

$$E[X] = \sum_{\omega \in S} X(\omega) Pr\{\omega\}$$

$$E[X] = \sum_{x \in \mathbb{R}} x Pr\{X=x\}$$

$$Pr\{X=x\} = \sum_{\substack{\omega \in S \\ X(\omega)=x}} Pr\{\omega\}$$

$$x(a+b) = xa + xb$$

$$= \sum_{x \in \mathbb{R}} x \left( \sum_{\substack{\omega \in S \\ X(\omega)=x}} Pr\{\omega\} \right)$$

$$= \sum_{x \in \mathbb{R}} \left( \sum_{\substack{\omega \in S \\ X(\omega)=x}} x Pr\{\omega\} \right)$$

$$= \sum_{x \in \mathbb{R}} \left( \sum_{\substack{\omega \in S \\ X(\omega)=x}} X(\omega) Pr\{\omega\} \right) = \sum_{\omega \in S} X(\omega) Pr\{\omega\}$$

$$E[X] = \sum_{\omega \in S} X(\omega) \Pr\{\omega\}$$

$$(X+Y)(\omega) = X(\omega) + Y(\omega)$$

$$E[X+Y] = \sum_{\omega \in S} ((X+Y)(\omega)) \Pr\{\omega\}$$

$$= \sum_{\omega \in S} (X(\omega) + Y(\omega)) \Pr\{\omega\}$$

$$= \sum_{\omega \in S} X(\omega) \cdot \Pr\{\omega\} + \sum_{\omega \in S} Y(\omega) \cdot \Pr\{\omega\}$$

$$= E[X] + E[Y]$$