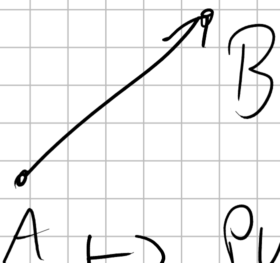
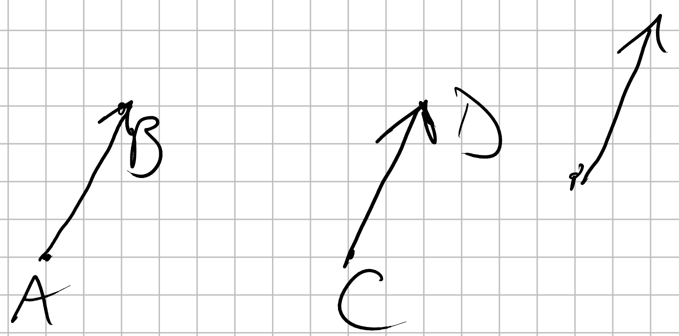
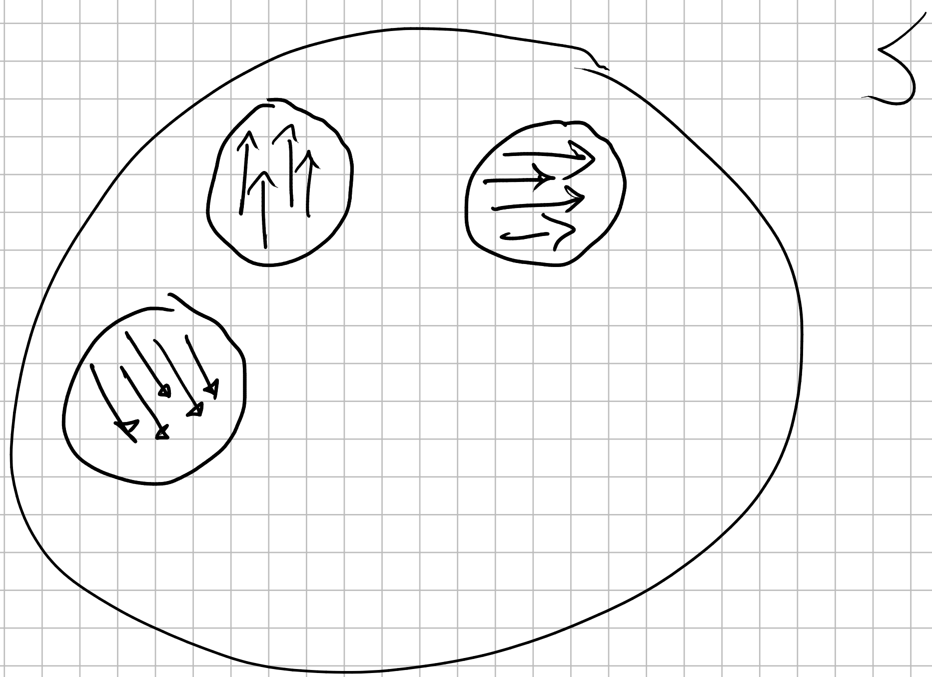


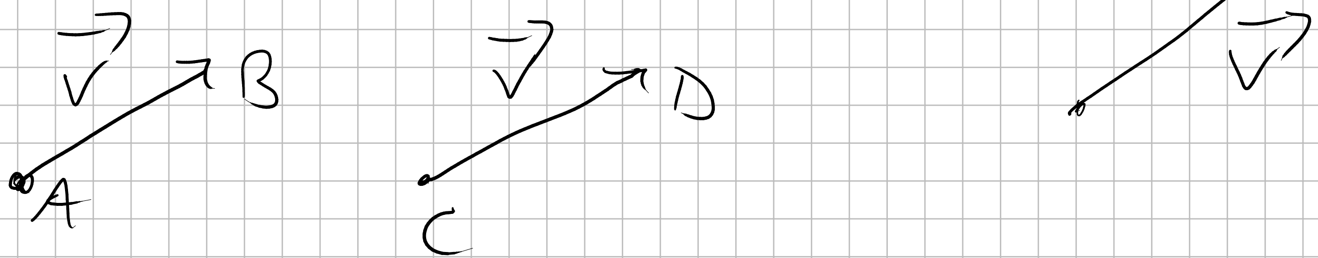
$B \mapsto$ PUNTO FINALE

$A \mapsto$ PUNTO DI APPLICAZIONE o INIZIALE



VETTORI EQUIVALENTI





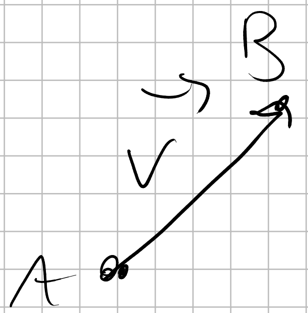
$\vec{v} = \vec{AB}$ $\qquad$ $\vec{v} = \vec{CD}$

SOMMA DI VETTORI

Prendo $\vec{v}$ e $\vec{w}$.

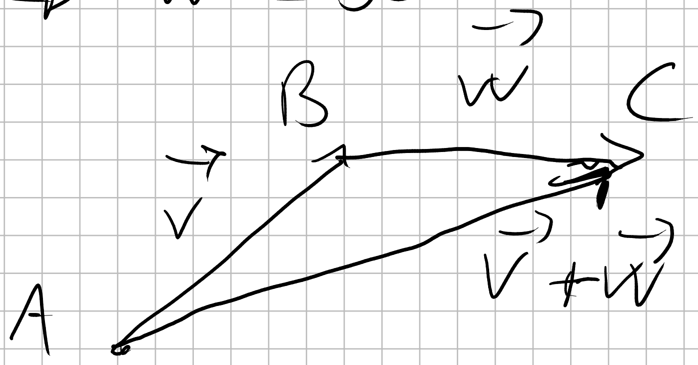
Scelgo $\overrightarrow{AB}$ rappresentante di $\vec{v}$.

$\Rightarrow \vec{v} = \overrightarrow{AB}$

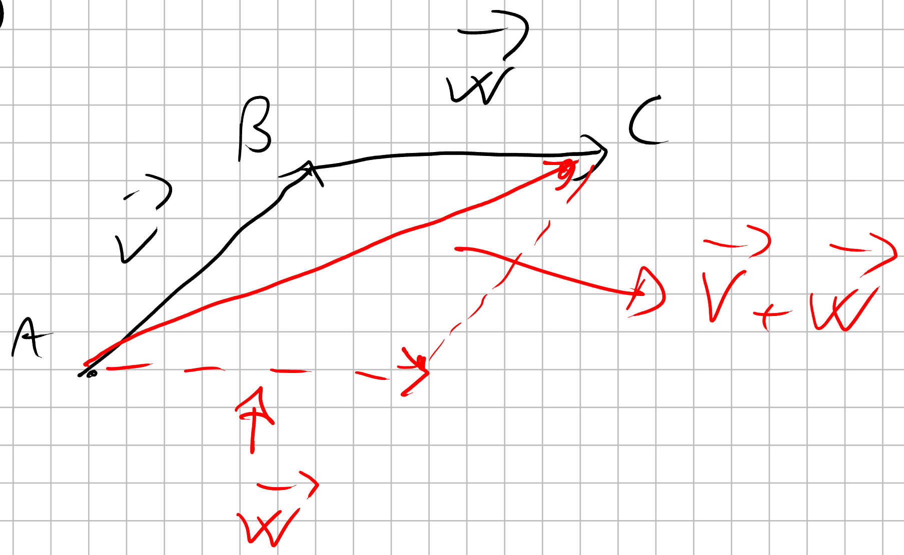


Prendo $\overrightarrow{BC}$ rappresentante di $\vec{w}$

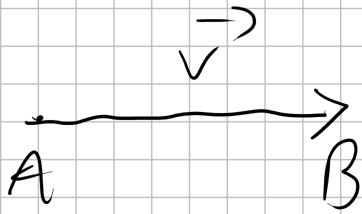
$\Rightarrow \vec{w} = \overrightarrow{BC}$



$\vec{v} + \vec{w} = \overrightarrow{AC}$

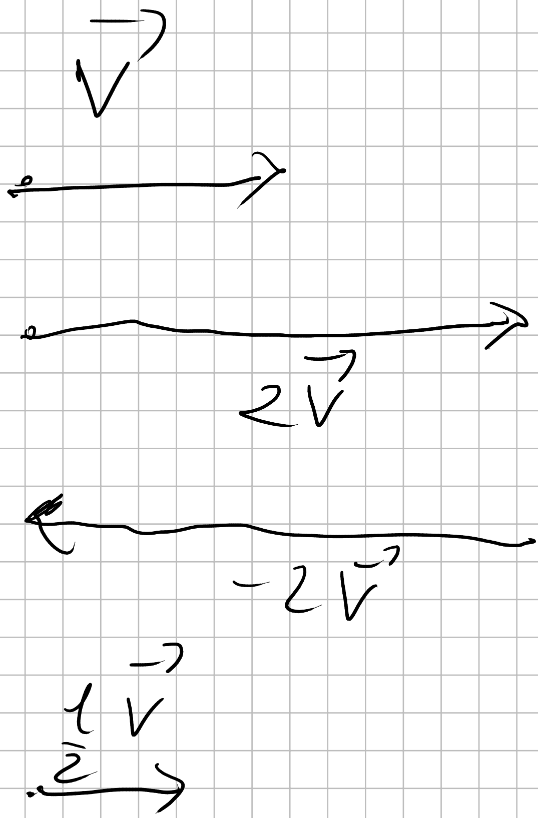


$\vec{v} + \vec{0} = \vec{v}$



$\Rightarrow -\vec{v} = \overrightarrow{BA}$

$\vec{v} + (-\vec{v}) = \vec{0}$



VALORE ASSOLUTO

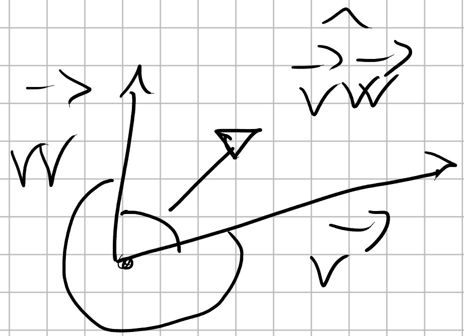
MODULO

modulo di $a\vec{v}$ è $|a| \cdot |\vec{v}|$

Se $a\vec{v} \neq \vec{0} \Rightarrow a\vec{v}$ è parallelo a $\vec{v}$

Se $\vec{w} \parallel \vec{v} \Rightarrow \vec{v} = \lambda \cdot \vec{w}$, $\lambda \neq 0$

$\vec{v}, \vec{w} \neq \vec{0}$



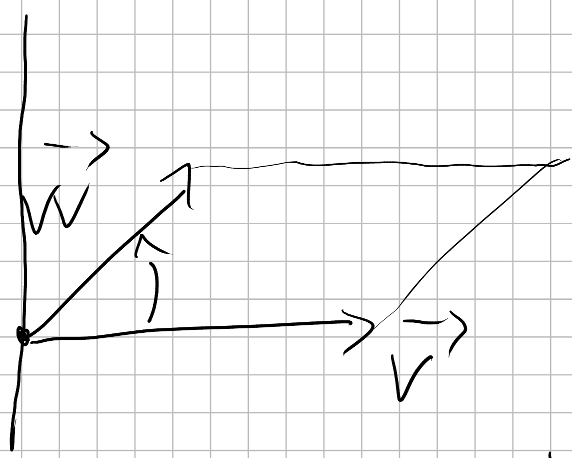
Se $\vec{v} = \vec{0}$ o $\vec{w} = \vec{0}$ $\Rightarrow$ $\vec{v} \cdot \vec{w} = 0$

Altrimenti $\Rightarrow$ $\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cos \widehat{\vec{v}\vec{w}}$

Supponiamo $\boxed{\vec{v}, \vec{w} \neq \vec{0}}$

$$\vec{v} \cdot \vec{w} = 0 \Leftrightarrow \underbrace{|\vec{v}|}_{\neq 0} \cdot \underbrace{|\vec{w}|}_{\neq 0} \cos \widehat{\vec{v}\vec{w}} = 0$$

$$\Leftrightarrow \cos \widehat{\vec{v}\vec{w}} = 0 \Leftrightarrow \widehat{\vec{v}\vec{w}} = \frac{\pi}{2} \Leftrightarrow \vec{v} \perp \vec{w}$$



$\vec{v} \wedge \vec{w}$ $\qquad$ $\vec{v} \times \vec{w}$

$$|\vec{v} \wedge \vec{w}| = |\vec{v}| \cdot |\vec{w}| \operatorname{sen} \widehat{\vec{v}\vec{w}}$$

$\vec{u}, \vec{v}, \vec{w}$

$$\vec{u} \cdot \vec{v} \wedge \vec{w} = \vec{u} \cdot (\vec{v} \wedge \vec{w})$$ NUMERO

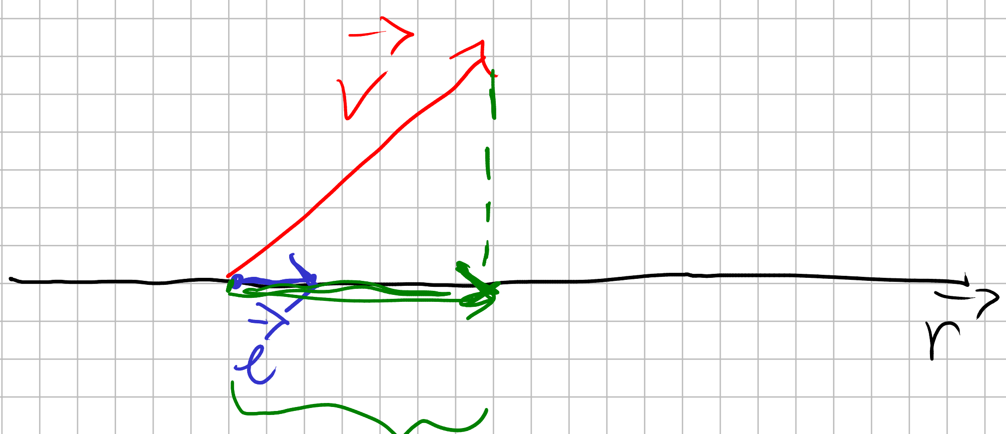
$$\left| \vec{u} \cdot \vec{v} \wedge \vec{w} \right|$$

$\boxed{\text{VERSORE}}$ $|\vec{v}| = 1$

$$\vec{v} \cdot \vec{v} = |\vec{v}| \cdot |\vec{v}| \cdot \cos 0 = |\vec{v}|^2$$

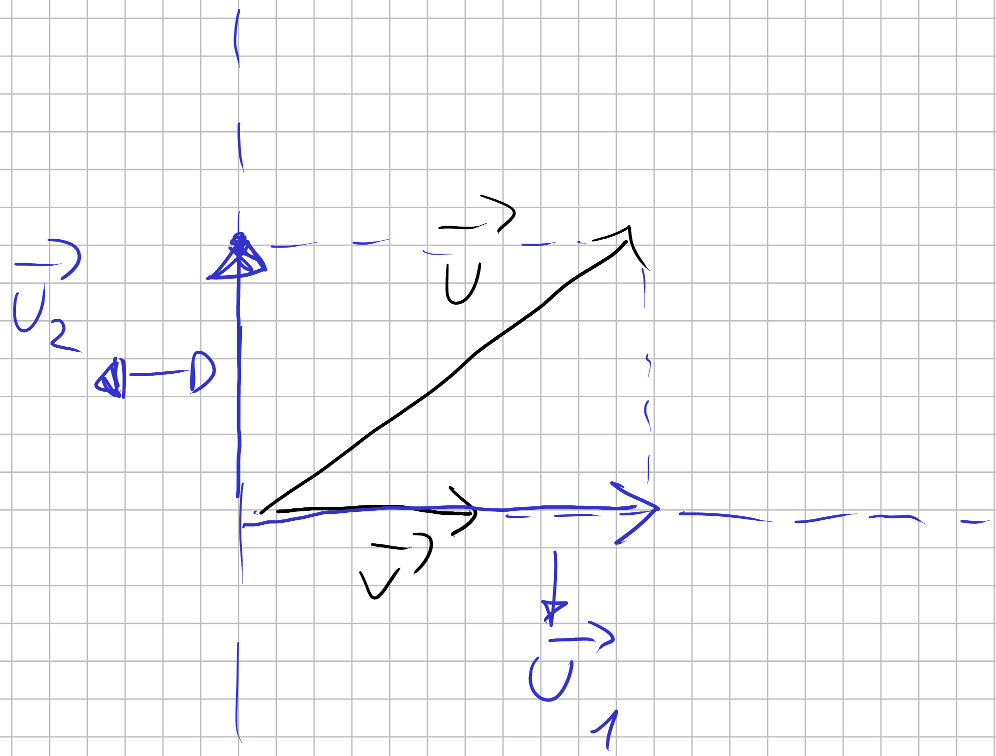
se $\vec{v}$ e' un versore $\vec{v} \cdot \vec{v} = 1$

$\longleftrightarrow$



$(\vec{v} \cdot \vec{e})\, \vec{e}$

PROIEZIONE ORTOGONALE DI $\vec{v}$ SU $\vec{r}$



SISTEMI DI RIFERIMENTO

$\vec{i} \mapsto \vec{x}$

$\vec{j} \mapsto \vec{y}$

$\vec{k} \mapsto \vec{z}$ $\qquad \vec{k} = \vec{i} \wedge \vec{j}$

ESEMPI

$\vec{v} = 2\vec{i} + \vec{j} - 3\vec{k} \qquad (2, 1, -3)$

$\vec{w} = -4\vec{i} + 3\vec{j} + 2\vec{k} \qquad (-4, 3, 2)$

$\vec{u} = 5\vec{i} + 7\vec{k} \qquad (5, 0, 7)$

$$|\vec{v}| = \sqrt{2^2+1^2+(-3)^2} = \sqrt{4+1+9} = \sqrt{14}$$

$(2,1,-3)$

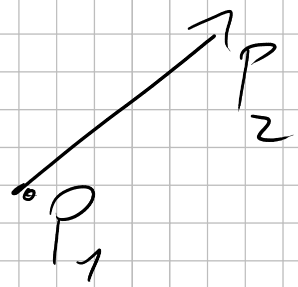
$$|\vec{w}| = \sqrt{(-4)^2+3^2+2^2} = \sqrt{16+9+4} = \sqrt{29}$$

$\downarrow$

$(-4,3,2)$

$\longleftrightarrow$

$$P_1, P_2 \longmapsto \overrightarrow{P_1P_2}$$



$$\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2}$$

$$\Rightarrow \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

<u>ESEMPI</u>

1) $P_1 = (2,1,4)$, $P_2 = (3,2,-5)$

$$\overrightarrow{P_1P_2} = (3-2)\,\vec{i} + (2-1)\,\vec{j} + (-5-4)\,\vec{k} =$$

$= \vec{i} + \vec{j} - 9\vec{k} \quad (1, 1, -9)$

(2) $P_1 = (1, 2, -3)$, $P_2 = (3, -4, -2)$

$\overrightarrow{P_1P_2} \mapsto (3-1, -4-2, -2+3) = (2, -6, 1)$

SOMMA DI VETTORI

$\vec{v} = 3\vec{i} + 4\vec{j} - 6\vec{k} \quad (3, 4, -6)$

$+ \ \vec{w} = 2\vec{i} + \vec{j} + \vec{k}$

$\vec{v} + \vec{w} = 5\vec{i} + 5\vec{j} - 5\vec{k}$

$+ \ \vec{u} = -\vec{i} + 2\vec{j} + 7\vec{k} \ ] \mapsto (-1, 2, 7)$

$\vec{w} + \vec{u} = \vec{i} + 3\vec{j} + 8\vec{k}$

$\Longleftrightarrow$

$2\vec{v} = 6\vec{i} + 8\vec{j} - 12\vec{k} \quad (6, 8, -12)$

$-3\vec{w} = -6\vec{i} - 3\vec{j} - 3\vec{k}$

$-\vec{u} = \vec{i} - 2\vec{j} - 7\vec{k}$

ESEMPIO

$\vec{v} = 4\vec{i} - \vec{j} + 2\vec{k}$ $\quad (4, -1, 2)$

$\vec{w} = -3\vec{i} + \vec{j} + 5\vec{k}$ $\quad (-3, 1, 5)$

$$\vec{v} \cdot \vec{w} = 4 \cdot (-3) + (-1) \cdot 1 + 2 \cdot 5 =$$

$$= -12 - 1 + 10 = -3$$

$\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$ $\quad (1, 2, -1)$

$$\vec{v} \cdot \vec{u} = 4 \cdot 1 + (-1) \cdot 2 + 2 \cdot (-1) = 0$$

$\Rightarrow \quad \vec{v} \perp \vec{u}$

$\vec{v} \mapsto (4, -1, 2)$

I vettori paralleli a $\vec{v}$ hanno componenti proporzionali a quelle di $\vec{v}$.

$(4\lambda, -\lambda, 2\lambda)$

$(8, -2, 4)$, $(12, -3, 6)$

$$\vec{v} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k} \qquad (v_x, v_y, v_z)$$

$$\vec{i} = 1 \cdot \vec{i} + 0 \cdot \vec{j} + 0 \cdot \vec{k} \qquad (1, 0, 0)$$

$$\cos(\widehat{\vec{v}\vec{i}}) = \frac{\vec{v} \cdot \vec{i}}{|\vec{v}| \cdot \underbrace{|\vec{i}|}_{=1}} = \frac{v_x}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

$$\vec{v} = v_y \vec{y} + v_z \vec{z} \qquad (0, v_y, v_z)$$

$\vec{v}$ ASSE y $\qquad (0, 1, 0)$

$$\cos \widehat{\vec{v}\vec{y}} = \frac{v_y}{\sqrt{v_y^2 + v_z^2} \cdot 1} = \frac{v_y}{\sqrt{v_y^2 + v_z^2}}$$

<u>ESEMPIO</u>

$$\vec{v} = \vec{i} + 2\vec{j} - \vec{k}$$

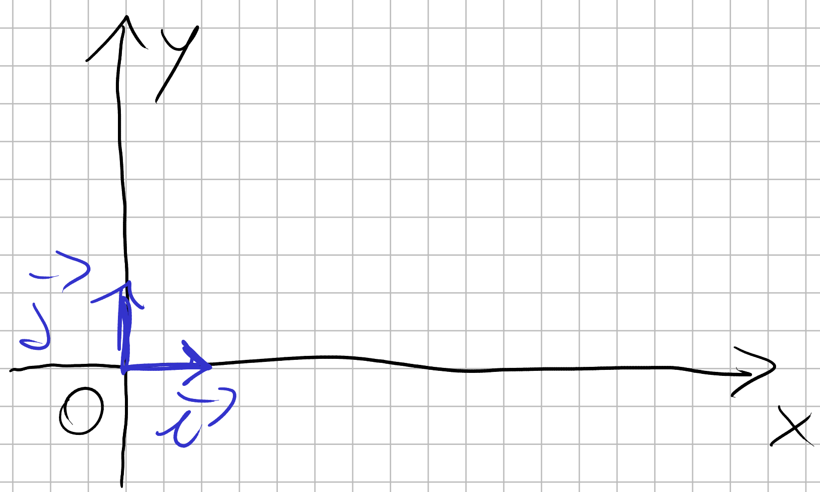
$$\vec{w} = 3\vec{i} + 4\vec{j} + 2\vec{k}$$

$$\vec{v} \wedge \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 3 & 4 & 2 \end{vmatrix} =$$

$$= 4\vec{i} - 3\vec{j} + 4\vec{k} - 6\vec{k} - 2\vec{j} + 4\vec{i} =$$

$$= 8\vec{i} - 5\vec{j} - 2\vec{k}$$

SISTEMI DI RIFERIMENTO NEL PIANO



$$\vec{v} = v_x \vec{i} + v_y \vec{j}$$

ESEMPIO

$$\vec{v} = 2\vec{i} + 3\vec{j} \mapsto (2, 3, 0)$$

$$\vec{w} = -\vec{i} + 4\vec{j} \mapsto (-1, 4, 0)$$

$$\vec{v} \wedge \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 4 & 0 \end{vmatrix} =$$

$$= 8\vec{k} + 3\vec{k} = 11\vec{k}$$

ESERCIZI

Sono dati i vettori

$\vec{v} = 3\vec{i} + 2\vec{j} + 2\vec{k}$

$\vec{w} = -\vec{i} - \vec{j} + 2\vec{k}$

Calcolare:

1. $2\vec{v} - 3\vec{w}$
2. $\vec{v} \cdot \vec{w}$
3. $\vec{v} \wedge \vec{w}$

④ Determinare $\widehat{\vec{v}\vec{w}}$

⑤ L'area del parallelogramma individuato da $\vec{v}$ e $\vec{w}$

⑥ Un vettore parallelo a $\vec{v}$ e uno ortogonale a $\vec{v}$.

⑦ $\vec{k} \cdot \vec{v} \wedge \vec{w}$

$\longleftrightarrow$

$$\vec{v} = 3\vec{i} + 2\vec{j} + 2\bar{k} \qquad (3, 2, 2)$$

$$\vec{w} = -\vec{i} - \vec{j} + 2\vec{k} \qquad (-1, -1, 2)$$

① $2\vec{v} - 3\vec{w} = 2 \cdot (3\vec{i} + 2\vec{j} + 2\vec{k}) +$
$- 3 \cdot (-\vec{i} - \vec{j} + 2\vec{k}) = 9\vec{i} + 7\vec{j} - 2\vec{k}$

② $\vec{v} \cdot \vec{w} = 3 \cdot (-1) + 2 \cdot (-1) + 2 \cdot 2 =$

$= -3 - 2 + 4 = -1$

3 $\vec{v} \wedge \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 2 \\ -1 & -1 & 2 \end{vmatrix} =$

$= 4\vec{i} - 2\vec{j} - 3\vec{k} + 2\vec{k} - 6\vec{j} + 2\vec{i} =$

$= 6\vec{i} - 8\vec{j} - \vec{k}$

4 $\cos \widehat{\vec{v}\vec{w}} = \dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} =$

$$= \frac{-1}{\sqrt{3^2+2^2+2^2} \cdot \sqrt{(-1)^2+(-1)^2+2^2}} = -\frac{1}{\sqrt{17} \cdot \sqrt{6}} =$$

$= -\dfrac{1}{\sqrt{102}} \Rightarrow$

$\widehat{\vec{v}\vec{w}} = \arccos\left(-\dfrac{1}{\sqrt{102}}\right) =$

$= \pi - \arccos\left(\dfrac{1}{\sqrt{102}}\right)$

(5) $|\vec{v} \wedge \vec{w}| = \sqrt{6^2+(-8)^2+(-1)^2} =$

$= \sqrt{36+64+1} = \sqrt{101}$

(6) $\vec{v} = 3\vec{i} + 2\vec{j} + 2\vec{k}$

$2\vec{v} = 6\vec{i} + 4\vec{j} + 4\vec{k}$

$\vec{u} = u_x\vec{i} + u_y\vec{j} + u_z\vec{k} \perp \vec{v}$

$\vec{u} \cdot \vec{v} = 0 \Rightarrow 3u_x + 2u_y + 2u_z = 0$

$(0, 1, -1) \Rightarrow \vec{u} = \vec{j} - \vec{k}$

$(2, -3, 0) \quad \vec{u} = 2\vec{i} - 3\vec{j}$

(7) $\vec{k} \cdot \vec{v} \wedge \vec{w} =$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 3 & 2 & 2 \\ -1 & -1 & 2 \end{vmatrix} = -3+2 = -1$$

$$\vec{n} \mapsto (0,0,1)$$

$$\vec{v} \wedge \vec{w} \mapsto (6, -8, -1)$$

$$\vec{n} \cdot \vec{v} \wedge \vec{w} = 0 \cdot 6 + 0 \cdot (-8) + 1 \cdot (-1) = -1$$