On the unique independent set of four elements in PSL(2,31)

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A subset $S$ of a group $G$ is called **independent** if we have $s \not\in \langle S \setminus \{s\} \rangle$ for each $s \in S$.

In [2], useful connections between independent sets of a given group and incidence geometries on which that group acts flag-transitively are described. It is therefore meaningful to investigate independent sets for well-known families of groups. Such investigations were started in [1, 2, 3]. In the last paper the authors prove that an independent set in $PSL(2,p)$ has at most 4 elements for $p$ prime. They also show that the size of a maximal independent set is actually 3 when $p \not\equiv \pm 1 \mod 8$ and $p \not\equiv \pm 1 \mod 10$.

Investigating small primes which are not covered by [3] I found that $PSL(2,11)$ and $PSL(2,19)$ had many independent sets of size 4. Also that $PSL(2,29)$ has no independent set of size 4 and $PSL(2,31)$ has an independent set of size 4 which is unique up to conjugacy.

This unique independent set of size 4 for $PSL(2,31)$ gives rise to a rank 4 geometry which has many nice properties. We study this geometry and explain its connection with independent sets.

References

