The existence of $4 \times 4$ grid-block designs

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Fu, Hwang, Jimbo, Mutoh and Shiue (2004) introduced the notion of a grid-block design, which is defined as follows: For a $v$-set $V$, let $\mathcal{A}$ be a collection of $k_1 \times k_2$ arrays with elements in $V$. Each array is called a grid-block. A pair $(V, \mathcal{A})$ is called a $k_1 \times k_2$ grid-block design if every two distinct points $x$ and $y$ in $V$ occurs exactly once in the same row or in the same column. The necessary conditions for the existence of a $k_1 \times k_2$ grid-block design are (i) $v - 1 \equiv 0 \pmod{k_1 + k_2 - 2}$ and (ii) $v(v - 1) \equiv 0 \pmod{k_1 k_2 (k_1 + k_2 - 2)}$. They gave some general constructions and proved that the necessary conditions $v \equiv 1, 9 \pmod{36}$ for the existence of $3 \times 3$ grid-block designs are sufficient. Meanwhile, Mutoh, Morihara, Jimbo and Fu (2003) solved that the necessary condition $v \equiv 1 \pmod{32}$ for the existence of $2 \times 4$ grid-block designs is sufficient.

In this talk, we show that the necessary condition $v \equiv 1 \pmod{96}$ for the existence of $4 \times 4$ grid-block designs is sufficient except for $v = 865$ and 1057 whose existence have not been known, yet. The proof is similar to the case of $2 \times 4$. Some $4 \times 4$ grid-block designs are constructed by the constructions in Fu et al. (2004) and another construction given by finite fields. By these constructions, we can shown the existence of $4 \times 4$ grid-block designs. Moreover, we mention a partial result of $3 \times 4$ grid-block designs. The necessary conditions for the existence of $3 \times 4$ grid-block designs are $v \equiv 1, 16, 21, 36 \pmod{60}$. It can be shown that the necessary condition $v \equiv 1 \pmod{60}$ is sufficient by the same method of $4 \times 4$. 