Corso di laurea triennale in Fisica
Prova scritta di Meccanica Analitica
Appello del 23.12.2022

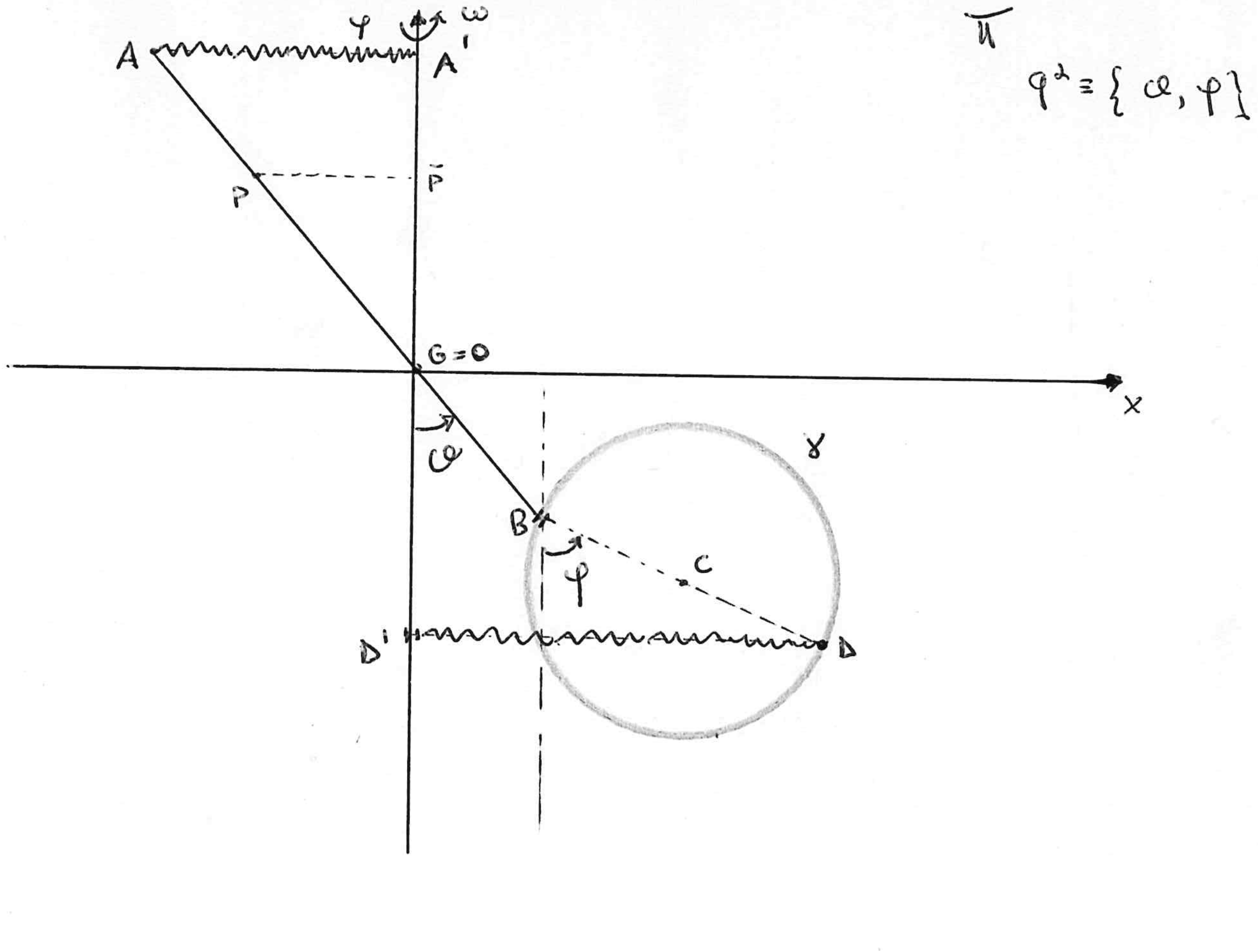
In un piano verticale Π sia dato un sistema di riferimento $\{O, \vec{x}, \vec{y}\}$ (vedi figura) con $\vec{y}$ verticale ascendente. Su Π si abbia un sistema materiale costituito da un'asta materiale AB di massa m e lunghezza $4r$, con densitá non omogenea che, in un suo punto P, é espressa dalla relazione $\rho(P) = \rho_0 \, (PA)^2/r^2$ (con ρ_0 e r costanti), che puó ruotare attorno al suo baricentro G concidente con l'origine O del riferimento. L'estremo B dell'asta é collegato tramite una cerniera con un disco materiale omogeneo γ, mobile in Π, di raggio r e massa m.
Sul sistema agiscono, oltre le due forze peso, le due forze elastiche

$$\{F_1 = -k\,(A - A'),\ A\} \qquad \text{e} \qquad \{F_2 = -k\,(D - D'),\ D\} \qquad \text{con} \quad k > 0,$$

essendo D il punto sul bordo di γ diametralmente opposto a B ed i punti A' e D' le proiezioni ortogonali di A e D sulla verticale y. Supposto che il piano Π sia posto in rotazione uniforme con velocitá angolare ω attorno alla verticale y, che tutti i vincoli siano realizzati senza attrito, ed utilizzando come coordinate Lagrangiane l'angolo ϑ che $\overline{GB}$ forma con l'asse delle y negativa e l'angolo φ che $\overline{BD}$ (diametro di γ) forma con la verticale discendente passante per B, si chiede di determinare nel riferimento relativo:

1. Le condizioni a cui debbono soddisfare i parametri affinché sia configurazione di equilibrio relativo per il sistema quella per cui $\overline{\vartheta} = (2/3)\pi$ e $\overline{\varphi} = 0$.

2. Determinare nelle condizioni di cui al punto 1. :
   a) Tutte le configurazioni di equilibrio, discutendone la stabilitá-instabilitá.
   b) Le equazioni del moto relativo e gli eventuali integrali primi.
   c) Studiare i moti in prima approssimazione, attorno alla configurazione di equilibrio del sistema di cui al punto 1.



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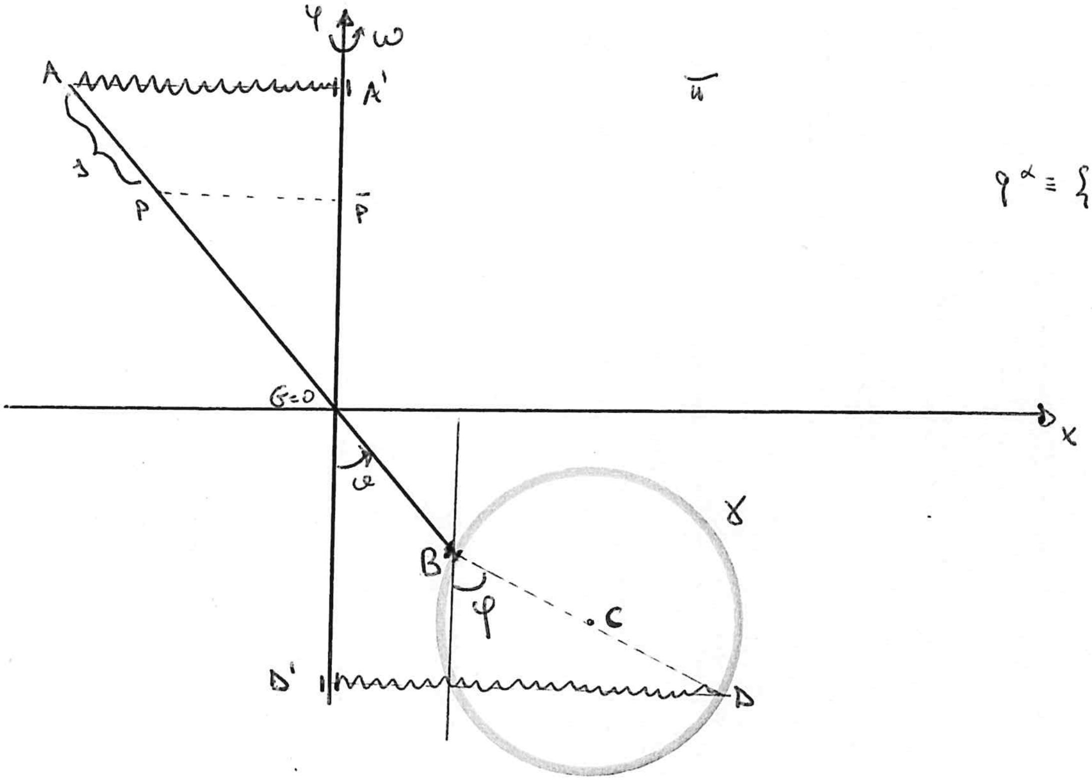


π

$q^\alpha \equiv \{\varphi, \psi\}$

$\overline{AB} =$ asta non omogenea $\begin{cases} m & \text{(massa)} \\ 4r & \text{(lunghezza)} \\ \rho(P) = \rho_0 \dfrac{\overline{PA}^2}{r^2} \end{cases}$

$\gamma \equiv \begin{cases} \text{disco omogeneo} \\ m & \text{(massa)} \\ r & \text{(raggio)} \end{cases}$

FORZE: FORZE PESO, FORZE ELASTICHE $\begin{cases} \{F_1 = -k(A-A'), A\} \\ \{F_2 = -k(D-D'), D\} \end{cases}$ con $k>0$

FORZE APPARENTI: FORZE CENTRIFUGHE "NOTA" (LA FIGURA E' NEL PIANO π QUINDI NON CONSIDERIAMO CORIOLIS)

BARICENTRO:

$$G-A = \frac{1}{m}\int_0^{4r} s\, dm = \frac{1}{m}\int_0^{4r} s\left(\rho_0 \frac{s^2}{r^2}\right) ds = \frac{1}{m}\frac{\rho_0}{r^2}\int_0^{4r} s^3\, ds =$$

$$= \frac{1}{m}\frac{\rho_0}{r^2}\frac{(4r)^4}{4} = \frac{1}{m}\rho_0 4^3 r^2$$

$$m = \int_0^{4r} dm = \frac{\rho_0}{r^2}\int_0^{4r} s^2\, ds = \frac{\rho_0}{r^2}\cdot\frac{(4r)^3}{3} = \rho_0 \frac{4^3}{3} r$$

da cui $G-A = \dfrac{3}{\cancel{4^3}\cancel{r}}\dfrac{1}{\cancel{\rho_0}}\cancel{\rho_0}\,\cancel{4^3}\cancel{r^2} = 3r$

QUINDI:

$A = \{-3\ell\,\mathrm{sen}\,\alpha,\ 3\ell\cos\alpha\}$ $\qquad B \equiv \{\ell\,\mathrm{sen}\,\alpha,\ -\ell\cos\alpha\}$

$C \equiv \{\ell\,\mathrm{sen}\,\alpha + \ell\,\mathrm{sen}\,\varphi,\ -\ell\cos\alpha - \ell\cos\varphi\}$; $\qquad G = (0,0)$.

$D \equiv \{\ell\,\mathrm{sen}\,\alpha + 2\ell\,\mathrm{sen}\,\varphi,\ -\ell\cos\alpha - 2\ell\cos\varphi\}$;

CALCOLIAMO IL POTENZIALE

$\{F^{(PESO,\gamma)} = -mg(0,1),\ C\} \Rightarrow U^{\gamma}_{PESO} = -mg(0,1)\cdot(\ell(\mathrm{sen}\,\alpha + \mathrm{sen}\,\varphi), -\ell(\cos\alpha+\cos\varphi))$

$= mg\ell(\cos\alpha + \cos\varphi)$.

FORZA: $\quad F_1 = -K(A-A'),\ A \Rightarrow U_{F_1} = -\frac{1}{2}K(A-A')^2 = -\frac{1}{2}K(3\ell\,\mathrm{sen}\,\alpha)^2 =$

$= -\frac{9}{2}K\ell^2\,\mathrm{sen}^2\alpha$

$\{F_2 = -K(D-D'),\ D\} \Rightarrow U_{F_2} = -\frac{1}{2}K(D-D')^2 = -\frac{1}{2}K(\ell\,\mathrm{sen}\,\alpha + 2\ell\,\mathrm{sen}\,\varphi)^2$

$= -\frac{1}{2}K\ell^2(\mathrm{sen}^2\alpha + 4\,\mathrm{sen}^2\varphi + 4\,\mathrm{sen}\,\alpha\,\mathrm{sen}\,\varphi)$

$dF^{(CENT,\gamma)} = \omega^2(P-\bar{P})\,dm \Rightarrow U^{\gamma}_{CENT.} = \frac{1}{2}\omega^2\int(P-\bar{P})^2\,dm = \frac{1}{2}\omega^2 I^{\gamma}_{y,O}$.

$I^{\gamma}_{y,O} = I^{\gamma}_{y,C} + m(C-\bar{C})^2$ $\qquad$ MA IL MOMENTO DI INERZIA $I^{\gamma}_{y,C} = \mathrm{cost}$

$= m\ell^2(\mathrm{sen}\,\alpha + \mathrm{sen}\,\varphi)^2 + \mathrm{Kost}$.

DA CUI $\quad U^{\gamma}_{CENT} = \frac{1}{2}m\omega^2\ell^2(\mathrm{sen}\,\alpha + \mathrm{sen}\,\varphi)^2 + \mathrm{Kost}$.

PER L'ASTA $\overline{AB}$ $\qquad P \equiv \{(3\ell - s)\,\mathrm{sen}\,\alpha,\ (3\ell - s)\cos\alpha\}$

$U^{AB}_{CENT} = \frac{1}{2}\omega^2\int(P-\bar{P})^2\,dm = \frac{1}{2}\omega^2\int_0^{4\ell}(3\ell-s)^2\,\mathrm{sen}^2\alpha\cdot\left(\rho_0\frac{s^2}{\ell^2}\right)ds$

$= \frac{1}{2}\omega^2\frac{\rho_0}{\ell^2}\,\mathrm{sen}^2\alpha\int_0^{4\ell}(3\ell-s)^2 s^2\,ds = \frac{1}{2}\frac{\omega^2}{\ell^2}\cdot\left(\frac{3}{4^3}\frac{m}{\ell}\right)\mathrm{sen}^2\alpha\int_0^{4\ell}(3\ell-s)^2 s^2\,ds$

$$\int_0^{4z} (9z^2 + s^2 - 6zs)\, s^2 ds = 9z^2 \int_0^{4z} s^2 ds + \int_0^{4z} s^4 ds - 6z \int_0^{4z} s^3 ds$$

$$= 9z^2 \frac{(4z)^3}{3} + \frac{(4z)^5}{5} - 6z \frac{(4z)^4}{4} = (4z)^3 \left\{ 3z^2 + \frac{16z^2}{5} - 6z \cdot \frac{4z}{4} \right\}$$

$$= (4z)^3 z^2 \left\{ 3 + \frac{16}{5} - 6 \right\} = (4z)^3 \frac{z^2}{5}$$

$$U^{AB}_{cont} = \frac{3}{2} \frac{m}{z^3} \frac{1}{\cancel{4^3}} \omega^2 \operatorname{sen}^2\alpha \cdot \cancel{(4)^3} \frac{z^5}{5} = \frac{3}{10} m z^2 \omega^2 \operatorname{sen}^2\alpha.$$

Quindi:

$$\boxed{\begin{aligned} U_{TOT} = {} & mgz(\cos\alpha + \cos\varphi) - \frac{9}{2} Kz^2 \operatorname{sen}^2\alpha - \frac{1}{2} Kz^2 (\operatorname{sen}\alpha + 2\operatorname{sen}\varphi)^2 \\ & + \frac{1}{2} m\omega^2 z^2 (\operatorname{sen}\alpha + \operatorname{sen}\varphi)^2 + \frac{3}{10} m\omega^2 z^2 \operatorname{sen}^2\alpha. \end{aligned}}$$

da cui

$$Q_\alpha = \frac{\partial U}{\partial \alpha} = -mgz \operatorname{sen}\alpha - 9Kz^2 \operatorname{sen}\alpha \cos\alpha - Kz^2 (\operatorname{sen}\alpha + 2\operatorname{sen}\varphi)\cos\alpha$$
$$+ m\omega^2 z^2 (\operatorname{sen}\alpha + \operatorname{sen}\varphi)\cos\alpha + \frac{3}{5} m\omega^2 z^2 \operatorname{sen}\alpha \cos\alpha$$

$$Q_\varphi = \frac{\partial U}{\partial \varphi} = -mgz \operatorname{sen}\varphi - Kz^2(\operatorname{sen}\alpha + 2\operatorname{sen}\varphi)\, 2\cos\varphi$$
$$+ m\omega^2 z^2 (\operatorname{sen}\alpha + \operatorname{sen}\varphi)\cos\varphi$$

Calcoliamo le sollecitazioni per $\bar\alpha = \pi - \pi/3 \qquad \bar\varphi = 0$

da cui $\operatorname{sen}\bar\alpha = \operatorname{sen}\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2} \qquad \cos\bar\alpha = \cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$

$$Q_\alpha \big|_{\bar\alpha,\bar\varphi} = +2mgz\left(\cancel{-\frac{\sqrt{3}}{4}}\right) - 9Kz^2\left(\cancel{-\frac{\sqrt{3}}{4}}\right) - Kz^2\left(\cancel{\frac{\sqrt{3}}{2}}\right)\left(-\frac{1}{2}\right)$$
$$+ m\omega^2 z^2 \left(\cancel{\frac{\sqrt{3}}{2}}\right)\left(\cancel{-\frac{1}{2}}\right) + \frac{3}{5} m\omega^2 z^2 \left(\cancel{\frac{\sqrt{3}}{2}}\right)\left(\cancel{-\frac{1}{2}}\right) = 0$$

da cui

$$\boxed{Q_\alpha \big|_{\bar\alpha,\bar\varphi} = 2mgz - 10Kz^2 + \frac{8}{5} m\omega^2 z^2 = 0} \qquad (1)$$

ANALOGAMENTE

$$Q_\varphi\Big|_{\bar{\alpha},\bar{\varphi}} = -K z^2 \left(\cancel{\frac{\sqrt{3}}{2}}\right)\cdot 2 + m\omega^2 z^2 \left(\cancel{\frac{\sqrt{3}}{2}}\right) = 0$$

CIOÈ LA CONDIZIONE $\quad m\omega^2 \cancel{z^2} - 2K\cancel{z^2} = 0 \quad \Rightarrow \quad \boxed{m\omega^2 = 2K}$ (2)

CHE SOSTITUITA NELLA (1) CI DARÀ:

$$2mg\cancel{z} - 10Kz^{\cancel{2}} + \frac{16}{5}Kz^{\cancel{2}} = 0 \quad \Rightarrow \quad \boxed{mg = \frac{17}{5}Kz}$$

DA CUI SOSTITUENDO QUESTI DUE RISULTATI AVREMO

$$Q_\alpha = -\frac{17}{5}Kz^2 \operatorname{sen}\alpha - 9Kz^2 \operatorname{sen}\alpha\cos\alpha - Kz^2(\operatorname{sen}\alpha + \cancel{2\operatorname{sen}\varphi})\cos\alpha$$
$$+ 2Kz^2(\operatorname{sen}\alpha + \cancel{\operatorname{sen}\varphi})\cos\alpha + \frac{6}{5}Kz^2\operatorname{sen}\alpha\cos\alpha$$

$$Q_\varphi = -\frac{17}{5}Kz^2\operatorname{sen}\varphi - 2Kz^2(\cancel{\operatorname{sen}\alpha} + 2\operatorname{sen}\varphi)\cos\varphi +$$
$$+ 2Kz^2(\cancel{\operatorname{sen}\alpha} + \operatorname{sen}\varphi)\cos\varphi$$
$$= -\frac{17}{5}Kz^2\operatorname{sen}\varphi - 2Kz^2\operatorname{sen}\varphi\cos\varphi =$$
$$= -Kz^2\operatorname{sen}\varphi\left\{+\frac{17}{5} + 2\cos\varphi\right\} = 0$$

DA $Q_\alpha = -\frac{17}{5}Kz^2\operatorname{sen}\alpha + Kz^2\left[-9 - 1 + 2 + \frac{6}{5}\right]\operatorname{sen}\alpha\cos\alpha$

$$= -\frac{17}{5}Kz^2\operatorname{sen}\alpha - \frac{34}{5}Kz^2\operatorname{sen}\alpha\cos\alpha =$$
$$= -\frac{17}{5}Kz^2\operatorname{sen}\alpha\left\{1 + 2\cos\alpha\right\} = 0$$

CIOÈ IL SISTEMA:

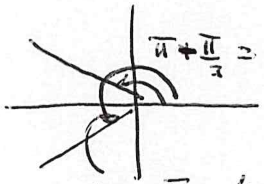
$$\begin{cases} Q_\varphi = -Kz^2\operatorname{sen}\varphi\left\{\frac{17}{5} + 2\cos\varphi\right\} = 0 \\ Q_\varphi = -\frac{17}{5}Kz^2\operatorname{sen}\alpha\left\{1 + 2\cos\alpha\right\} = 0 \end{cases}$$

DALLA 1^a RELAZIONE

$$\operatorname{sen}\varphi = 0 \Rightarrow \varphi = 0, \pi \quad \left(\text{OPPURE } \cos\varphi = -\frac{17}{10} \text{ "ASSURDO"}\right)$$

DALLA II^a RELAZIONE

$$\operatorname{sen}\alpha = 0 \Rightarrow \alpha = 0, \pi \quad \text{OPPURE } \cos\alpha = -\frac{1}{2} \Rightarrow \alpha = \begin{cases} \frac{2}{3}\pi \\ \frac{4}{3}\pi \end{cases}$$



DA CUI LE CONFIGURAZIONI DI EQUILIBRIO

$$(\alpha, \varphi) \equiv \{ S_1 \equiv (0, 0) \ ; \ S_2 = (0, \pi) \ ; \ S_3 = (\pi, 0) \ ; \ S_4 = (\pi, \pi)$$
$$S_5 = \left(\frac{2}{3}\pi, 0\right) \ ; \ S_6 = \left(\frac{2}{3}\pi, \pi\right) \ ; \ S_7 = \left(\frac{4}{3}\pi, 0\right) \ ; \ S_8 = \left(\frac{4}{3}\pi, \pi\right) \}$$

"STABILITÀ"

$$\frac{\partial^2 U}{\partial \varphi^2} = \frac{\partial Q_\varphi}{\partial \varphi} = -K\ell^2 \cos\varphi \left\{ \frac{17}{5} + 2\cos\varphi \right\} + 2K\ell^2 \operatorname{sen}^2\varphi$$

$$\frac{\partial^2 U}{\partial \alpha^2} = \frac{\partial Q_\alpha}{\partial \alpha} = -\frac{17}{5} K\ell^2 \cos\alpha \left\{ 1 + 2\cos\alpha \right\} + \frac{34}{5} K\ell^2 \operatorname{sen}^2\alpha$$

$$\frac{\partial^2 U}{\partial \varphi \partial \alpha} = \frac{\partial^2 U}{\partial \alpha \partial \varphi} = 0$$

$$H(\alpha, \varphi) = \frac{17}{5}(K\ell^2)^2 \left\{ \left[2\operatorname{sen}^2\varphi - \cos\varphi\left(\frac{17}{5} + 2\cos\varphi\right)\right] \left[2\operatorname{sen}^2\alpha - \cos\alpha\,(1 + 2\cos\alpha) \right] \right\}$$

) $S_1 = (\alpha = 0, \varphi = 0)$

$$\left.\frac{\partial^2 U}{\partial \varphi^2}\right|_{S_1} = -\frac{27}{5} K\ell^2 < 0$$

$$\Rightarrow \text{STABILE} \quad (\text{MAX})$$

$$H|_{S_1} = \frac{17}{5}(K\ell^2)^2 \left\{ \left(-\frac{27}{5}\right)(-3) \right\} > 0$$

) $S_2 \equiv (\alpha = 0, \varphi = \pi)$

$$H|_{S_2} = \frac{17}{5}(K\ell^2)^2 \left\{ \left(\frac{7}{5}\right)(-1) \right\} < 0 \quad \Rightarrow \quad \text{"INSTABILE"} \quad (\text{NO MAX})$$

3) $S_3 = (\vartheta = \pi\ ,\ \varphi = 0)$

$\left.\frac{\partial^2 U}{\partial \varphi^2}\right|_{S_3} = -\frac{27}{5} K z^2 < 0$ $\Rightarrow$ "STABILE" (MAX)

$H|_{S_3} = \frac{17}{5}(Kz^2)^2 \left\{ \left(-\frac{27}{5}\right)(-1) \right\} > 0$

4) $S_4 \equiv (\vartheta = \pi\ ,\ \varphi = \pi)$

$\Rightarrow$ "INSTABILE" (NO MAX)

$H|_{S_4} = \frac{17}{5}(Kz^2)^2 \left\{ \left(\frac{7}{5}\right)(-1) \right\} < 0$

5) $S_5 = (\vartheta = \frac{2}{3}\pi\ ,\ \varphi = 0)$

$\Rightarrow$ "INSTABILE" (NO MAX)

$H|_{S_5} = \frac{17}{5}(Kz^2)^2 \left\{ \left(-\frac{27}{5}\right) \left[2\left(\frac{3}{4}\right) + \frac{1}{2}\cancel{(1-1)} \right] \right\} < 0$

6) $S_6 = (\vartheta = \frac{2}{3}\pi\ ,\ \varphi = \pi)$

$\left.\frac{\partial^2 U}{\partial \varphi}\right|_{S_6} = Kz^2 \left\{ \frac{7}{5} \right\} > 0$ $\Rightarrow$ "INSTABILE" (NO MAX)

$H|_{S_6} = \frac{17}{5}(Kz^2)^2 \left\{ \left(\frac{7}{5}\right) \cdot \frac{3}{2} \right\} > 0$

7) $S_7 = (\vartheta = \frac{4}{3}\pi\ ,\ \varphi = 0)$

$\left.\frac{\partial^2 U}{\partial \varphi^2}\right|_{S_7} = Kz^2 \left(-\frac{27}{5}\right) < 0$

$\Rightarrow$ "INSTABILE" (NO MAX)

$H|_{S_7} = \frac{17}{5}(Kz^2)^2 \left\{ \left(-\frac{27}{5}\right) \left[2\ \frac{3}{4} \right] \right\} < 0$

8) $S_8 = (\vartheta = \frac{4}{3}\pi\ ,\ \varphi = \pi)$

$H|_{S_8} = \frac{17}{5}(Kz^2)^2 \left\{ \left(\frac{7}{5}\right) \left[\frac{3}{2} \right] \right\} > 0$

$\Leftrightarrow$ INSTABILE (NO MAX)

$\left.\frac{\partial^2 U}{\partial \varphi^2}\right|_{S_8} = Kz^2 \cdot \frac{7}{5} > 0$

QUINDI $S_1 = (0,0)$ $S_3 = (\pi, 0)$ SONO STABILI

$S_2, S_4, S_5, S_6, S_7, S_8$ SONO INSTABILI

— o —

$\boxed{\text{"ENERGIA CINETICA"}}$

$$T = T_{AB} + T_\gamma$$

$$T_{AB} = \frac{1}{2} I^{AB}_{z,G} \dot{\alpha}^2$$

$$I^{AB}_{z,G} = \int (P-O)^2 dm = \int (3\ell - s)^2 dm =$$

$$= \underbrace{\rho_0 \int_0^{4\ell} \frac{s^2}{\ell^2} (3\ell - s)^2 ds}_{\text{CONE A PAG. 2 A PARTE IL FATTORE } \frac{1}{2}\omega^2 \operatorname{sen}^2\alpha} = \frac{3}{5} m \ell^2$$

DA CUI $\boxed{T_{AB} = \frac{3}{10} m \ell^2 \dot{\alpha}^2}$

$$T_\gamma = \frac{1}{2} m \dot{C}^2 + \frac{1}{2} I^\gamma_{z,C} \dot{\varphi}^2$$

$$I^\gamma_{z,C} = \sigma \iint \rho^2 \rho \, d\rho \, d\alpha = \frac{m}{\pi \ell^2} \int_0^R \rho^3 d\rho \int_0^{2\pi} d\alpha = \frac{m}{\cancel{\pi} \ell^2} \cdot \frac{\ell^4}{4} \cdot \cancel{2\pi} = \frac{1}{2} m \ell^2$$

$$\dot{C} = \{ \ell \cos\alpha \, \dot{\alpha} + \ell \cos\varphi \, \dot{\varphi}, \ \ell \operatorname{sen}\alpha \, \dot{\alpha} + \ell \operatorname{sen}\varphi \, \dot{\varphi} \}$$

$$\dot{C}^2 = \ell^2 \dot{\alpha}^2 + \ell^2 \dot{\varphi}^2 + 2\ell^2 \dot{\alpha}\dot{\varphi} \underbrace{[\cos\alpha \cos\varphi + \operatorname{sen}\alpha \operatorname{sen}\varphi]}_{\cos(\alpha - \varphi)}$$

$$T_\gamma = \frac{1}{2} m \ell^2 \{ \dot{\alpha}^2 + \dot{\varphi}^2 + 2\dot{\alpha}\dot{\varphi}\cos(\alpha - \varphi) \} + \frac{1}{4} m \ell^2 \dot{\varphi}^2$$

$$\boxed{T_\gamma = \frac{1}{2} m \ell^2 \{ \dot{\alpha}^2 + \frac{3}{2}\dot{\varphi}^2 + 2\dot{\alpha}\dot{\varphi}\cos(\alpha - \varphi)}$$

$$\boxed{T = \frac{1}{2} m r^2 \left\{ \frac{8}{5} \dot{\alpha}^2 + \frac{3}{2} \dot{\varphi}^2 + 2 \dot{\alpha} \dot{\varphi} \cos(\alpha - \varphi) \right]}$$

DA CUI LE EQUAZIONI DEL MOTO:

$$\frac{\partial T}{\partial \dot{\alpha}} = \frac{8}{5} m r^2 \dot{\alpha} + m r^2 \dot{\varphi} \cos(\alpha - \varphi)$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} = \frac{8}{5} m r^2 \ddot{\alpha} + m r^2 \ddot{\varphi} \cos(\alpha - \varphi) - m r^2 \sin(\alpha - \varphi)(\dot{\alpha} - \dot{\varphi}) \dot{\varphi}$$

$$\frac{\partial T}{\partial \alpha} = - m r^2 \sin(\alpha - \varphi) \dot{\varphi} \dot{\alpha}$$

DA CUI

$$\boxed{\frac{8}{5} m r^2 \ddot{\alpha} + m r^2 \ddot{\varphi} \cos(\alpha - \varphi) + m r^2 \dot{\varphi}^2 \sin(\alpha - \varphi) = Q_\alpha = - \frac{17}{5} K r^2 \sin\alpha \left[1 + 2 \cos\alpha \right]}$$

$$\frac{\partial T}{\partial \dot{\varphi}} = \frac{3}{2} m r^2 \dot{\varphi} + m r^2 \dot{\alpha} \cos(\alpha - \varphi)$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\varphi}} = \frac{3}{2} m r^2 \ddot{\varphi} + m r^2 \ddot{\alpha} \cos(\alpha - \varphi) - m r^2 \sin(\alpha - \varphi)(\dot{\alpha} - \dot{\varphi}) \dot{\alpha}$$

$$\frac{\partial T}{\partial \varphi} = m r^2 \dot{\alpha} \dot{\varphi} \sin(\alpha - \varphi)$$

DA CUI:

$$\boxed{\frac{3}{2} m r^2 \ddot{\varphi} + m r^2 \ddot{\alpha} \cos(\alpha - \varphi) - m r^2 \dot{\alpha}^2 \sin(\alpha - \varphi) = Q_\varphi = - K r^2 \sin\varphi \left[\frac{17}{5} + 2 \cos\varphi \right]}$$

"INTEGRALI PRIMI"

LE FORZE SONO CONSERVATIVE QUINDI $E = T - U =$ COSTANTE

DOVE
$$\boxed{U_{TOT} = \frac{17}{5} K r^2 (\cos\alpha + \cos\varphi) - \frac{9}{2} K r^2 \sin^2\alpha - \frac{1}{2} K r^2 (\sin\alpha + 2\sin\varphi)^2 + K r^2 (\sin\alpha + \sin\varphi)^2 + \frac{3}{5} K r^2 \sin^2\alpha}$$

NON ABBIAMO VARIABILI CICLICHE

LE EQUAZIONI SONO ACCOPPIATE

LA CONFIG. DI EQUILIBRIO DI CUI AL PUNTO 1 E' DATA DA

$$S_5 \equiv \left\{\vartheta = \frac{2}{3}\pi \quad , \varphi = 0\right\}$$

DA CUI LINEARIZZANDO AVREMO:

$$\frac{8}{5} m\ell^2 \ddot{\vartheta} - \frac{1}{2} m\ell^2 \ddot{\varphi} = \cancel{Q_\vartheta}\Big|_{S_5} + \overbrace{\frac{\partial Q_\vartheta}{\partial \vartheta}\Big|_{S_5}}^{\frac{51}{10}k\ell^2 = \frac{3}{2}mg\ell} \left(\vartheta - \frac{2}{3}\pi\right)$$

$$\frac{3}{2} m\ell^2 \ddot{\varphi} - \frac{1}{2} m\ell^2 \ddot{\vartheta} = \cancel{Q_\varphi}\Big|_{S_5} + \underbrace{\frac{\partial Q_\varphi}{\partial \varphi}\Big|_{S_5}}_{-\frac{27}{5}k\ell^2 = -\frac{27}{17}mg\ell} \varphi$$

DA CUI LE 2 EQUAZIONI:

$$\begin{cases} \frac{8}{5} m\cancel{\ell^2}\ddot{\vartheta} - \frac{1}{2} m\cancel{\ell^2}\ddot{\varphi} - \frac{51}{10}k\cancel{\ell^2}\left(\vartheta - \frac{2}{3}\pi\right) = 0 \\ -\frac{1}{2} m\cancel{\ell^2}\ddot{\vartheta} + \frac{3}{2} m\cancel{\ell^2}\ddot{\varphi} + \frac{27}{5}k\cancel{\ell^2}\varphi = 0 \end{cases}$$

CERCHIAMO SOLUZIONI NON BANALI DEL TIPO $\begin{cases} \left(\vartheta - \frac{2}{3}\pi\right) = \vartheta_0 e^{\lambda t} \\ \varphi = \varphi_0 e^{\lambda t} \end{cases}$

DA CUI IL SISTEMA

$$\left[\frac{8}{5} m\lambda^2 - \frac{51}{10}k\right]\vartheta_0 + \left[-\frac{1}{2}m\lambda^2\right]\varphi_0 = 0$$

$$\left[-\frac{1}{2} m\lambda^2\right]\vartheta_0 + \left[\frac{3}{2}m\lambda^2 + \frac{27}{5}k\right]\varphi_0 = 0$$

CHE AMMETTE SOLUZIONI NON BANALI SE λ SODDISFA L'EQUAZIONE SECOLARE.

$$\left[\frac{24}{10}m^2 - \frac{1}{4}m^2\right]\lambda^4 + \left[\frac{27}{5}\cdot\frac{8}{5}mk - \frac{51}{10}\cdot\frac{3}{2}mk\right]\lambda^2 - \frac{51}{10}\cdot\frac{27}{5}k^2 = 0$$

$$\left(\frac{43}{20}m^2\right)\lambda^4 + \left(\frac{99}{100}mk\right)\lambda^2 - \frac{1377}{50}k^2 = 0$$

BANALMENTE $\Delta > 0$ $\quad a z^2 + b z + c = 0 \quad a>0,\ b>0\ \ c<0$

UTILIZZANDO CARTESIO. $z_1 + z_2 = -\frac{b}{a} < 0,\ z_1 \cdot z_2 = \frac{c}{a} < 0 \Rightarrow z_1 > 0 \text{ e } z_2 < 0$

$\lambda_{1,2}^2 = z_1 > 0$ $\lambda_{1,2} = \pm\sqrt{z_1}$ $\Rightarrow$ MOTI IPERBOLICI

$\lambda_{3,4}^2 = z_2 < 0$ $\lambda_{3,4} = \pm i\sqrt{|z_2|}$ $\Rightarrow$ MOTI ARMONICI

QUINDI INSTABILE

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