Explicit Constructive Logic ECL: a new representation of construction and selection of logical information by an epistemic agent

Paolo Gentilini e Maurizio Martelli

CILC 2013

gentilini@ge.imati.cnr.it, martelli@unige.it
Formal Inferential semantics for higher order logic (HOL) :

- a semantics for HOL having a constructive character
- distinction between meaning and truth value of a sentence
- a constructive semantics of proofs
- typed formulas are interpreted into an algebra of sequent-trees, reductions between trees correspond to the logical connectives of the interpreted formula
- **to interpret formulas (statements, sentences,...) of predicate logic into quantification acts (cognitive level) i.e. into quantification rules for $\exists$-R and $\forall$-L**

\[
\Gamma \vdash \Delta, [t_a/x_a]A \quad \exists-R \\
\Gamma \vdash \Delta, \exists x_a A
\]

CILC 2013 – Gentilini, Martelli
a semantics resembling S-Semantics

a more constructive proof of the completeness of HOL w.r.t. Henkin Models

scientific program: investigate what happens to our inferential models in various logics (intuitionistic, first order, paraconsistent, uniform, ....)

looking into intuitionistic logic, trying to explain the algebra of the sequent trees, some ideas about the nature of constructivity came out

ECL was conceived
• only atomic formulas occur in the logical axioms
• weakening rules are admitted only to introduce atomic formulas (not logical information)
• cut rule is admitted only with atomic cut formulas, i.e. cut cannot eliminate logical information
• each logical rule is always thought as occurring in some proof $P$, and have constraints on its auxiliary formulas that take into account their introduction in the whole proof-segment above the premise(s) of the rule occurrence in $P$. That is, they are global and not local constraints.
• i.e. only elementary data can be used without having been constructed
ECL (Z\(\omega\))

- Logical axioms \(A^+ \vdash A^+\), \(\vdash T\), \(\bot \vdash\)
- a propositional strong logical rule:
- \(A, B, \varphi \vdash \varphi \quad \varphi - L\)
  \[A \supset B, \varphi \vdash \]
- Weak logical rules and structural rules for the controlled use of weakening and cut
- global constraints.
- Z1 and ZP
ECL: intuitionistic or paraconsistent?

- ECL is orthogonal to the two other logics
- there are intuitionistic proofs that have a corresponding ECL proof and some that have not
- there are paraconsistent proofs that have a corresponding ECL proof and some that have not
- the “¬” is not constrained
- \( P \supset P \) is not in general valid: it depends on \( P \)
ECL and Logic Programming

• The types of proofs in LP seem, quite naturally, to have some relations with ECL.

• Facts are at the base of all the reasoning and there is a controlled use of implication.

• A first result:

  Given a refutation of a goal G in a program P (in definite clauses) it is possible to construct a corresponding ECL proof in Z1.

  The sequent proved is \( (C_i \text{ instances of clauses in } P, a_j \text{ in } G) \)

  \[ C_1, \ldots, C_n \vdash a_1 \lor \ldots \lor a_k \]

  [Given a clause C in P there are m instances of the n Cn if the refutation of G has used m times the clause C]

CILC 2013 – Gentilini, Martelli