Searching for minimal blocking sets in generalized quadrangles

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A (finite) generalized quadrangle (GQ) of order \((s,t)\) is an incidence structure \(S = (P,B,I)\) in which \(P\) and \(B\) are disjoint (nonempty) sets of objects called points and lines (respectively), and for which \(I\) is a symmetric point-line incidence relation satisfying the following axioms: (i) each point is incident with \(1 + t\) lines \((t \geq 1)\) and two distinct points are incident with at most one line; (ii) each line is incident with \(1 + s\) points \((s \geq 1)\) and two distinct lines are incident with at most one point; (iii) if \(x\) is a point and \(L\) is a line not incident with \(x\), then there is a unique pair \((y,M) \in P \times B\) for which \(x I M I y I L\).

A blocking set \(B\) is a set of points, such that every line of the GQ meets \(B\) in at least one point, \(\mathcal{B} = st + 1 + \delta\), \(\delta > 0\) (for a GQ of order \((s,t)\)).

We will discuss algorithmic techniques for searching blocking sets in generalized quadrangles. This problem (cover problem) is well known to be NP-hard. We use standard strategies combined with techniques based on specific properties of the given generalized quadrangle. Non-trivial results obtained by this search will be presented.