A generalized system of Coupled KdV-like Equations

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We consider the generalized system of KdV-like equations

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\begin{align*}
  u_t + u_{xxx} + 2u u_x + 2e_1 v v_x + e_2 (u_x v + uv_x) + e_3 v_{xxx} &= 0, \\
  c_1 v_t + v_{xxx} + 2v v_x + c_2 v_x + c_3 [e_1 (u_x v + uv_x) + 2e_2 u u_x + e_3 u_{xxx}] &= 0,
\end{align*}
\]

(1)

coupled through both dispersive and nonlinear terms, which was derived by Gear and Grimshaw [1]. In (1), \(u(t,x)\) and \(v(t,x)\) are the dependent variables and subscripts denote partial derivatives with respect to the independent variables \(t\) and \(x\); moreover, \(e_i\) \((i = 1, 2, 3)\) and \(c_j\) \((j = 1, 2, 3)\) represent arbitrary constants.

The model describes the strong interaction of two-dimensional, long, internal gravity waves propagating on neighboring pycnoclines in a stratified fluid. Some mathematical questions related to the Cauchy Problem (CP) of (1) have been studied by Bona et al. [2] who showed that the (CP) is globally well-posed in suitably strong function spaces. Moreover, Bona and Saut have also stated that the system (1) is susceptible of experiencing the dispersive blow-up phenomenon.

We present some solutions [3]-[5] of physical relevance in closed form, in order to characterize a complete hierarchy of traveling wave solutions admitted by the model (1). Also, we determine the conditions on the parameters suitable to guarantee the possibilities that the model equations are self-adjoint in order to construct the associated conservation laws.


