DEFINIZIONE PER CASI

- SIANO $M_1(\vec{x}), \dots, M_k(\vec{x})$ PREDICATI DECIDIBILI TALI CHE

  - $M_1(\vec{x}) \vee \dots \vee M_k(\vec{x}) \equiv \underline{true}, \quad \forall \vec{x}$

  - $M_i(\vec{x}) \wedge M_j(\vec{x}) \equiv \underline{false}, \quad \forall \vec{x} \quad \forall i \neq j$

  CIOE', PER OGNI $\vec{x}$, UNO ED UNO SOLO TRA $M_1(\vec{x}), \dots, M_k(\vec{x})$ E' VERO.

- SIANO $f_1(\vec{x}), \dots, f_k(\vec{x})$ FUNZIONI CALCOLABILI (ANCHE PARZIALI)

  [CON $\vec{x} = (x_1, x_2, \dots, x_m)$]

NELLE IPOTESI PRECEDENTI, LA FUNZIONE

$$g(\vec{x}) =_{df} \begin{cases} f_1(\vec{x}) & \text{SE} \quad M_1(\vec{x}) \\ f_2(\vec{x}) & \text{SE} \quad M_2(\vec{x}) \\ \vdots & \quad \vdots \qquad \vdots \\ f_k(\vec{x}) & \text{SE} \quad M_k(\vec{x}) \end{cases}$$

DEFINITA PER CASI E' CALCOLABILE

DIM. SIANO $e_1, e_2, \dots, e_k$ TALI CHE

$$f_1 = \phi_{e_1}^{(m)}$$

$$f_2 = \phi_{e_2}^{(m)}$$

$$\vdots$$

$$f_k = \phi_{e_k}^{(m)}$$

SUPPONIAMO CHE $g(\vec{c})\downarrow$, CON $\vec{c} = (c_1, c_2, \ldots, c_m)$.

ALLORA PER QUALCHE i

- $M_i(\vec{c}) = true$ (E QUINDI $M_j(\vec{c}) = false$, $\forall j \neq i$)
- $f_i(\vec{c})\downarrow$ (E QUINDI $(\exists y)(\exists t)\, S_m(e_i, \vec{c}, y, t)$)
- $g(\vec{c}) = f_i(\vec{c})$

SI HA: $(\exists y)(\exists t)\, S_m(e_i, \vec{c}, y, t)$

$$\longleftrightarrow (\exists w)\, S_m(e_i, \vec{c}, (w)_1, (w)_2)$$

$$\longleftrightarrow (\mu w)\,(S_m(e_i, \vec{c}, (w)_1, (w)_2))\downarrow$$

POSTO $\overline{w} = (\mu w)\left(S_m(e_i, \vec{c}, (w)_1, (w)_2)\right)$, SI HA

$$S_m(e_i, \vec{c}, (\overline{w})_1, (\overline{w})_2) = \underline{\text{true}}$$

DA CUI $g(\vec{c}) = f_i(\vec{c}) = \phi_{e_i}^{(m)}(\vec{c}) = (\overline{w})_1$

MA POICHÉ $- M_i(\vec{c}) = \underline{true}$, E

$- M_j(\vec{c}) = \underline{false}$, $\forall j \neq i$

SI HA, PER OGNI $w \in \mathbb{N}$,

$$S_m(e_i, \vec{c}, (w)_1, (w)_2)$$

$$\Leftrightarrow \left(S_m(e_1, \vec{c}, (w)_1, (w)_2) \wedge M_1(\vec{c})\right) \vee \dots$$

$$\dots \vee \left(S_m(e_k, \vec{c}, (w)_1, (w)_2) \wedge M_k(\vec{c})\right)$$

E QUINDI

$$g(\vec{c}) = (\overline{w})_1 = \left((\mu w)\left(S_m(e_i, \vec{c}, (w)_1, (w)_2)\right)\right)_1$$

$$= \left((\mu w)\left(\left(S_m(e_1, \vec{c}, (w)_1, (w)_2) \wedge M_1(\vec{c})\right) \vee \dots\right.\right.$$

$$\left.\left.\dots \vee \left(S_m(e_k, \vec{c}, (w)_1, (w)_2) \wedge M_k(\vec{c})\right)\right)\right)_1$$

D'ALTRA PARTE, SE $g(\vec{c})\uparrow$, ALLORA PER QUALCHE i

- $M_i(\vec{c}) = true$ (E QUINDI $M_j(\vec{c}) = false$, $\forall j \neq i$)
- $f_i(\vec{c})\uparrow$ (E QUINDI $(\exists y)(\exists t)\, S_m(e_i, \vec{c}, y, t) = \underline{false}$)

MA ALLORA

$$(\exists y)(\exists t)\, S_m(e_i, \vec{c}, y, t) = \underline{false}$$

$$\leftrightarrow (\exists w)\, S_m(e_i, \vec{c}, (w)_1, (w)_2) = \underline{false}$$

$$\leftrightarrow (\exists w)\Big(\big(S_m(e_1, \vec{c}, (w)_1, (w)_2) \wedge M_1(\vec{c})\big) \vee \dots$$
$$\dots \vee \big(S_m(e_k, \vec{c}, (w)_1, (w)_2) \wedge M_k(\vec{c})\big)\Big) = \underline{false}$$

$$\leftrightarrow \Big((\mu w)\Big(\big(S_m(e_1, \vec{c}, (w)_1, (w)_2) \wedge M_1(\vec{c})\big) \vee \dots$$
$$\dots \vee \big(S_m(e_k, \vec{c}, (w)_1, (w)_2) \wedge M_k(\vec{c})\big)\Big)\Big)_1 \uparrow$$

PERTANTO SI HA, PER OGNI $\vec{x}$,

$$g(\vec{x}) = \Big(\big(\mu w\big)\Big(\big(S_m(e_1, \vec{x}, (w)_1, (w)_2) \wedge M_1(\vec{x})\big) \vee \ldots$$

$$\ldots \vee \big(S_m(e_k, \vec{x}, (w)_1, (w)_2) \wedge M_k(\vec{x})\big)\Big)\Big)_1$$

DA CUI SEGUE LA CALCOLABILITÀ DELLA FUNZIONE $g(\vec{x})$. $\blacksquare$