Graph decompositions with some symmetry

Arrigo BONISOLI

Dipartimento di Scienze Sociali, Cognitive, Quantitative
Università di Modena e Reggio Emilia
via Giglioli Valle 9, 42100 Reggio Emilia (Italy)
e-mail: bonisoli.arrigo@unimore.it

A $\Gamma$–decomposition of the complete graph $K_v$, is a collection of subgraphs of $K_v$, all isomorphic to the given graph $\Gamma$, with the property that each edge of $K_v$ is to be found in precisely one subgraph of the collection.

Decompositions of various kinds are available in the literature. Here I want to assume that the automorphism group of such a decomposition is sufficiently rich. A typical assumption would be primitivity or multiple transitivity on vertices. It is clear for example that a bowtie or a kite decomposition of $K_v$ cannot admit an automorphism group acting doubly transitively on vertices (that would be true more generally for any $\Gamma$–decomposition in which $\Gamma$ is not a regular graph).

The idea of applying powerful classification theorems from finite group theory has been pursued in various situations with various degrees of success.

If one is happy with constructions, difference methods may be of help. I discuss the possibility of decomposing $K_v$ into Petersen graphs in such a way that the resulting decomposition admits an elementary abelian group of automorphisms acting transitively on vertices (here $v$ is a prime power).