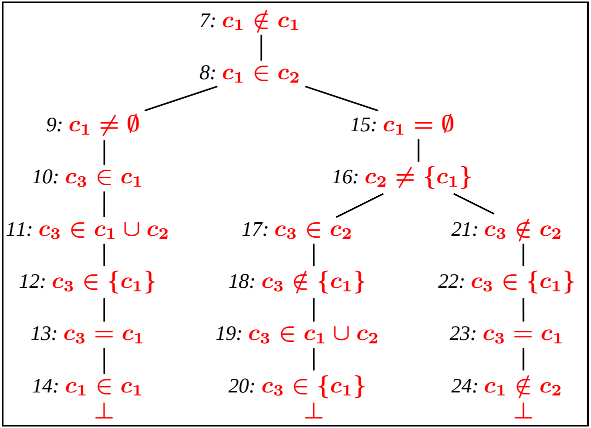
$\perp$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{23} is obtained from φ_{22} by an application of the $\{\_\}$-rule

$$\frac{s \in \{t_1\}}{s = t_1}$$

Example (cntd)



7: $c_1 \notin c_1$

8: $c_1 \in c_2$

9: $c_1 \neq \emptyset$

10: $c_3 \in c_1$

11: $c_3 \in c_1 \cup c_2$

12: $c_3 \in \{c_1\}$

13: $c_3 = c_1$

14: $c_1 \in c_1$

$\perp$

15: $c_1 = \emptyset$

16: $c_2 \neq \{c_1\}$

17: $c_3 \in c_2$

18: $c_3 \notin \{c_1\}$

19: $c_3 \in c_1 \cup c_2$

20: $c_3 \in \{c_1\}$

$\perp$

21: $c_3 \notin c_2$

22: $c_3 \in \{c_1\}$

23: $c_3 = c_1$

24: $c_1 \notin c_2$

$\perp$

Table 1: A $\mathfrak{T}_{\mathrm{MLSS}}$-tableau proof of $\{c_1\} = c_1 \cup c_2 \rightarrow (c_1 = \emptyset \wedge c_2 = \{c_1\})$ (cntd)

φ_{24} is obtained from φ_{23} and φ_{21} by an application of the $=$-rule

$$\frac{\begin{array}{c} t_1 = t_2 \\ \ell \end{array}}{\ell^{t_1}_{t_2}}$$