COVERING $K_{n,n} - M$ BY BICLIQUES

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The strong product $H = \boxtimes_{i=1}^k H_i$ of graphs $H_1, H_2, \ldots, H_k$ is the graph defined on the Cartesian product of the vertex sets of the factors, two distinct vertices $(u_1, u_2, \ldots, u_k)$ and $(v_1, v_2, \ldots, v_k)$ being adjacent if and only if $u_i$ is equal or adjacent to $v_i$ in $H_i$ for $i = 1, 2, \ldots, k$. The strong isometric dimension, $\text{idim}(G)$, of a graph $G$ is the least number $k$ such that there is a set of paths $\{P^{(1)}, P^{(2)}, \ldots, P^{(k)}\}$ with the property that $G$ isometrically embeds into $\boxtimes_{i=1}^k P^{(i)}$.

Jerebic and Klavzar conjectured that if $k \geq 1$ and \( \left\lfloor \frac{k}{2} \right\rfloor < n \leq \left\lfloor \frac{k+1}{2} \right\rfloor \) then $\text{idim}(K_2 \square K_n) = k + 1$. They also proved that the conjecture is equivalent to the following

Conjecture (Jerebic, Klavzar 2003).

Let $k \geq 1$ and let $n = \left\lfloor \frac{k+1}{2} \right\rfloor + 1$. Then the edges of $K_{n,n} - M$, where $M$ is the perfect matching, cannot be covered by $k$ complete bipartite graphs.

We prove the conjecture. If time permits, we also present some related results.