First Workshop on
“Variational inequalities, Nash equilibrium problems and applications”

*Dipartimento di Matematica e Informatica - Università di Catania*
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**ABSTRACTS**

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**Titles and abstracts**

**Samir Adly** (University of Limoges)  
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**HOW ELEGANT MODERN CONVEX ANALYSIS WAS INFLUENCED BY MOREAU’S SEMINAL WORK**

We show in this presentation how elegant modern convex analysis was influenced by Moreau’s seminal work. We will review some major contributions of J.J. Moreau in convex analysis and nonsmooth mechanics. A large analysis in the talk will be devoted to the sweeping process introduced by J.J. Moreau in 1971 with concrete motivations in elastoplasticity, contact dynamics, friction dynamics and granular materials. The sweeping process model is also of great interest in nonsmooth mechanics, convex optimization, mathematical economics and more recently in the modeling and simulation of switched electrical circuits and crowd motion.

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**EVOLUTIONARY QUASI-VARIATIONAL INEQUALITIES AND APPLICATIONS TO A GENERAL COURNOT-NASH PRINCIPLE**

The aim of the talk is to study the dynamic oligopolistic market equilibrium problem in the realistic case in which we allow the presence of capacity constraints and production excesses and, moreover, we assume that the production function depends not only on the time but also on the equilibrium distribution. As a consequence, we introduce the generalized dynamic Cournot-Nash principle in the elastic case and prove the equivalence between this equilibrium definition and a suitable evolutionary quasi-variational inequality. For completeness we make the analysis of existence, regularity, and sensitivity of the solution. In the end, a numerical example is provided.

**Giancarlo Bigi** (University of Pisa)  
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**DESCENT METHODS FOR QUASI EQUILIBRIUM PROBLEMS**

The quasi-equilibrium problem (QEP) is a rather general format that is modelled upon generalized Nash games, i.e., noncooperative games in which the feasible strategies of one player depend upon the strategies chosen by the others (it happens, for instance, if players share resources). Descent methods have been widely studied for equilibrium problems (EPs), exploiting reformulations of EPs as optimization problems. The aim of this talk is to show how this kind of approach can be applied also for solving QEPs by introducing a suitable gap function. An overview of the basic ideas and features of this approach is provided, underlying how to overcome the difficulties due to the dependence of the feasible region upon the candidate solution. In particular, continuity and regularity properties of the gap function, the relationships between its stationary points and equilibria are analysed and error bounds are given.
Based on a joint work with M. Passacantando.

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**Existence results for nonlinear variational-hemivariational inequalities**

Some existence results for nonlinear variational-hemivariational inequalities are presented. The approach adopted is based on critical point theory for non-smooth functionals.

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**On a class of nonlinear elliptic variational-hemivariational inequalities**

The aim of this talk is to present some results on the existence of infinitely many solutions for a class of nonlinear elliptic variational-hemivariational inequalities. The approach is based on a result of infinitely many critical points. Moreover as a special case, a result of existence of infinitely many solutions to an elliptic Neumann problem involving the p-Laplacian is pointed out.

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**A necessary and sufficient optimality condition in vector optimization problems in infinite dimensional spaces**

In this paper a nonconvex vector optimization problem among infinite dimensional spaces is presented. In particular, a generalized Lagrange multiplier rule is formulated as necessary and sufficient optimality condition for weakly minimal solutions of a constrained vector optimization problem, without requiring that the ordering cone which defines the inequality constraints has nonempty interior. This result is achieved by using the tangent cone and by assuming the Hadamard differentiability of the maps.

**Roberto Livrea** (Mediterranea University of Reggio Calabria)
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**Critical point theory and variational-hemivariational inequalities**
The aim of the talk is to show some critical points results for possible non-differentiable functionals, useful to study classes of variational and variational-hemivariational inequalities. Some applications to concrete differential problems will be also presented.

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**Some multiplicity results for elliptic variational-hemivariational inequalities**

The existence of multiple solutions $u \in H_0^1(\Omega)$ to a differential inclusion of the type $-\Delta u \in \partial J(x,u)$ in $\Omega$, where $\partial J(x,\cdot)$ denotes the generalized sub-differential of $J(x,\cdot)$, is investigated through critical point theorems for locally Lipschitz continuous functionals on closed convex sets of a Hilbert space.

Antonino Maugeri (University of Catania)
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**On the multipliers associated with the equilibrium problems**

We prove that the Lagrange multiplier associated with the elastic-plastic torsion problem according to von Mises is always a Radon measure. If data have some kind of regularity, it is possible to show that the Lagrange multiplier is an $L_p$ function.

Monica Milasi (University of Messina)
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**Quasi concavity and existence result for competitive equilibrium**

In this talk, we are interested in approximating quasiconvex functions by means of strictly quasiconvex functions. We provide such an approximation, in a general Banach space. Next, the above result is applied in order to get a general theorem of existence of an equilibrium for market economies. In particular, we see how it is possible to weaken the assumption of strictly concave utility functions for the agents, by allowing them to have local maxima.

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During the last decades, Game Theory has found several applications in telecommunication systems and in particular in wireless networking problems. A recent interesting application proposes a bargaining game modeling for the design of radio resource management policies for an effective multicast service delivery in LTE-A (Long Term Evolution - Advanced) cellular systems. This service requires particular attention in handling fairness and service efficiency requirements of multicast users experiencing different channel conditions. Bargaining solutions show inherent nice properties, which make them very attractive for resource allocation and sharing problems in general. Moreover, they offer a mathematical characterization and a precise framework in which different solutions can be compared. A radio resource management is proposed for the considered multicast service, relying on game theoretic bargaining solutions such as the Nash Bargaining solution, the Kalai-Smorodinsky bargaining solution, the Egalitarian and the Utilitarian solutions. An optimal trade-off algorithm between fairness and efficiency allows the system to select the most appropriate solution on all available component carriers in the system.

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GENERAL FINANCIAL EQUILIBRIUM PROBLEM

We consider a general equilibrium model of financial flows and prices. The model is assumed evolving in time. Then equilibrium conditions are considered in dynamic sense. We present the governing variational inequality formulation and we study the dual Lagrange problem. From the Lagrange formulation we derive the Deficit Formula, Balance Law and Liability Formula which enable us to give some suggestion for the achievement of the world financial equilibrium.

Based on a joint work with A. Barbagallo, P. Daniele, M. Lorino and A. Maugeri.

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A CLOSED-LOOP SUPPLY CHAIN NETWORK UNDER ENVIRONMENTAL REGULATION: A VARIATIONAL INEQUALITIES APPROACH

Global climate change has encouraged international and regional adoption of pollution taxes and carbon emission reduction policies. These environmental policies have significantly affected the production choices of European energy and industrial sectors. In this paper, we propose a closed-loop supply chain network design problem that includes raw material suppliers, manufacturers, consumers, and recovery centers. The objective of this paper is to formulate and optimize the equilibrium state of this network assuming that manufacturers are subject to environmental regulation and a
pollution tax is imposed on transportation. The model is optimized and solved by using the theory of variational inequalities.

Based on a joint work with E. Allevi.

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SOLUTION METHODS FOR QUASI VARIATIONAL INEQUALITIES AND GENERALIZED NASH EQUILIBRIUM PROBLEMS

This talk deals with solution methods for two problems which are strictly connected: quasi variational inequalities (QVI), (i.e. a variational inequalities in which the feasible region depends on the variable) and the generalized Nash equilibrium problem (GNEP), (i.e. a noncooperative game in which the strategy set of each player, as well as his payoff function, depends on the strategies of all players). Recently, QVI and GNEP have emerged as an effective and powerful tool for modelling a wide class of problems arising in many fields (electricity, telecommunications, transportation and others). We consider the role of optimization for analyzing properties of some solution methods for QVI and GNEP: projection, sequential VI, Kuhn-Tucker approach, and gap functions. Finally, we propose a derivative-free descent type method (in the class of gap function methods) for GNEP with inexact line search and we prove that our algorithm is globally convergent. The convergence analysis is not based on conditions guaranteeing that every stationary point of the optimization problem is a solution of GNEP.

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CUTTING PLANE METHODS FOR EQUILIBRIUM PROBLEMS

The abstract equilibrium problem (EP) provides a rather general setting which includes several mathematical models such as optimization, variational inequalities, fixed point and complementarity problems, Nash equilibria in noncooperative games. It is well-known that a pseudomonotone EP is equivalent to minimize the so-called Minty gap function. Though it is a convex function, it can be difficult to evaluate its values since the evaluation requires to solve non-convex optimization problems. The aim of this paper is to develop cutting plane methods for solving EP via the Minty gap function, relying on lower convex approximations which are easier to compute. These methods require to solve a convex optimization problem at each iteration and their convergence is proved under suitable monotonicity assumptions.

Based on a joint work with G. Bigi and G. Mastroeni.
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Stochastic Nash equilibrium problems with applications to oligopoly models

We use the theory of stochastic variational inequalities to investigate Nash equilibrium problems with uncertain data. We apply our theory to study in detail a Cournot oligopoly model.

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Compactness and Structural Stability of Nonlinear Flows

After Fitzpatrick’s seminal work (S. Fitzpatrick: Representing monotone operators by convex functions. Workshop/Miniconference on Functional Analysis and Optimization (Canberra, 1988), 59–65, Proc. Centre Math. Anal. Austral. Nat. Univ., 20, Austral. Nat. Univ., Canberra, 1988), it is known that in a Banach space $V$ any maximal monotone operator $\alpha : V \to \mathcal{P}(V')$ may be given a variational representation, even if $\alpha$ is no subdifferential. This is here illustrated on some examples.

On this basis De Giorgi’s $\Gamma$-convergence is then applied to the analysis of monotone inclusions. Via Fitzpatrick’s theory, compactness and structural stability of the Cauchy problem

$$\frac{du}{dt} + \alpha(u) \ni h \quad \text{in } V', \text{ a.e. in } [0, T], \quad u(0) = u^0$$

is studied, with respect to variations not only of the data $h \in L^2(0, T; V')$ and $u^0 \in V'$, but also of the maximal monotone operator $\alpha$. See e.g. A. Visintin: Variational formulation and structural stability of monotone equations. Calc. Var. Partial Differential Equations 47 (2013), 273–317.