Minimal covering of all chords of a conic in $PG(2, q)$, $q$ even

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In this paper we determine the minimal blocking sets of chords of an irreducible conic $C$ in the desarguesian projective plane $PG(2, q)$, $q$ even. The procedure for the construction of minimal blocking sets of chords is the following.

Assume that the conic $C$ has (affine) equation $Y = X^2$, that is, $C$ is a parabola in the affine plane $AG(2, q)$. For every $a \in GF(q)$, $\varphi_a : (X, Y) \rightarrow (X + a, Y + a^2)$, is a translation of the affine plane $AG(2, q)$. The center of $\varphi_a$, viewed as an elation in the projective closure $PG(2, q)$ of $AG(2, q)$, is the infinite point $B_a = (1, a, 0)$.

The translation group of $C$ is $T = \{\varphi_a \mid a \in GF(q)\}$ and it is isomorphic to the additive group $(GF(q), +)$ of $GF(q)$. Take a subgroup $G = \{\varphi_a \mid a \in H\}$ of $T$ where $H$ is a subgroup in $(GF(q), +)$, and define $\Gamma$ to be the set of the centers of all nontrivial translations in $G$. If $P = (u, u^2)$ is an affine point in $C$, the orbit of $P$ under $G$ is $\Delta_u = \{(a + u, (a + u)^2) \mid a \in H\}$. Then, $B(G, u) = (C \setminus \Delta_u) \cup \Gamma$ is a blocking set of chords of $C$. We prove the following theorem.

**Theorem.** Let $C$ be an irreducible conic of $PG(2, q)$, with $q = 2^h$, and let $\mathcal{L}$ be the set of all chords of $C$. Any pointset $B$ of $PG(2, q)$ meeting every line of $\mathcal{L}$ has size at least $q$. If equality holds then $B = B(G, u)$ for some $B(G, u)$ arising from an additive subgroup $H$ of $GF(q)$ as in the above construction.