ESERCIZIO

Si enuncino il Teorema Master ed il suo Corollorio, quindi si risolva la seguente equazione di ricorrenza al variare del parametro $\alpha \geq 1$:

$$T(n) = \alpha \cdot T\left(\frac{n}{2}\right) + n^2 \log^2 n \,. \qquad (*)$$

Per quali valori di α si ha: (a) $T(n) = \mathcal{O}(n^3)$; (b) $T(n) = \Omega(n^2 \log^3 n)$; (c) $T(n) = \Omega(n^2 \log^4 n)$?

- PER COMINCIARE, RISOLVIAMO L'EQUAZIONE DI RICORRENZA PARAMETRICA (*).

- APPLICANDO DIRETTAMENTE IL COROLLARIO, SI HA:

$$T(n) = \begin{cases} \Theta\left(n^{\lg \alpha}\right) & \text{SE } \lg \alpha > 2 \\ \Theta\left(n^2 (\lg n)^3\right) & \text{SE } \lg \alpha = 2 \\ \Theta\left(n^2 (\lg n)^2\right) & \text{SE } 0 \leq \lg \alpha < 2 \end{cases}$$

POICHÉ

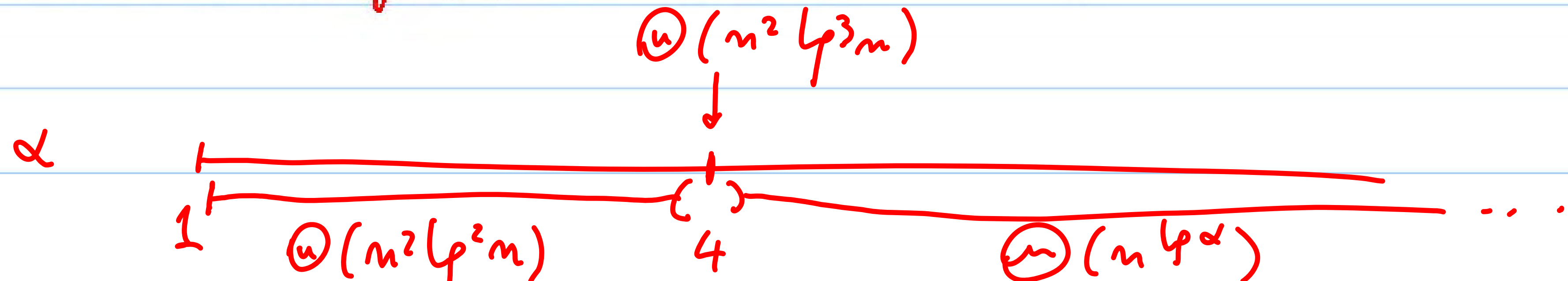
$$\lg \alpha > 2 \iff \alpha > 4$$

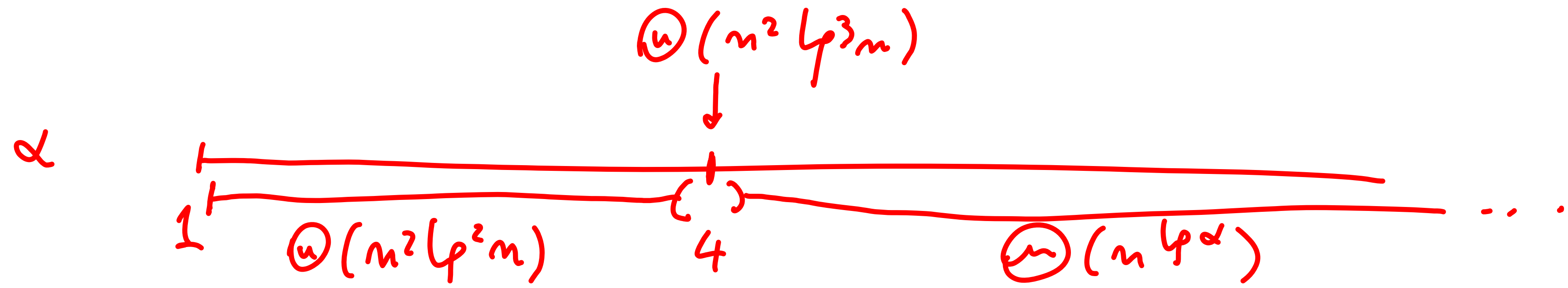
$$\lg \alpha = 2 \iff \alpha = 4$$

$$0 \leq \lg \alpha < 2 \iff 1 \leq \alpha < 4$$

LA SOLUZIONE TROVATA PUÒ ESSERE RISCRITTA COSÌ:

$$T(n) = \begin{cases} \Theta(n^{\lg \alpha}) & \text{SE } \alpha > 4 \\ \Theta(n^2 (\lg n)^3) & \text{SE } \alpha = 4 \\ \Theta(n^2 (\lg n)^2) & \text{SE } 1 \leq \alpha < 4 \end{cases}$$





$$T(n) \overset{?}{=} O(n^3)$$

$$\underbrace{1 \le \alpha < 4, \quad \alpha = 4, \quad 4 < \alpha \le 8}_{1 \le \alpha \le 8}$$

$$\log \alpha \le 3 \iff \alpha \le 2^3 = 8$$

$T(n)$

α

$\Theta(n^2 \log^3 n)$

1 $\Theta(n^2 \log^2 n)$ 4 $\Theta(n \log^{\alpha})$ . . .

$$T(n) = \Omega(n^2 \log^3 n)$$

$$\underbrace{\alpha = 4 \ , \ 4 < \alpha}_{4 \leq \alpha}$$

$\boxed{n^2} \log^3 n$ $\boxed{n^\beta}$ $\beta > 2$

$T(n)$

α

$\Theta(n^2 \log^3 n)$

1 $\quad \Theta(n^2 \log^2 n) \quad$ 4 $\quad \omega(n \log^{\alpha}) \quad \ldots$

$\Omega(n^2 \log^4 n) \quad : \quad 4 < \alpha$