TO LEARN MATHEMATICS: MAYAN MATHEMATICS IN BASE 10

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Abstract

It is currently accepted that, around 600 B.C. the Mayan civilization developed for the first time the mathematical abstraction of zero. To them zero, had a cosmic meaning also. They developed the zero six hundred years before the ancient cultures of India.

The Mayan civilization is also known for its outstanding achievements in astronomy, architecture, medicine, and agriculture. Today it is considered one of the most important pre-Columbian cultures of America. They developed a vigesimal numerical system (base of 20) which has very advanced features, such as the positional system with zero as a place marker. They used only three symbols to construct a method for all the arithmetic operations. This system was so powerful that it enabled them to make predictions, with astonishing precision, of astronomical events and to calculate a calendar, which has a larger precision than the Gregorian calendar.

They used a positional system of numeration similar to the one we use today. This means that each sign has a value depending on the position that it occupies in the representation of the number. The numbers are set in a vertical position to represent powers of twenty (running from bottom up). They used only dots, bars and zero to represent numbers. The Mayas represented the Zero as an empty seashell.

There are large advantages in using dots, bars, and seashells to perform mathematical operations. The resulting method does not require tables of any kind, and it is powerful, dynamic, and ludic. It is an excellent tool for teaching since it gives intuitive knowledge of basic algorithms.

In the following, we describe the Mayan system. We also describe the transformation to a numerical system of base 10. Then, we show the fundamental arithmetic operations to use as an educational proposal in mathematics.

Keywords: Mayan mathematics, arithmetic operations, base 20, base10.

1 INTRODUCTION

Let us look at the representation of numbers in base 10. In our decimal system, we write 2345 = 2 X 103 + 3 X 102 + 4 X 101 + 5 X 100. That is, we have an implicit sum. The Mayas do the same, but vertically and using dots, bars and seashells. They used a base of 20. In figure 1, we show the first 21 Mayan numerals in base 20 [1-8]. In figure 2, we show the representation of the number 2015 using the Mayan system of base 20. Notice the blocks of powers of 20 in the vertical direction and notice that 5 points are equivalent to a bar.

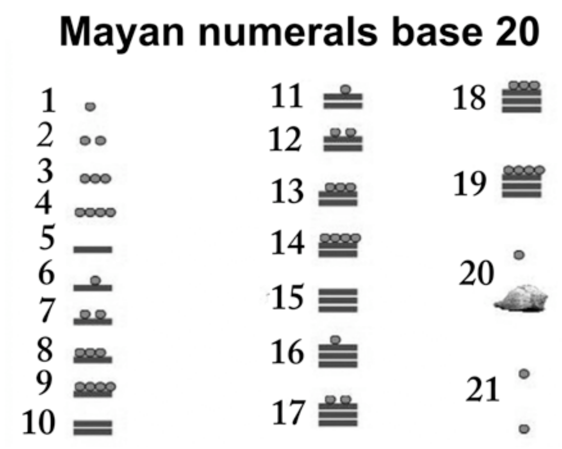

Figure 1

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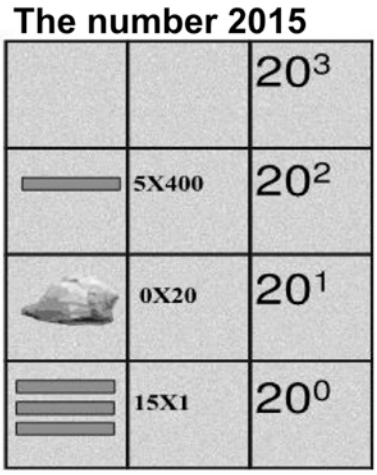


Figure 2

We will use the methodology that, more likely, used the Maya to conduct their operations, but we will do it in base 10 for reasons of clarity. We will use the following rules [2, 6].

I. Two bars in a level are equivalent to one point in the superior immediate level.

II. One point in a level is equivalent to two bars in the inferior immediate level.

III. Five points are equivalent to one bar in the same level.

IV. One bar is equivalent to five points in that level.

In Figures 3 and 4 we show the Mayan numeration in base 10.

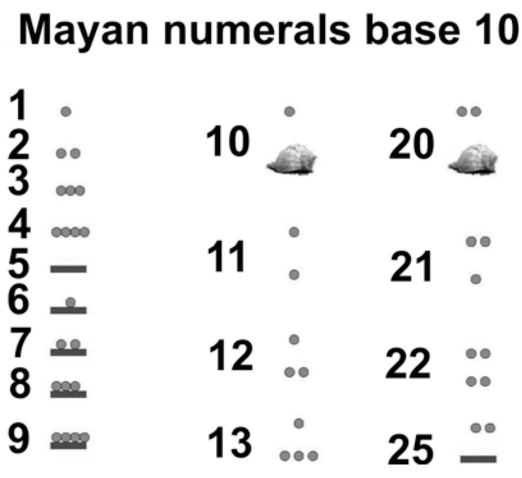


Figure 3

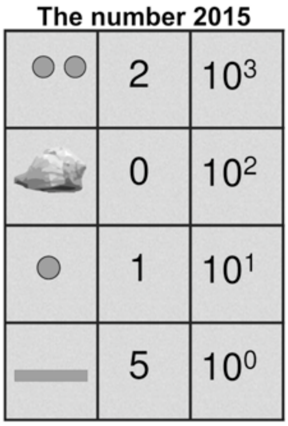


Figure 4.

2 ADDITION

The addition is striking simple. In figures 5 to 10 we show the operation 197 + 834. We make the addition in figure 5, grouping points and bars on each level and transforming bars into points as shown figure 6. For every two bars, we put a point in the superior immediate level, like in figures 7, 8 and 9. The result is given in figure 10 and is 1031. For adding several numbers the procedure is the same. The addition of many numbers can be done very quickly.

Figure 5

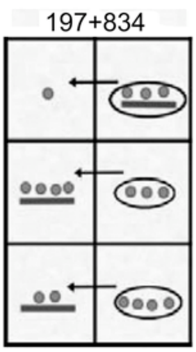
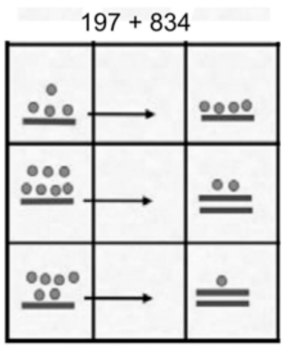


Figure 6



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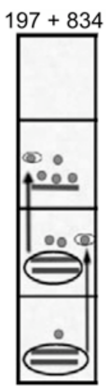

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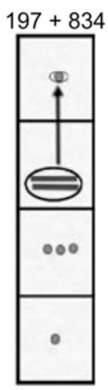


Figure 9

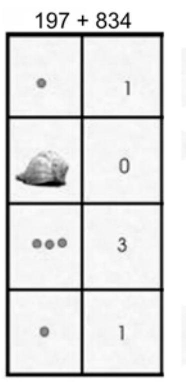


Figure10

3 SUBTRACTION

For the subtraction, one point annihilates one point and one bar annihilates one bar. This is done for each level, starting from the lowest one. In figures 11 to 16 we show the accomplishment of the subtraction 862-643. Notice that in figure 11, the number 862 is on the column of the left and the number 643 is on the right. We work on the minuend without altering the subtrahend (see figures 11 to 16).

In order to have enough dots to be annihilated on the lowest level, we start by passing one dot to this level, taking it from the second level. This dot transforms into two bars, as it is seen in figure 12. Furthermore, we replace bars by dots, as we do in figure 13, to arrive at figure 14. The dots and bars locked up in figure 15 are eliminated. We can read the result in the column of the left of figure 16, and is 219. The test is straightforward. We simply add the numbers in figure 16, and we obtain the minuend.

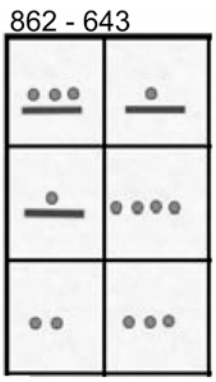
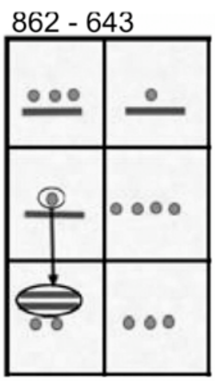


Figure 11



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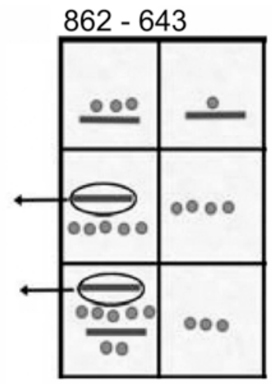
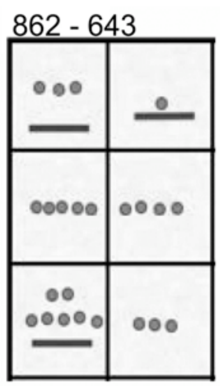


Figure 13



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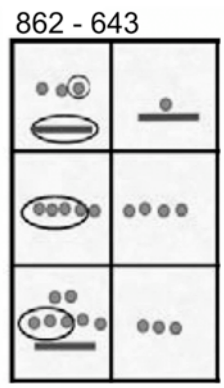


Figure 15

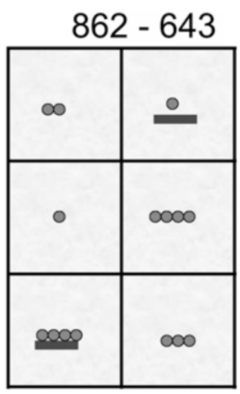
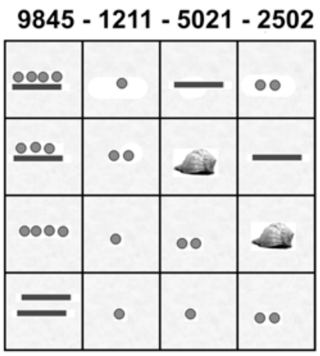


Figure 16

3.1 Multiple subtractions

One clear advantage of this method of subtraction is the fast and easy way to perform multiple subtractions. The rules are the same as in section 3. Let us consider the subtraction 9845 -1211- 5021 - 2502. We put the largest of these numbers (the minuend) on the first column of the left. However, we can place it on any column. We can place the other numbers (the subtrahends) on the columns of our choice, see figure 17.

Then, we work on the minuend without altering the subtrahends (see figures 11 to 16). We annihilate on the largest number and on each level the corresponding dots and bars of the other numbers. We show the procedure in figures 19 and 20. We can read the result on the same column where he had placed the largest number. The result is 1111. Notice that adding all the numbers again in figure 20, we obtain the test of the subtraction. We obtain 9845, as we should.



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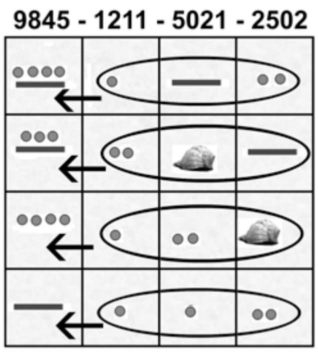


Figure 18

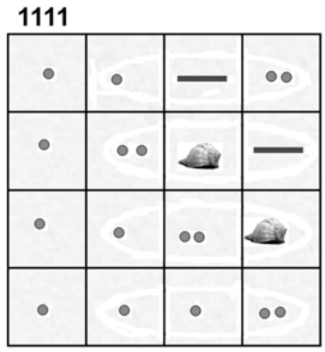
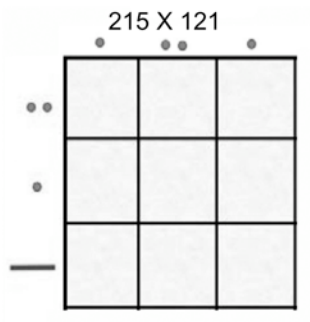


Figure 19

4 MULTIPLICATION

To perform the multiplication, we do not need tables. Let us make the product 215X121. We put the factors outside the board; one (215), vertically and the other horizontally, like we show in figure 17. We are going to reproduce in each square the figure which we have to the left by outside the board, so many times as the number of the superior part indicates. Clearly, we can perform the reciprocal operation too. We do the easier of these two possibilities. In this manner, in the first square of the left column we put one pair of dots, or twice a dot. In this way, we solve the squares of the first column of the left, as we show in figure 18. We solve the squares of the following columns in a similar fashion, as we show in figure 19. We have almost finished the multiplication.

Then, we start the final part to obtain the product. We group diagonally, as we show in figure 20. Each diagonal corresponds to a power of 10. Afterwards, we use the rules of which each group of five dots is transformed into a bar and that every two bars become a dot in the superior immediate level, leaving a zero (this is a seashell) in its place. After doing this, we read the result directly along the diagonal. The square of the right inferior corner corresponds to the units. We show the sequence in figures 21 to 25. Notice in figure 25 that we have left a seashell in the square at the center of the board. Now we read in figure 25 the result: 26015.



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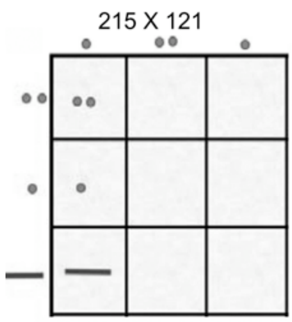
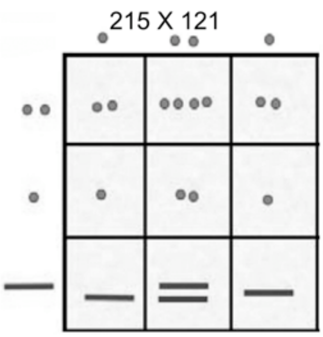


Figure 21

Figure 22



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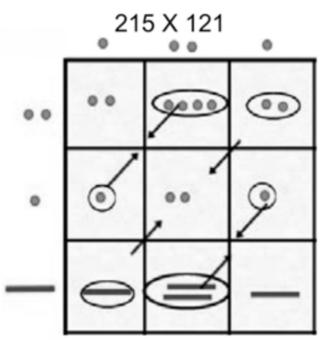

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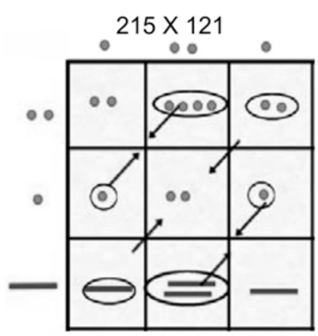

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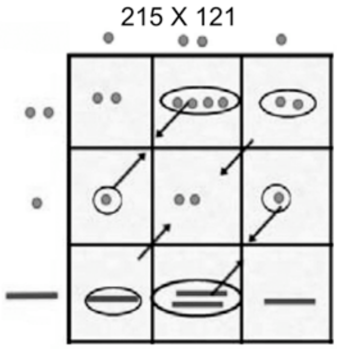

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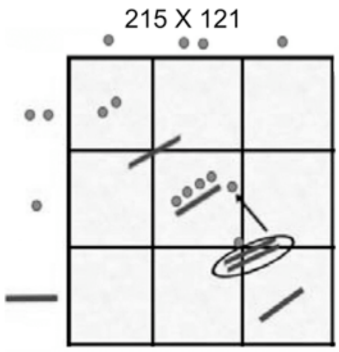

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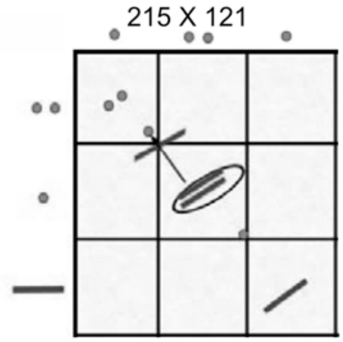

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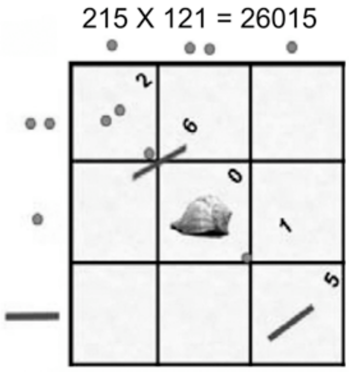


Figure 29

4.1 One square multiplication board

Let us now consider the case of a simple multiplication: seven times nine. This is a typical case when students memorize without understanding the meaning of what they are doing. With this method, it is not necessary to memorize the product. They can reproduce it as many times as they need. However, what is more likely is that at large, they will memorize the result after going through the procedure enough number of times.

For this case, our board is made of only one square. We place the factors outside of our board, as we show in figure 31. In the only square of our board we reproduce seven times one bar and four dots, as we show in the figure 32.

Then, we exchange every five dots with one bar. This means that we have to put five additional bars, as we show in the figure 33. Afterwards, we substitute every pair of bars for one dot on the immediate superior level. We have six pairs of bars, which we show encircled in the figure 34. These six pairs correspond to six dots in the immediate superior level, as we show in the figure 35.

Finally, we substitute five dots with one bar and we can read the result: 63, as we show in the figure 36.

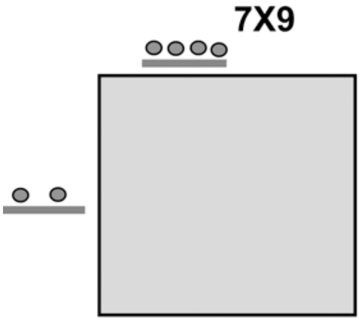


Figure 30

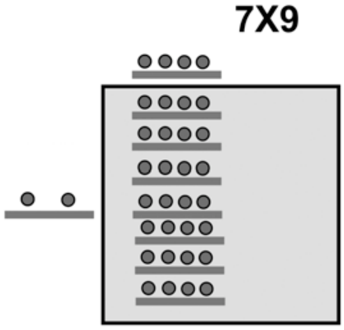


Figure 31

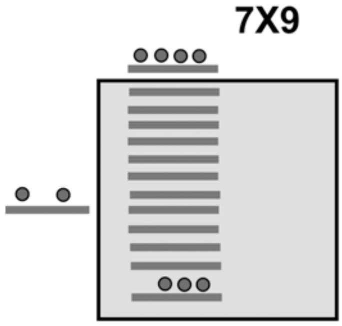


Figure 32

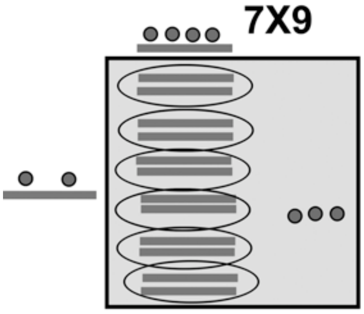


Figure 33

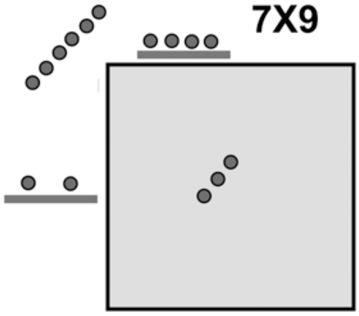


Figure 34

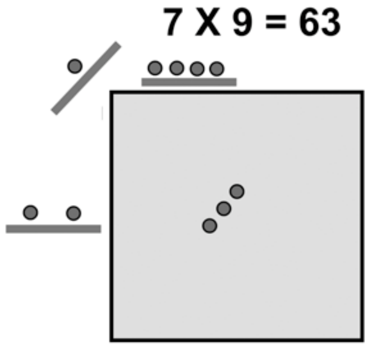


Figure 35

5 DIVISION

This is the inverse operation of the multiplication, and thus we will solve it. The dividend is conceived like the product of two numbers, one of them is the divisor and the other, unknown, is the quotient. In this way, the dividend is placed on the diagonal of the board. Let us consider 180 ÷ 12, see figure 26. We will put the divisor in a vertical form and by outside of the board. The quotient will be in a horizontal form and by outside of the board too. These positions can be exchanged with no problem. We begin the division by trying to find the number that we have to put in the external part, by above of the square of the left corner, so that, reproducing the first figure external to the left (one dot) as many times as the number that we are looking for, we obtain the dot of the left superior square of the board. Thus we are following an inverse way from the multiplication.

We find that we have to put one dot, as in figure 27. With this we satisfy the first square of the first column. In order to complete the inferior immediate square of the same column we see that we need two points. We take those two points from the number that is in the diagonal, as we show in figures 27 and 28. In this way, we have completed satisfactorily the first column.

Now, we work with the second column. We must find the following number of the quotient that goes in the external part of the board by above of the second column. If we use a number six (i. e, one bar and one dot) we complete the first square of this column, but we exceed what we have in the inferior square of this column. In this way, we try a solution by using one bar, as we show in figure 30. In the first square of the second column, the dot is in excess, thus we lower it to the inferior immediate square like two bars. We show this in figure 29, and we ignore the seashell that corresponds to the zero. These two bars are exactly what we need in the last square. Thus the division has concluded and is exact: 180÷12=15, as we show in figure 30.

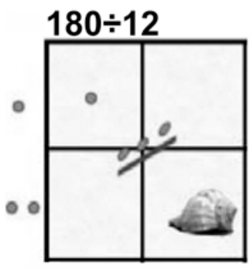
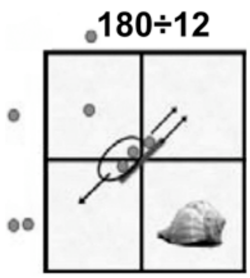


Figure 36



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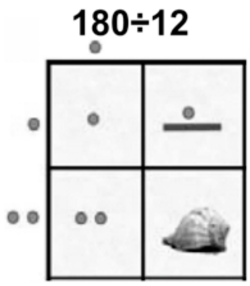

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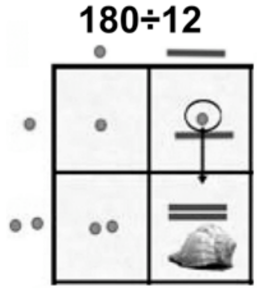

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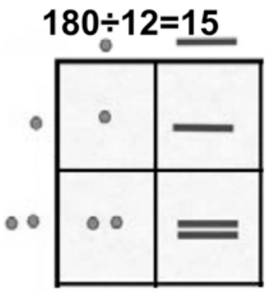


Figure 40

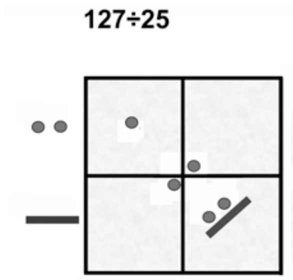
5.1 Divisions and decimal fractions

Let us consider the division 127÷25. Again, we place the dividend on the diagonal of the board, and we place the divisor in a vertical form and by outside of the board, see figure 31. We have to put the seashell as the first number of the quotient, as we show in the figure 32. In this way, the dot in the first square must pass to the inferior immediate level as two bars. Thus, they join to the two dots which were previously there, as we show in the same figure 32.

Then, we displace these two bars and two dots to the first square of the second column, see figure 33. After doing this, we proceed to find the second number of the quotient. We can try with a bar, as we show in figure 34. In the first square of the second column, we should have only two bars, so that the two dots must go down one level as four bars. These four bars join one bar and two dots, which were there previously.

In the second square of the second column, we must have only five bars. The two dots are in excess. Thus we translate the pair of dots to the first square of the third column, as we show in figures 34 and 35. We can see that the division at this point has a quotient of 5 and a remainder of 2. At this moment, we use the little star to separate integers from decimal fractions in the quotient.

Then, we proceed to obtain the decimal fractions. We try with a seashell (i.e. zero), as the following number in the quotient. This means that in both squares of the third column must have seashells. Thus, the two dots must pass to the immediate inferior level as four bars as we show in figure 36. At this point, the quotient is 5.0, and we proceed to find the following number in the quotient. We try with a bar and three dots (i.e. 8). With this trial, we must have in the first square of the fourth column only three bars and one dot. The remaining four dots must pass to the immediate inferior level as eight bars, as we show in the figure 37. We see that in the second square of the fourth column we should have eight bars. We have exactly eight bars. Thus, the division is finished, and it is exact. The result is 5.08, as we show in the figure 37.



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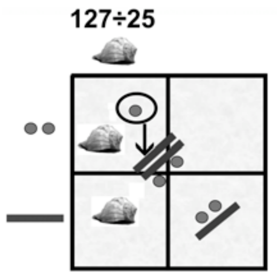

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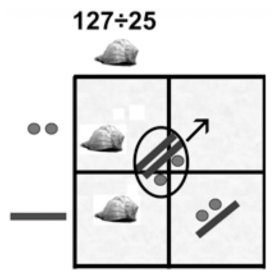

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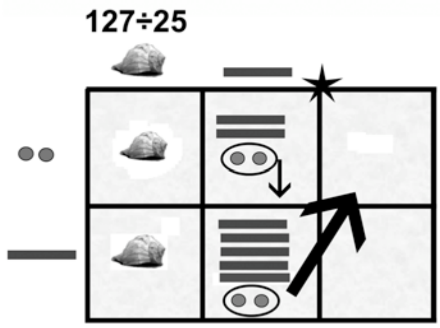

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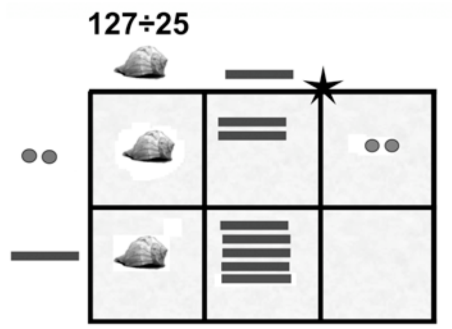

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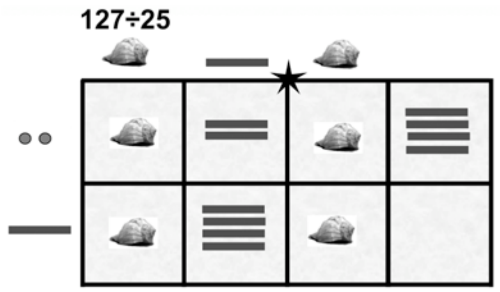

Figure 46

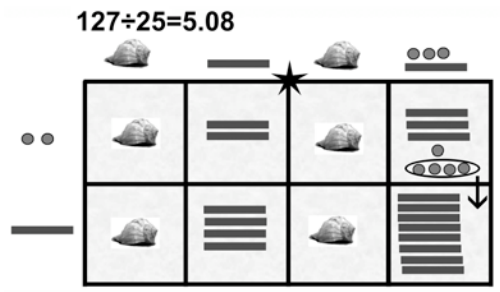

Figure 47

6 SQUARE ROOT

To find the solution of the square root, we consider it like a division. The radicand is the dividend. Of course that we do not know the divisor or the quotient, but we know that they are the same, and we use this fact to find the solution. Let us find the square root of 144.

As we are considering it as a division, we place the radicand on the diagonal of the board, as we show in figure 31. We proceed like we do in the division, but knowing that the quotient must be equal to the divisor. In order to have one dot in the square of the left corner of the board, we need one dot like first number of the divisor and of the quotient, as we show in figure 32.

After finding the solution for the first square, we follow an additional rule for the square root. We have to distribute symmetrically as possible, the number of the radicand located on the following inferior level. This distribution is performed among the squares located along the diagonal that corresponds to the same power of 10. We show this distribution in figures 32 and 33. Now we proceed to find the following number of the solution. We may try with a pair of dots in the external superior part of the second column and simultaneously, in the external part of the second row of the board, see figure 34. We see that with this trial of the pair of dots, we require to have a pair of dots on the first square of the second column, which we do. We also require having two dots on the first square of the second row, which we do.

Finally, we need to solve the second square of the second column. In this square, we have four dots. These four dots correspond to two pairs, which are what we need according to our trial of a pair of dots. In this manner, the square root is solved, see figure 34. The result is exact and is 12.

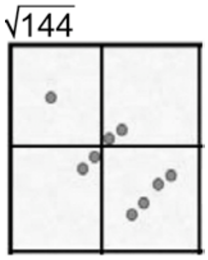

Figure 48

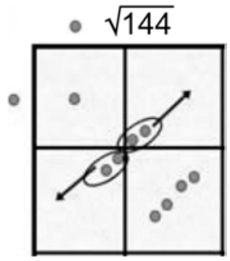

Figure 49

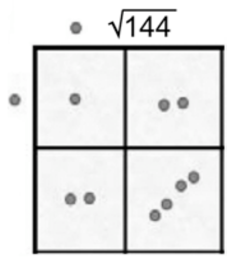

Figure 50

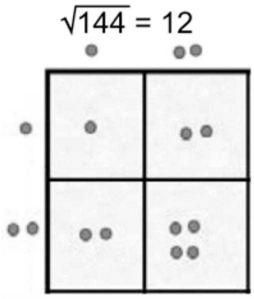

Figure 51

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