Classes of balance laws reducible to conservation laws by means of equivalence transformations

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A class of partial differential equations (a conservation law and two balance laws), with three independent variables and involving six arbitrary continuously differentiable functions, is considered in the framework of equivalence transformations [1, 2, 3, 4, 5, 6, 7, 8]. The form of balance laws considered is

$$\frac{\partial u_1}{\partial t} + \frac{\partial u_2}{\partial x} + \frac{\partial u_3}{\partial y} = 0,$$

$$\frac{\partial u_2}{\partial t} + \frac{\partial p_1}{\partial x} + \frac{\partial p_2}{\partial y} = p_5,$$

$$\frac{\partial u_3}{\partial t} + \frac{\partial p_3}{\partial x} + \frac{\partial p_4}{\partial y} = p_6,$$

where $t, x$ and $y$ are the independent variables, $u_1, u_2$ and $u_3$ the dependent variables, whereas $p_i \equiv p_i(t, x, y, u_1, u_2, u_3)$ ($i = 1, \ldots, 6$) stand for arbitrary continuously differentiable functions of the indicated arguments; when $p_i(t, x, y, u_1, u_2, u_3) \equiv 0$ ($i = 5, 6$), we have a system of conservation laws.

The equivalence transformations are point transformations of differential equations involving arbitrary elements and live in an augmented space of independent, dependent and additional variables representing values taken by the arbitrary elements. Projecting the admitted symmetries into the space of independent and dependent variables, we determine some finite transformations mapping the system of balance laws to an equivalent one with the same differential structure but involving different arbitrary elements; in particular, we are interested in finding an equivalent autonomous system of conservation laws.

A similar approach has been used recently [9] for a $2 \times 2$ first order quasilinear system of partial differential equations.

Some examples of physical relevance (2D Euler equations of gas dynamics and rotating shallow water equations), where the above procedure can be applied, are discussed.