UNIVERSITÀ DI CATANIA
Corso di Laurea Magistrale in Matematica
Algebraic Geometry
ACADEMIC YEAR 2016-2017
Docente: Prof. Francesco Russo

I) – Revision on projective spaces: parametric and cartesian equations of linear spaces, dual projective space. Linear systems of hypersurfaces and applications to the first enumerative problems with special regards to conics and hyperquadrics. Lines incident four general lines in the space. Klein quadric as parameter space of lines in projective space. Planes in the Klein’s quadric via the geometry of lines.

II) – Affine and projective algebraic sets. Zariski topology on affine and projective spaces. Correspondence between affine algebraic sets and radical ideal in a polynomial ring (algebraically closed field). Irreducible algebraic sets and correspondence with prime ideals. Coordinate ring of an affine algebraic variety and of a projective variety. Decomposition of an algebraic set into irreducible components and its relations with primary decomposition of an ideal. Dimension of an algebraic variety: topological and algebraic definition.

III) – Regular functions on a quasi-projective variety: definition and first properties. Examples and applications. Morphisms between varieties: definition and first properties. Examples and applications. Local ring of regular functions on a variety: definition and first properties. Rational functions on a variety: definition and first properties. Rational (and birational) maps between algebraic varieties: definitions and first examples. Correspondence between dominant rational maps and homomorphisms of their function fields. Regular functions on a projective varieties and applications.


V) – Non-singular point on an algebraic variety: extrinsic and intrinsic definition. Singular locus. Blow-up of a variety at a point. Tangent cone and tangent space to a variety at a point: extrinsic and intrinsic definition. Examples and applications. Definition of multiplicity of a point on a variety. Comparison between the tangent cone and the tangent space at a point: non-singularity criterion.

VI) – Theorem on the dimension of the fibers of a morphism. Applications. Irreducibility Criterion. Applications to the study of lines on surfaces in projective space with special regard to the case of cubics. Dual variety and Bertini Theorem. Secant and plurisecant spaces to a variety: definitions and examples. Gauss map: definition and examples.

It time allows some topics will be chosen between the following ones:


References


Exercises and Lecture Notes:

– Appunti Lezioni del Corso, disponibili sulla pagina internet http://www.dmi.unict.it/~frusso/DMI/Geometria_Algebrica.html


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Web site: http://www.dmi.unict.it/~frusso/DMI/Geometria_Algebrica.html