Interface energy notion revisited with the d’Alembert-Lagrange principle

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In this talk, we consider fluids and solids in presence of interfaces. Our aim is to emphasize the case when interfaces are endowed with non-constant surface energies; the surfaces have their own material properties independent of the bulks and are embedded in the physical space which is a three dimensional metric space. The equation governing the motions and the conditions on surfaces and edges between fluids and solids in presence of non-constant surface energies, as in the case of surfactants attached to the fluid particles at the interfaces, are revisited with the principle of virtual works. The principle of virtual works or the d’Alembert-Lagrange principle allows us to deduce both the equation of motion and conditions on surfaces and line by means of variational analysis. The surface energy density is taken into account and naturally intervenes in the boundary conditions as the Laplace and the Young-Dupré equations by using variations associated with virtual displacement fields. In the simplest cases two notions of surface energy and surface tension are mingled but it is not generally the case when the surface energy is non-constant along the interfaces: we point out that adequate behaviors of surface concentrations may drastically modify the surface tension which naturally appears in the Laplace and the Young-Dupré equations and strongly differs from the surface energy. As an example, when capillary forces operate and surfactant molecules are attached to the fluid molecules at the interfaces, the conditions on surfaces and lines reveal a fundamental difference between the concepts of surface energy and surface tension.

Complex behaviors can also change the form of the variation of the integral of the free energy which is related with the virtual work functional as in the case of shells or in second gradient models. We consider and discuss also the case when the surface energy depends on the surface curvature tensor as in membranes and vesicles which appear in biomechanics.