

**ADDENDUM TO
REMARKS ON WEAK COMPACTNESS OF OPERATORS
DEFINED ON CERTAIN INJECTIVE TENSOR PRODUCTS**

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ADDENDUM TO
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The idea (used in [1]) of embedding $X \otimes_{\epsilon} Y$ into $X^{**} \otimes_{\epsilon} Y$, when X is a \mathcal{L}_{∞} -space can also be used to get the following results.

Theorem 1. *Let X be a \mathcal{L}_{∞} -space and Y be a Banach space. Then $X \otimes_{\epsilon} Y$ contains a complemented copy of l_1 iff Y does the same.*

Proof. If l_1 embeds complementably into $X \otimes_{\epsilon} Y$, then c_0 embeds into $(X \otimes_{\epsilon} Y)^*$, a closed subspace of $(X^{**} \otimes_{\epsilon} Y)^*$; then l_1 embeds complementably into $X^{**} \otimes_{\epsilon} Y$, that is a complemented subspace of some $C(K, Y)$ space (see Theorem 2 of [1]). Hence, it is enough to apply the main result of [2] to get our thesis.

Theorem 2. *Let X be a Banach space with X^* isometric to an L^1 space. If Y is a reflexive Banach space, then $X \otimes_{\epsilon} Y$ has property (V) of Pelczynski.*

Proof. Let T be an unconditionally converging operator on $X \otimes_{\epsilon} Y$. The results of [3] imply that T^{**} is unconditionally converging, too. Hence T^{**} restricted to $X^{**} \otimes_{\epsilon} Y$ is unconditionally converging; since $X^{**} \otimes_{\epsilon} Y$ is complemented in some $C(K, Y)$ space, it inherits property (V) of Pelczynski. Hence T^{**} , and so T , are weakly compact. We are done.

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