Smooth Shock Wave Profiles with Maximum Entropy Gas Dynamics

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A shock wave is a nonlinear travelling wave in a gas that connects two equilibrium states with a steep transition layer. To predict a smooth and accurate shock wave profile within a continuum theory is a long standing problem of gas dynamics. The hyperbolic models of extended thermodynamics seem to be doomed to introduce sub-shocks into the profile as shown in [1]. These artefacts appear as soon as the shock wave moves faster than the fastest propagation speed of the theory in front of the shock.

These theoretical limitations have also been confirmed numerically, but typically based on continuum theories like Grad’s moment method that are linear in non-equilibrium fields. A fully nonlinear and globally hyperbolic theory is given within the framework of entropy maximization, see e.g., [2]. Even though these theories contain severe analytical and computational obstacles when including moments beyond the pressure tensor, it is possible to obtain numerical solutions based on suitable approximations as shown in [3].

It turns out that these maximum-entropy-models give smooth shock wave profiles even for very high shock speeds, seemingly contradicting the theory of [1]. However, a singularity of the flux function at equilibrium is responsible for generating arbitrarily high signal speeds in the system which ensure a smooth profile. In this talk we will study a system with maximum-entropy closure for a fixed number of variables. Details of the closing flux function as well as the Hugoniot-loci and wave speeds are given for the shock wave setting and special Riemann problems. The system is based on a kinetic equation with 1-dimensional velocity space. In [4] it is shown that a similar approach is also possible for the more realistic case of 3-dimensional velocity space.