

# Dataflow models, control flow

## Lecture 03 on Dedicated systems

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outline:

- dataflow graphs
- synchronous dataflow graphs (SDF)
- PASS algorithm for SDF analysis
- limitations of SDF graphs for control flow
- SDF graph extensions for performance analysis
- SDF graph transformations

high-level models for system design: block diagrams

- aka filters and pipes architectures
- each block (filter) processes an incoming data stream from its input channels and produces an outgoing data stream on its output channels
- functional partitioning
- uncommitted to hardware vs. software implementation

dataflow graphs: mathematical model of block diagrams

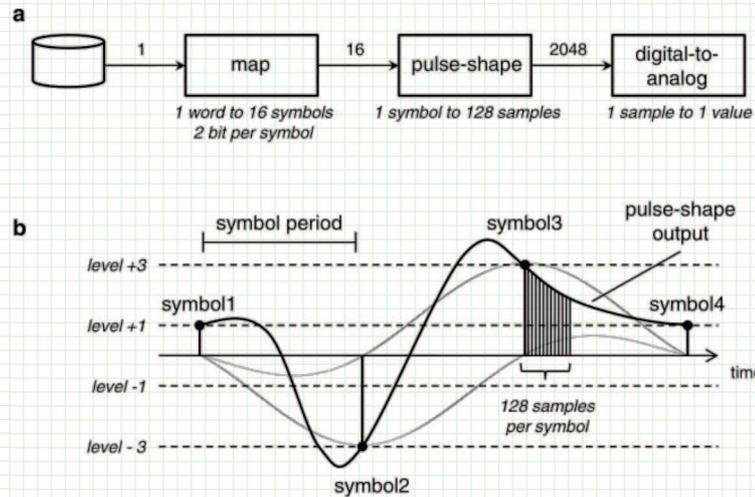
- a model of computation (MoC)
- widely used in signal processing
- other related models: actor systems (Hewitt), Kahn networks, Petri nets, ...

example: block diagram of a 4-level pulse-amplitude modulation system (PAM-4)

see next

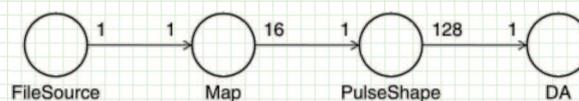
block diagram of a signal processing system

example: 4-level pulse-amplitude signal modulation (PAM-4)



Schaumont, Figure 2.1 - (a) Pulse-amplitude modulation system. (b) Operation of the pulse-shaping unit

usefulness of dataflow models for codesign



Schaumont, Figure 2.2 - Data flow model for the pulse-amplitude modulation system

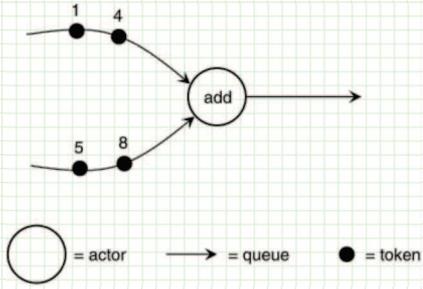
a software model of the PAM-4 system is feasible, see for example the C program in Schaumont, Listing 2.1, where each block is represented as a function, yet:

- the software model forces a sequential order of function execution
- hardware implementations are feasible with parallel function execution, with their respective components operating in pipeline
- a dataflow model allows both solutions

in the model in figure, functions are represented by *actors* linked through *communication channels* modeled as *FIFO queues*

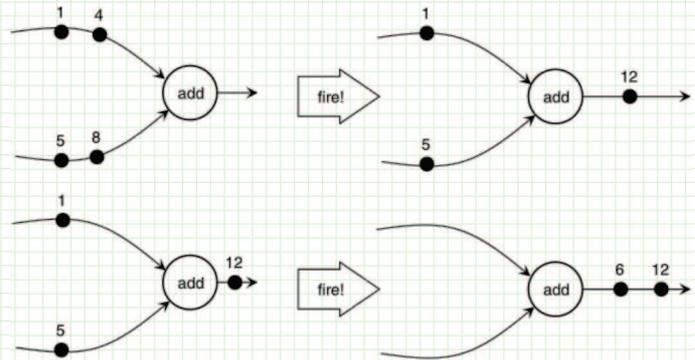
- every actor operates as an independent processing unit, with its inputs and outputs marked by the respective *data consumption and production rates* at every activation of the actor
- strong features: dataflow graphs are a concurrent, distributed, modular MoC
- thanks to their mathematical nature, dataflow graphs are amenable to analysis, allowing the designer to assess useful properties of an abstract architecture, e.g. deadlock freedom, well before its implementation

constituents of dataflow models



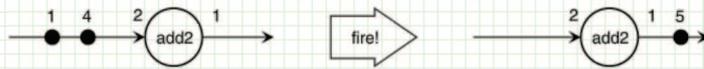
Schaumont, Figure 2.3 - Data flow model of an addition

- actors: iterative execution, each firing is subject to availability of data on all input channels
- channels: unbounded FIFO queues
- data: represented by values of tokens in the channels



Schaumont, Figure 2.4 - The result of two firings of the add actor, each resulting in a different marking

marking: state of a dataflow graph, consisting of an assignment of tokens to channels



Schaumont, Figure 2.7 - Example of a multi-rate data flow model

synchronous dataflow (SDF) graphs

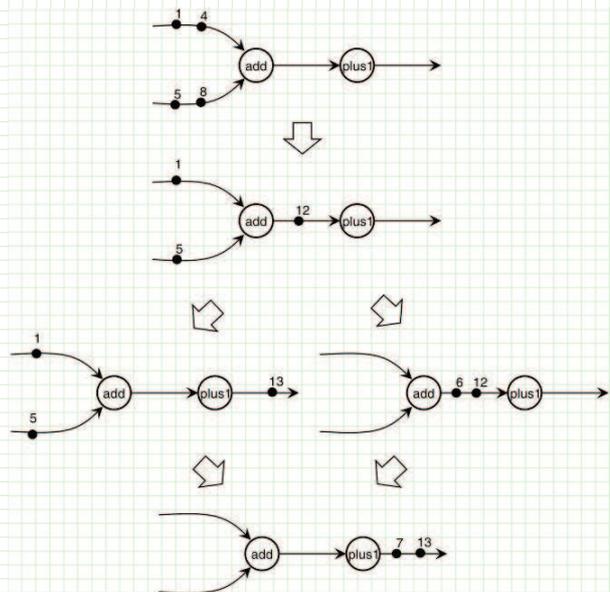
actors are functional processing units with no internal state

the only observable state information of a dataflow graph is its marking

in multirate dataflow graphs each actor may consume more than one token on each input channel and may produce more than one token on each output channel, at every firing

graphs with fixed production and consumption rates are said synchronous dataflow graphs (SDF)

- SDF graphs cannot express data-dependent execution
- however, mathematical analysis of their properties is easier
- and, they are determinate (see figure)

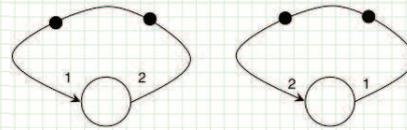


Schaumont, Figure 2.8 - SDF graphs are determinate

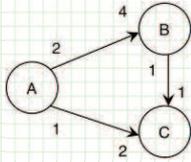
## SDF graph analysis

desirable properties of an SDF graph:

- unbounded execution
- bounded buffers



Schaumont, Figure 2.9 - Which SDF graph will deadlock, and which is unstable?



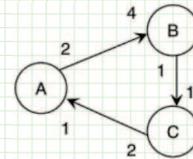
Schaumont, Figure 2.10 - Example SDF graph for PASS construction

PASS: periodic admissible sequential schedule

- sequential: firing of an actor at a time
- admissible: deadlock-free + bounded buffers
- periodicity of the state sequence → unbounded execution

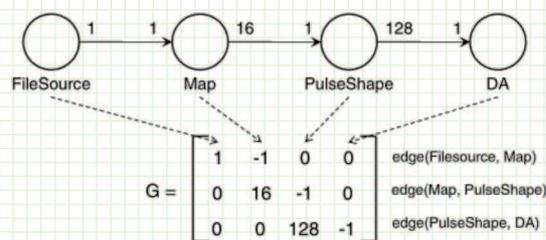
PASS construction algorithm, if one exists (Lee):

1. create the topology matrix  $G$  of the SDF graph
2. verify rank of matrix = n. of actors minus one
3. determine a firing vector  $q$  such that  $Gq = 0$
4. try round robin firing of each actor until count in firing vector



Schaumont, Figure 2.11 - A deadlocked graph

## PASS example for the signal processing model



Schaumont, Figure 2.12 - Topology matrix for the PAM-4 system

application of the method by Lee:

1. see figure
2. OK, the three rows of  $G$  are linearly independent
3.  $q^T = [1 \ 1 \ 16 \ 2048]$
4. the firing schedule described by  $q^T$  is a PASS, though it requires a 2048-position storage in the rightmost FIFO queue  
a more economical solution is the schedule  $(1, 1, 16*(1, 128))$ , which only needs 128

modeling control flow is not easy with SDF graphs...

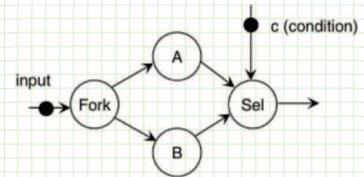
a few problematic issues:

- stopping and restarting
- dynamic network topology
- exception handling
- run-time conditions, such as conditional execution

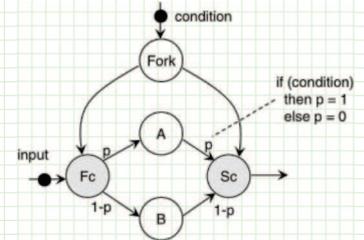
a simple if-then-else is troublesome with SDF graphs

workarounds:

- emulation in SDF with Fork-Sel actors
- extension of SDF semantics to Boolean Data Flow (BDF)



Schaumont, Figure 2.13 - Emulating if-then-else conditions in SDF

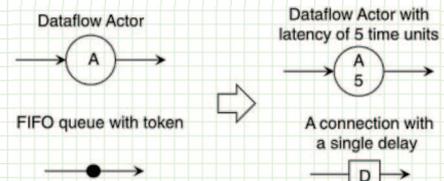


Schaumont, Figure 2.14 - Implementing if-then-else using Boolean Data Flow

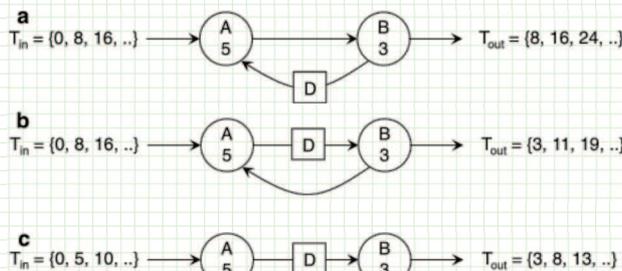
minimal extensions, preserving SDF semantics:

- time: actor latency
- space: bounded queues

useful for performance analysis and to evaluate performance-enhancing transformations



Schaumont, Figure 2.15 - Enhancing the SDF model with resources: execution time for actors, delays for FIFO queues



Schaumont, Figure 2.16 - Three data flow graphs

in the initial state, buffers always hold a token

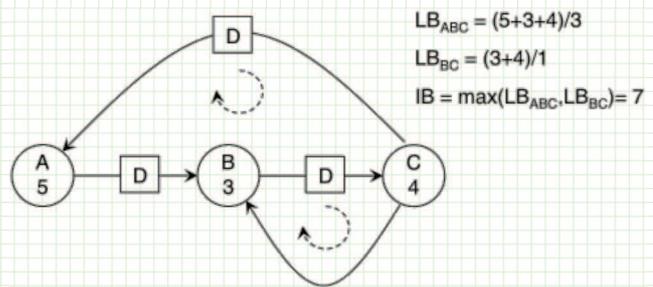
a buffer may hold a token for the duration of a single activation of the next actor  
by adding buffers, pipelining may increase the throughput of an SDF graph

## analysis of throughput limits

number and distribution of buffers in an SDF graph affect its throughput

loops also affect its latency and throughput; two concepts to see this:

- loop bound: latency along a given loop divided by the number of buffers in that loop
- iteration bound: highest loop bound in the graph



Schaumont, Figure 2.17 - Calculating loop bound and iteration bound

the throughput of an SDF graph cannot be higher than (the reciprocal of) its iteration bound  
linear graphs have an iteration bound, too (implicit feedback from output to input)



Schaumont, Figure 2.18 - Iteration bound for a linear graph

## dataflow model transformations

functionality-preserving transformations aimed at improving the performance of SDF graphs

to increase the throughput and/or reduce the latency

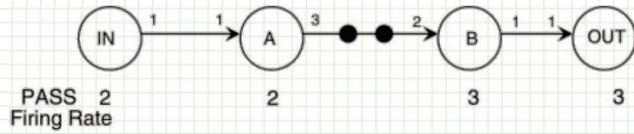
four are most used:

- *multirate expansion*: transforms a multirate SDF graph into a single-rate one  
useful because the other three are defined for single-rate SDF graphs
- *retiming*: buffer redistribution over the graph edges  
to improve the throughput, no effect on latency nor on transient behaviour
- *pipelining*: introduction of additional buffers to improve the iteration bound  
affects throughput and transient behaviour
- *unfolding*: actor replication to increase the computational parallelism of the graph  
may affect the throughput, but not transient behaviour

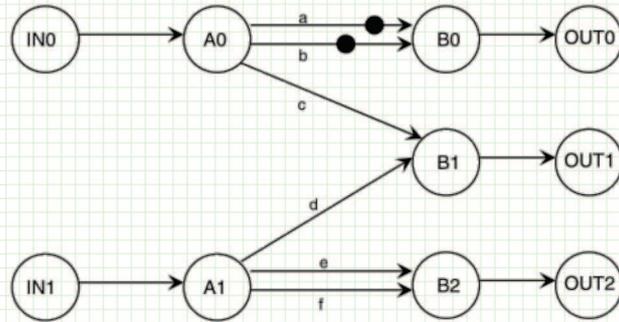
multirate SDF graph expansion

transformation of a multirate SDF graph to single-rate, in 5 steps:

1. compute the actors' firing vector
2. replicate each actor as many times as indicated by its firing rate
3. replicate each I/O of each actor's replica into as many single-rate I/O's as indicated by its consumption/production rate
4. re-introduce the queues to connect all actors in the single-rate SDF
5. re-introduce the initial tokens, distributing them sequentially over the single-rate queues



Schaumont, Figure 2.19 - Multi-rate data flow graph

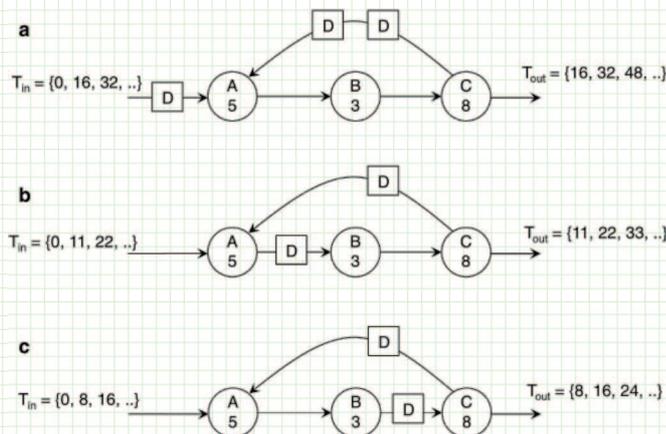


Schaumont, Figure 2.20 - Multi-rate SDF graph expanded to single-rate

retiming

this transformation redistributes the buffers without changing their total number

treat the buffers as tokens, moving them along the queues according to the actors' firings, and evaluate the SDF graph performance in the different states of the succession to choose the one which exhibits the best performance

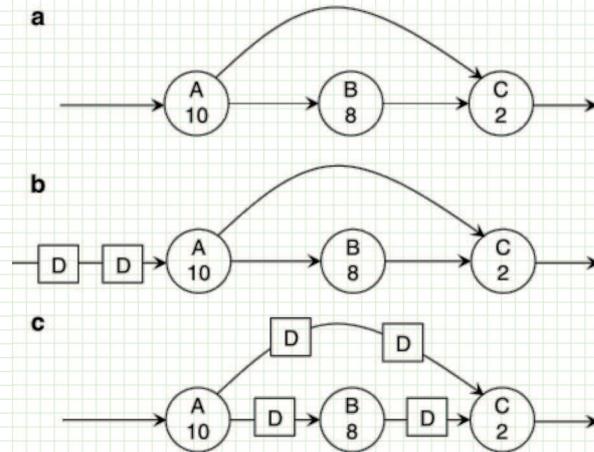


Schaumont, Figure 2.21 - Retiming: (a) Original Graph. (b) Graph after first re-timing transformation. (c) Graph after second re-timing transformation

## pipelining

pipelining = additional buffers + retiming

this transformation improves the throughput at the cost of an increased latency



Schaumont, Figure 2.22 - Pipelining: (a) Original graph. (b) Graph after adding two pipeline stages. (c) Graph after retiming the pipeline stages

## unfolding

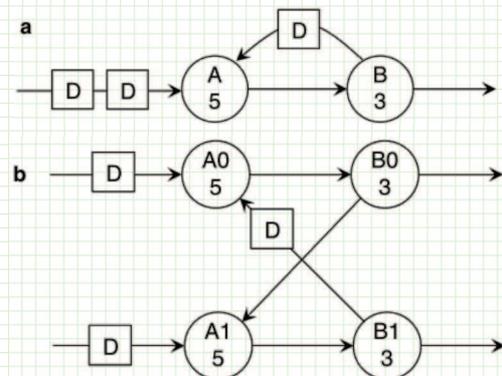
replication of a given SDF graph into several copies in order to increase the parallelism of input stream processing

rules are partly similar to those of multirate expansion

the  $V$ -unfolding of an SDF graph yields a graph with  $V$  copies of each actor and of each edge

if edge  $AB$  connects actors  $A$  and  $B$  in the original graph, where it carries  $n$  buffers, then in the transformed graph, for  $i = 0..V-1$ :

- edge  $(AB)_i$  connects actor  $A_i$  with actor  $B_k$  where  $k = (i + n) \bmod V$
- edge  $(AB)_i$  carries  $(i + n) / V$  buffers (with  $V-n$  edges without buffer if  $n < V$ )



Schaumont, Figure 2.23 - Unfolding: (a) Original graph. (b) Graph after two-unfolding

recommended readings:

Schaumont (2012) Ch. 2

for further consultation:

E.A. Lee & S.A. Seshia (2015) Ch. 6, Sect. 6.3

P. Marwedel (2011) Ch. 2, Sect. 2.5

A.H Ghamarian *et al.* (2007)

S. Stuijk *et al.* (2006)