Almost self-complementary graphs

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Abstract

A graph $X$ is called almost self-complementary if it is isomorphic to one of its almost complements $X^c - I$, where $X^c$ denotes the complement of $X$ and $I$ a perfect matching (1-factor) in $X^c$. Thus, an almost self-complementary graph of order $2n$ is equivalent to an index-2 isomorphic factorization of the “cocktail party graph” $K_{2n} - nK_2$.

Almost self-complementary circulant graphs were first studied by Dobson and Šajna (2004). In this presentation we describe some of the properties and constructions of general almost self-complementary graphs. In particular, we give necessary and sufficient conditions on the order of an almost self-complementary regular graph, and construct infinite families of almost self-complementary regular graphs, almost self-complementary vertex-transitive graphs, and non-cyclically almost self-complementary circulant graphs.

Keywords: Self-complementary graph, almost self-complementary graph, homogeneously almost self-complementary graph, non-cyclically almost self-complementary circulant graph, isomorphic factorization, homogeneous factorization.