THE LARGEST EIGENVALUE OF CATERPILLARS

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Abstract The eigenvalues of a (simple) graph are the eigenvalues of its adjacency matrix. The largest eigenvalue (or index) is very much studied in literature. In this paper we consider trees with fixed number of vertices and fixed diameter. In this situation the largest index is attained within caterpillars. (Recall, a caterpillar is a tree in which the removal of all vertices of degree one makes it a path.) Every caterpillar (of diameter $d$) can be denoted by $T(m_0, m_1, ..., m_{d-1}, m_d)$, which means a path with $d + 1$ vertices (say $0, 1, \ldots, d - 1, d$) to which there are attached stars $K_{1,m_i} ((m_i \geq 0, i = 0, 1, \ldots, d - 1, d)$ and $m_0 = m_d = 0$) by identifying the center of the $i$th star with the $i$th vertex the path.

It is easy to prove that a caterpillar with the largest index having $n$ vertices and diameter $d$ is equal to $T(0, \ldots, m_{\lfloor \frac{d+1}{2} \rfloor}, 0, \ldots, 0)$, where $m_{\lfloor \frac{d+1}{2} \rfloor} = n - d - 1$ (namely, it is a path on $d + 1$ vertices with a star $K_{1,n-d-1}$ attached to a vertex belonging to the center of the path).

A more difficult question arises if we keep fixed, besides $n$ and $d$, the stars of the caterpillars, and then to ask for those with the largest index. Let $T(0, m_1, ..., m_{d-1}, 0)$ be the corresponding class of caterpillars. In other words, $T(0, m_1, ..., m_{d-1}, 0)$ is the class of all caterpillars which differ from $T(0, m_1, ..., m_{d-1}, 0)$ only by a permutation of the values $m_1, ..., m_{d-1}$. In this paper we identify in $T(0, m_1, ..., m_{d-1}, 0)$ those caterpillars having the largest index.

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2000 Mathematical Subject Classification: 05C50
Key words and phrases: Graph eigenvalues, Graph index, Trees, Caterpillars