Ideals and Graph Theory

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There are several polynomial ideals one can associate to a simple undirected graph, both binomial and monomial ones. The aim of this work is to use these algebraic tools and in particular Gröbner bases, to discover properties of a graph and to implement procedures (we did it with Maple 8) in order to obtain these properties automatically.

What is known is a correspondence between even cycles and polynomials in a certain binomial ideal. Here we find correspondences between odd cycles and polynomials in an extended binomial ideal. Such results are used in order to show decision procedures for bipartite graphs, that are different from the usual approaches.

Finally topics on monomial ideals and known results in combinatorics are used in order to show decision procedures for minimal vertex covers and cliques of a graph with commutative algebra tools.