$$\sum_{\Delta=1}^{n-1}\sum_{i=1}^{n-\Delta}\sum_{k=i}^{\Delta+i-1} 1 = \sum_{\Delta=1}^{n-1}\sum_{i=1}^{n-\Delta} \Delta = \sum_{\Delta=1}^{n-1} (n-\Delta)\Delta$$

$$= \sum_{\Delta=1}^{n-1} (n\Delta - \Delta^2) = n\sum_{\Delta=1}^{n-1}\Delta - \sum_{\Delta=1}^{n-1}\Delta^2 = \frac{(n-1)n^2}{2} - \sum_{\Delta=1}^{n-1}\Delta^2$$

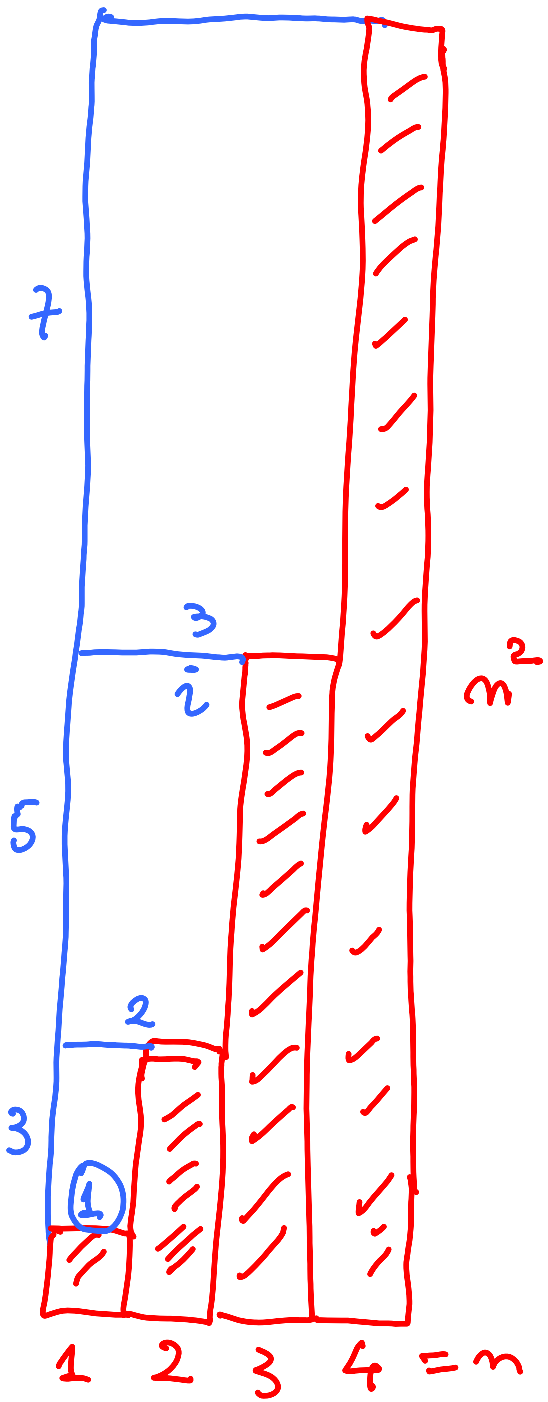
$$= \frac{(n-1)n^2}{2} - \frac{(n-1)n(2n-1)}{6} = \frac{3n^3 \cancel{-3n^2} - 2n^3 \cancel{+3n^2} - n}{6}$$

$$= \frac{n^3-n}{6} = \Omega(n^3)$$

$$O(n^3) \longrightarrow \Theta(n^3)$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$



$$\sum_{i=1}^{n} i^2 = n^3 - \sum_{i=1}^{n-1} i(2i+1)$$

$$= n^3 - 2\sum_{i=1}^{n-1} i^2 - \sum_{i=1}^{n-1} i$$

$$= n^3 - 2\sum_{i=1}^{n} i^2 + 2n^2 - \sum_{i=1}^{n-1} i$$

$$3\sum_{i=1}^{n} i^2 = n^3 + 2n^2 - \frac{(n-1)n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{2n^3 + 4n^2 - n^2 + n}{2 \cdot 3}$$

$$= \frac{2n^3 + 3n^2 + n}{6}$$

$$= \frac{n(2n^2 + 3n + 1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$