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Giovanni Emmanuele

A bounded sequence (x_n) in a Banach space X is said to *pseudo-converge* to a point x_0 , called a pseudo limit, if x_0 minimizes the function

$$f_s(x) = \limsup_m \|y_m - x\|$$

for every subsequence $S = (y_n)$ of (x_n) . In the recent paper [2] the author put the following question: *Is it true that in a general Banach space X , if (x_n) pseudo-converges to θ , then there exist a sequence (z_n) in X and a sequence (z_n^*) in X^* such that $z_n^* \in J(z_n)$ for all $n \in \mathbb{N}$, $z_n^* \xrightarrow{w^*} \theta$ and $\lim_n \|x_n - z_n\| = 0$? (here J denotes a duality map; see [2]).*

In this short note we want to show that when $X = \ell_\infty$ the answer to the above question is negative. Let us choose $x'_n = e_n$, the unit vector basis of c_0 ; it is well known that it converges weakly to θ . Furthermore, it is simple to see that it pseudo-converges to more than one point of ℓ_∞ ([2]). Choose one of its non-zero pseudo-limits x'_0 and put $x_n = x'_n - x'_0$ for all $n \in \mathbb{N}$. Let us assume that there exist (z_n) and (z_n^*) as in the question above. It is clear that $z_n \xrightarrow{w} -x'_0$, too. Furthermore, $z_n^* \xrightarrow{w} \theta$ in $(\ell_\infty)^*$ (see [1, p. 103, Theorem 15]). Hence, $(z_n^*, z_n) = (z_n^*, z_n + x'_0) + (z_n^*, -x'_0)$; using well-known results about $C(K)$ spaces (ℓ_∞ is isomorphic to $C(\beta\mathbb{N})!$) (see [1, p. 113, Exercise 1]) we obtain that $(z_n^*, z_n) \rightarrow 0$ and so $z_n \xrightarrow{s} \theta$; this easily implies that $x'_0 = \theta$. This contradiction concludes the proof.

At the end we observe that each space X with the Dunford-Pettis property and the Grothendieck property, too, can be used to answer in the negative Lim's question as

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done above (for these definitions and useful reformulations, we refer to [1]), provided there exists in X a w -null sequence that pseudo-converges to a non-zero pseudo-limit.

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1. J. Diestel, *Sequences and series in Banach spaces*, Grad. Texts Math. 92, Springer Verlag, 1984.
2. Teck-Cheong Lim, *Pseudo-convergence in normed linear spaces*, Rocky Mountain J. Math. 21 (1991), 1057-1070.

Department of Mathematics, University of Catania, Viale A.Doria 6, 95125 Catania, Italy
e-mail address: emmanuele@math.cineca.it