Number Systems and Binary Arithmetic

A.Y. 2020/2021
Prof. Alessandro Ortis

ortis@dmi.unict.it

Introduction to Numbering Systems

- We are all familiar with the decimal number system (Base 10). Some other number systems that we will work with are:
  - **Binary → Base 2**
  - **Octal → Base 8**
  - **Hexadecimal → Base 16**

Characteristics of Numbering Systems

1) The digits are consecutive.
2) The number of digits is equal to the size of the base.
3) Zero is always the first digit.
4) The base number is never a digit.
5) When 1 is added to the largest digit, a sum of zero and a carry of one results.
6) Numeric values are determined by the implicit positional values of the digits.

Significant Digits

Binary: 11101101

Most significant digit ***Least significant digit***

Hexadecimal: 1D63A7A

Most significant digit ***Least significant digit***

Binary Number System

- Also called the **"Base 2 system"**
- The binary number system is used to model the series of electrical signals computers use to represent information
- 0 represents the no voltage or an **off state**
- 1 represents the presence of voltage or an **on state**

Binary Numbering Scale

Base 2 Number	Base 10 Equivalent	Power	Positional Value
000	0	2^0	1
001	1	2^1	2
010	2	2^2	4
011	3	2^3	8
100	4	2^4	16
101	5	2^5	32
110	6	2^6	64
111	7	2^7	128

Decimal to Binary Conversion

- The easiest way to convert a decimal number to its binary equivalent is to use the ***Division Algorithm***
- This method repeatedly divides a decimal number by 2 and records the quotient and remainder
  - ***The remainder digits (a sequence of zeros and ones) form the binary equivalent in least significant to most significant digit sequence***

Division Algorithm

Convert 67 to its binary equivalent:

$67_{10} = x_2$

Step 1: 67 / 2 = 33 R 1 ***Divide 67 by 2. Record quotient in next row***

Step 2: 33 / 2 = 16 R 1 ***Again divide by 2; record quotient in next row***

Step 3: 16 / 2 = 8 R 0 ***Repeat again***

Step 4: 8 / 2 = 4 R 0 ***Repeat again***

Step 5: 4 / 2 = 2 R 0 ***Repeat again***

Step 6: 2 / 2 = 1 R 0 ***Repeat again***

Step 7: 1 / 2 = 0 R 1 ***STOP when quotient equals 0***

$1\ 0\ 0\ 0\ 0\ 1\ 1_2$

Binary to Decimal Conversion

- The easiest method for converting a binary number to its decimal equivalent is to use the ***Multiplication Algorithm***
- Multiply the binary digits by increasing powers of two, starting from the right
- Then, to find the decimal number equivalent, sum those products

Multiplication Algorithm

Convert $(10101101)_2$ to its decimal equivalent:

Binary	→	1	0	1	0	1	1	0	1
		x	x	x	x	x	x	x	x
Positional Values	→	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Products → 128 + 32 + 8 + 4 + 1

173_{10}

Octal Number System

- Also known as the Base 8 System
- Uses digits 0 - 7
- Readily converts to binary
- Groups of three (binary) digits can be used to represent each octal digit
- Also uses multiplication and division algorithms for conversion to and from base 10

Hexadecimal Number System

- Base 16 system
- Uses digits 0-9 &

letters A,B,C,D,E,F

- Groups of four bits represent each base 16 digit

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Decimal to Hexadecimal Conversion

Convert 830_{10} to its hexadecimal equivalent:

830 / 16 = 51 R 14 ← = E in Hex

51 / 16 = 3 R 3

3 / 16 = 0 R 3

$33E_{16}$

Hexadecimal to Decimal Conversion

Convert 3B4F16 to its decimal equivalent:

Hex Digits → 3 B 4 F

x x x x

Positional Values → 16^3 16^2 16^1 16^0

Products → 12288 +2816 + 64 +15

$15{,}183_{10}$

Binary to Hexadecimal Conversion

- The easiest method for converting binary to hexadecimal is to use a substitution code
- Each hex number converts to 4 binary digits

Substitution Code

0000 = 0	0100 = 4	1000 = 8	1100 = C
0001 = 1	0101 = 5	1001 = 9	1101 = D
0010 = 2	0110 = 6	1010 = A	1110 = E
0011 = 3	0111 = 7	1011 = B	1111 = F

Exercises