Quaestiones Mathematicae
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tqma20

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Version of record first published: 12 Nov 2009.

Quaestiones Mathematicae **26**(2003), 237–245.

GOOD PROJECTIONS OF SPACES OF VECTOR MEASURES ONTO SUBSPACES OF BOCHNER INTEGRABLE FUNCTIONS

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Abstract. We show that the complementability of $L_1(\mu, X)$ in $cabv(\mu, X)$ implies the complementability of $L_1(\mu, K(Z, X))$ in $cabv(\mu, K(Z, X))$, provided the projection from $cabv(\mu, X)$ onto $L_1(\mu, X)$ is "good", Z^* is separable and $K(Z, X) = L(Z, X)$. The projection got is also "good", so that it allows to construct a projection from the space $L(L_1(\mu), K(Z, X))$ onto the subspace $R(L_1(\mu), K(Z, X))$ of all representable operators

Mathematics Subject Classification (2000): 28B05, 46G10, 46B20, 46B28, 47L05.

Key words: Spaces of vector measures and vector functions, complementability, spaces of compact operators, quotient spaces, spaces of representable operators.

1. Introduction. Several papers have been devoted to the study of the position of the space $L_1(\mu, X)$ inside the space $cabv(\mu, X)$; in particular in [1] it was shown that if X is a Banach lattice not containing c_0, thus $L_1(\mu, X)$ is a projection band inside $cabv(\mu, X)$ (the result in [1] was recently rediscovered by F. Freniche and L. Rodriguez-Piazza, [9]), whereas in [4] L. Drewnowski and the author proved that if X is a Banach space containing a copy of c_0 then $L_1(\mu, X)$ is *not* complemented in $cabv(\mu, X)$. In the papers [6] and [14] it was also proved that the complementability of $L_1(\mu, X)$ in $cabv(\mu, X)$ together with that one of X in its bidual is equivalent to the complementability of $L_1(\mu, X)$ in the bidual $(L_1(\mu, X))^{**}$; this equivalence allowed those authors to present more families of Banach spaces for which complementability occurs.

The aim of this note is to show how other spaces (namely spaces of compact operators) may be constructed so that still complementability occurs.

On the other hand in the papers [7], [8] it was observed that if $L_1(\mu, X)$ is complemented in a "good" way in $cabv(\mu, X)$ thus the space $R(L_1(\mu), X)$ of all representable operators is complemented in the bigger space $L(L_1(\mu), X)$. Our main result may be also applied to this pair of spaces; in this way we improve theorems obtained by the author and T.S.S.R.K. Rao in [8].

*The author was supported in part by G.N.A.F.A. of Italy and by M.I.U.R. of Italy.

2. Complementability results. First of all we observe that through the paper we shall consider just finite measure spaces (S, Σ, μ). By $cabv(\mu, X)$ we shall denote the Banach space of all X-valued countably additive measures G that are absolutely continuous with respect to μ and have bounded variation (denoted by $\|G\|(S)$), equipped with the variation norm, whereas by $L_1(\mu, X)$ we shall denote the usual Banach space of Bochner integrable functions with values into X; such a space may be, of course, seen as a closed subspace of $cabv(\mu, X)$ by identifying Bochner integrable functions with their indefinite integrals.

Moreover, given two Banach spaces Z, X, $K(Z, X)$ and $L(Z, X)$ will denote the usual spaces of compact operators and linear, bounded operators from Z into X.

Suppose now that $P : cabv(\mu, X) \longrightarrow L_1(\mu, X)$ is a projection; we say that P is a *"good" projection* if there is a constant $L > 0$ such that the following inequality is verified

$$\|PG\|(E) \leq L\|G\|(E) \qquad \forall E \in \Sigma, G \in cabv(\mu, X).$$

Such an inequality is, for instance, true if P commutes with the projections P_E defined by the mapping $G \longrightarrow G_E$ for each $E \in \Sigma$ (here G_E denotes the measure defined by putting $G_E(A) = G(A \cap E), A \in \Sigma$); this last assumption is surely verified if P is an L-projection or if it is a band projection (when, of course, X is a Banach lattice), because also any P_E is easily seen to be a L-projection or a band projection (we refer to the papers [1], [6], [7] for examples of spaces X for which the corresponding projections from $cabv(\mu, X)$ onto $L_1(\mu, X)$ verify these last conditions; in such cases we may actually choose $L = \|P\|$); also in [8] the authors presented other occurrences where the projection P commutes with any P_E.

Such an assumption of goodness on the projection P is very important for our purpose as it can be seen from the proof of the next Theorem 1; indeed, starting from the existence of a "good" projection from $cabv(\mu, X)$ onto $L_1(\mu, X)$, we are able to construct a "good" projection from $cabv(\mu, K(Z, X))$ onto $L_1(\mu, K(Z, X))$; but also it allows to construct projections between certain spaces of operators defined on $L_1(\mu)$ with values into $K(Z, X)$ (see the discussion preceding the statement of Theorem 3 below).

We may now start proving the main result of the note; a part of its proof is performed following the same lines for the proof of the main theorem in [3], that in turn followed the proof of the famous result that any separable dual space has the (RNP) (see for instance [2]); but we observe that the result in [3] has a separability assumption on the range space X that we drop completely; moreover, from our proof it is clear that if X has the (RNP), then also $K(Z, X)$ has the same property.

Theorem 1. *Let Z be a Banach space with a separable dual. Suppose that X is another Banach space such that $L_1(\mu, X)$ is complemented in $cabv(\mu, X)$ by a "good" projection P. Suppose also that $L(Z, X) = K(Z, X)$. Then $L_1(\mu, K(Z, X))$ is complemented in $cabv(\mu, K(Z, X))$ by a "good" projection $\tilde{P}$. In particular, if P commutes with the projections P_E, so does $\tilde{P}$.*

Proof. Let $G \in cabv(\mu, K(Z, X))$. For each $z \in Z$ we consider the measure $Gz \in cabv(\mu, X)$, defined by $(Gz)(E) = G(E)z$, for all $E \in \Sigma$. Now we consider the projection $P : cabv(\mu, X) \longrightarrow L_1(\mu, X)$ and then $P[Gz]$ that belongs to $L_1(\mu, X)$; hence, there is $g_z \in L_1(\mu, X)$ such that

$$P[Gz](E) = \int_E g_z(s)d\mu \qquad \forall E \in \Sigma.$$

The fact that P is a "good" projection implies that there is $L > 0$ such that

$$\|P[Gz]\|(E) \leq L\|Gz\|(E) \qquad \forall E \in \Sigma, z \in Z.$$

With straightforward calculations we get also that

$$\|Gz\|(E) \leq \|z\|\|G\|(E) \qquad \forall E \in \Sigma, z \in Z.$$

The two inequalities above give that

$$\int_E \|g_z(s)\|d\mu \leq L\|z\|\|G\|(E) \qquad \forall E \in \Sigma, z \in Z.$$

Since $\|G\| \ll \mu$, there is $h_G \in L_1(\mu)$ such that

$$\|G\|(E) = \int_E h_G(s)d\mu \qquad \forall E \in \Sigma.$$

So we finally get

$$\int_E \|g_z(s)\|d\mu \leq \int_E [L\|z\|h_G(s)]d\mu \qquad \forall E \in \Sigma, z \in Z.$$

It follows that there is a μ-null set $S_z \in \Sigma$ such that

$$\|g_z(s)\| \leq L\|z\|h_G(s) \qquad \forall s \notin S_z. \tag{1}$$

Using the same reasonings of the main theorem in the paper [3] (and thanks to the separability of Z) we may construct a function $\tau_G : S \longrightarrow L(Z, X) = K(Z, X)$ such that

$$\tau_G(s)\left(\sum_{i=1}^{p} a_i z_{n_i}\right) = \sum_{i=1}^{p} a_i g_{z_{n_i}}(s)$$

for s off a suitable μ-null set, where (z_n) is a sequence dense in B_Z and the $a_i's, i = 1, 2...p$, run through $\mathbb{Q}$ when p runs through $\mathbb{N}$. We have now to show that such a τ_G is strongly measurable; the same procedure used in [3] will lead us to our goal, once we shall have proved that τ_G takes almost all values in a separable subspace of $K(Z, X)$. To this aim, observe that for each $n \in \mathbb{N}$ there is a μ-null set S_n such that $g_{z_n}(s)$ belongs to a separable subspace Y_n of X, for $s \notin S_n$; $Y_0 = \overline{\text{span}}(\cup_n Y_n)$ is clearly a separable subspace of X such that

$$g_{z_n}(s) \in Y_0 \qquad \forall s \notin S_0 = \cup_n S_n, n \in \mathbb{N}$$

and clearly S_0 is a μ-null set. Let now fix $z_0 \in B_Z$ and $s_0 \notin S_0$; there is a subsequence (z_{n_k}) converging to z_0 for which we have

$$g_{z_{n_k}}(s_0) = \tau_G(s_0)(z_{n_k}) \longrightarrow \tau_G(s_0)(z_0).$$

This implies that $\tau_G(s) \in K(Z, Y_0)$ for almost all $s \in S$; since Z^*, Y_0 are separable, $K(Z, Y_0)$ is ([3]) and we are done: τ_G is strongly measurable. Moreover, from (1) it follows that

$$\|\tau_G(s)\|_{K(Z,X)} \leq Lh_G(s) \qquad s \quad \text{a.e.} \quad \text{in} \quad S \tag{2}$$

from which it follows that τ_G is Bochner integrable. Hence the required projection $\tilde{P}$ defined by

$$[\tilde{P}(G)](E)(z) = (P[Gz])(E) \qquad \forall E \in \Sigma, z \in Z$$

is nothing else than the map assigning to any $G \in cabv(\mu, K(Z, X))$ the indefinite integral of the function τ_G. It is also clear that such a projection has a norm less or equal to L.

Furthermore, it is also a "good" projection as it follows immediately from inequality (2).

It remains to be shown that if P commutes with the projection P_E also $\tilde{P}$ does; to this aim, for $B \in \Sigma, z \in Z$ we have

$$\begin{gathered}[(P_E\tilde{P})(G)](B)(z) = [\tilde{P}(G)](B \cap E)(z) = [P(Gz)](B \cap E) = \\ [(P_EP)(Gz)](B) = [(PP_E)(Gz)](B) = [P((Gz)_E)](B) = \\ [P(G_Ez)](B) = [\tilde{P}(G_E)](B)(z) = [(\tilde{P}P_E)(G)](B)(z)\end{gathered}$$

where we have used the equality

$$G_Ez = (Gz)_E$$

that is true for all $G \in cabv(\mu, K(Z, X)), E \in \Sigma, z \in Z$; this easily follows from the following equality

$$G_E(B)(z) = G(B \cap E)(z) = (Gz)(B \cap E) = (Gz)_E(B) \qquad \forall B \in \Sigma.$$

We are done. $\square$

Question. Is it possible to weaken (or to drop) the assumption "Z^* is separable"?

There are several examples of pairs Z, X of Banach spaces verifying the assumptions of our main Theorem; for instance, one may choose $Z = l_p, \infty > p > 2$, and $X = L_1(\mu)$ (see [1], [6], [11], [14]) or Z such that Z^* has Schur property and is separable and X any weakly sequentially complete Banach space for which $L_1(\mu, X)$ is complemented in a "good" way in $cabv(\mu, X)$ (we recall that among such spaces one may found weakly sequentially complete Banach lattices or preduals of W^*-algebras, see [1], [6]); so our result is for instance applicable to $X = K(c_0, L_1(\mu))$ (that is weakly sequentially complete thanks to results in the paper [13]), but also to the still weakly sequentially complete space (use again the same results from [13]) $X = K(c_0, K(c_0, L_1(\mu)))$ (because the projection got in our result is "good" as pointed out at the end of the same Theorem 1) and to $X = K(c_0, K(c_0, K(c_0, L_1(\mu))))$ and so on.

COROLLARY 2. *Let Z be a Banach space with a separable dual. Suppose that X is a dual Banach space such that $L_1(\mu, X)$ is complemented in $cabv(\mu, X)$ by a "good" projection P. Suppose also that $L(Z, X) = K(Z, X)$. Thus $L_1(\mu, K(Z, X))$ is complemented in its bidual.*

Proof. Theorem 1 implies that $L_1(\mu, K(Z, X))$ is complemented in $cabv(\mu, K(Z, X))$; moreover, $L(Z, X) = K(Z, X)$ is a dual Banach space, since X is, which implies that it is complemented in its bidual. Hence the main result in [6] allows us to conclude. □

Corollary 2 can be applied with $Z = c_0$ and $X = (L_1(\mu))^{**}$.

After giving the definition of "good" projections we remarked that L-projections are "good"; we do not know if the projection $\tilde{P}$ constructed in Theorem 1 is an L-projection when the original projection P is; the best we can say here is that $\tilde{P}$ is a $\mathcal{U}$-projection, i.e. $\|Id - 2\tilde{P}\| = 1$ (see [10]), if P is a $\mathcal{U}$-projection also commuting with any projection P_E; before proving this statement we recall that such assumptions are trivially verified (see also the comments to the definition of "good" projection made at the beginning) if P is a L-projection, but also if P is the band projection of $cabv(\mu, X)$ onto $L_1(\mu, X)$ existing when X is a Banach lattice not containing c_0([1]); indeed, in such a case $L_1(\mu, X)$ is a $\mathcal{U}$-summand in $(L_1(\mu, X))^{**}$, because Banach lattice not containing c_0 (see [10], Example (1) in Section 4), and the band projection P from $cabv(\mu, X)$ onto $L_1(\mu, X)$ is nothing else than the restriction of the projection from $(L_1(\mu, X))^{**}$ onto $L_1(\mu, X)$ as observed in the papers [6] and [14]. Now we show that $\tilde{P}$ is a $\mathcal{U}$-projection if P is; for any norm one $G \in cabv(\mu, X)$ and any $\epsilon > 0$ there are $(A_i)_{i=1}^p$ from Σ and $(z_i)_{i=1}^p$ in the unit ball of Z such that

$$\|(I-2\tilde{P})(G)\|(S) \le \epsilon + \sum_{i=1}^p \|[(I-2\tilde{P})(G)](A_i)\| \le 2\epsilon + \sum_{i=1}^p \|[(I-2\tilde{P})(G)](A_i)(z_i)\| =$$

$$2\epsilon + \sum_{i=1}^p \|[(I-2P)(Gz_i)](A_i)\| = 2\epsilon + \sum_{i=1}^p \|[[P_{A_i}(I-2P)](Gz_i)](S)\| \le$$

$$2\epsilon + \sum_{i=1}^p \|[P_{A_i}(I-2P)](Gz_i)\|(S) \le 2\epsilon + \sum_{i=1}^p \|[(I-2P)P_{A_i}](Gz_i)\|(S) \le$$

$$2\epsilon + \sum_{i=1}^p \|P_{A_i}(Gz_i)\|(S) = \sum_{i=1}^p \|Gz_i\|(A_i) \le 2\epsilon + \sum_{i=1}^p \|G\|(A_i)\|z_i\| \le$$

$$2\epsilon + \sum_{i=1}^p \|G\|(A_i) \le 2\epsilon + \|G\|(S).$$

The arbitrariness of ϵ allows us to conclude that $\|Id - 2\tilde{P}\| = 1$. □

In [6] we also proved that the existence of a projection from $cabv(\mu, X)$ onto $L_1(\mu, X)$ sometimes allows us to construct a new projection from $cabv(\mu, X/H)$ onto $L_1(\mu, X/H)$ for suitable H; more precisely we proved the following result that we state here just for completeness

THEOREM 3. *Let X be a Banach space such that $L_1(\mu, X)$ is complemented in $cabv(\mu, X)$ by a projection $\tilde{P}$,H a closed subspace of X with the Radon-Nikodym property. Define $\tilde{Q} : cabv(\mu, X) \to cabv(\mu, X/H)$ by putting $\left[\tilde{Q}(\tilde{G})\right](E) = Q\left[\tilde{G}(E)\right]$ (here Q denotes the quotient map of X onto X/H), for all $E \in \Sigma$ and $G \in cabv(\mu, X)$. If $\tilde{Q}$ is a quotient map, then $L_1(\mu, X/H)$ is complemented in $cabv(\mu, X/H)$ by a projection P that may be defined as it follows*

$$[P(G)](E) = \tilde{Q}[\tilde{P}(\tilde{G})](E) \qquad \forall E \in \Sigma, \tilde{G} \in cabv(\mu, X), \tilde{Q}(\tilde{G}) = G.$$

In particular this result applies if X is a dual Banach space and H is a w*-closed subspace of X or X is arbitrary and H is reflexive, because for such pairs of spaces X, H we were able to show in [6] that $\tilde{Q}$ is a quotient map. Also in the quite recent paper [8] the authors extended the previous results by proving that $\tilde{Q}$ is a quotient map under the following more general assumption: suppose that X is contained in a dual Banach space Y and that H is a closed subspace of X that is w*-closed in Y. To apply this last result to our present situation we may choose $X = K(c_0, T) = L(c_0, T)$ where T is the weakly sequentially complete Banach lattice constructed by Talagrand in [16],$Y = L(c_0, T^{**}), H = K(c_0, F) = L(c_0, F)$ with F any reflexive subspace of T; T^{**} is a Banach lattice containing copies of c_0, otherwise by results in [15] $cabv(\Sigma, T)$ would be weakly sequentially complete,that is not the case ([16]); moreover, this also implies that $K(c_0, T^{**}) \neq L(c_0, T^{**})$ (see [5], [12]) so that our Theorem 1 cannot be applied to $X = K(c_0, T^{**})$; furthermore, it is not difficult to see that H is a w*-closed subspace in the dual space Y; also it has the (RNP) property because of results in [3]. Of course the quoted results from [6] cannot be used in this case to get a projection from $cabv(\mu, X/H)$ onto $L_1(\mu, X/H)$, since our X is not a dual space and our H is not reflexive.

Another occurrence in which the previous result may be applied is the following (see [8]): let T be the unit circle and let $\wedge$ be a Riesz subset of $\mathbb{Z}$ that is not nicely placed. Then $L^1_\wedge$ is a w^*-closed subspace of $C(T)^*$ having the Radon-Nikodym property. Therefore $Q : cabv(\mu, L^1) \to cabv(\mu, L^1/L^1_\wedge)$ is a quotient map and so $L^1(\mu, L^1/L^1_\wedge)$ is complemented in $cabv(\mu, L^1/L^1_\wedge)$ (actually L-complemented as remarked in [7]). Observe that we are assuming that $\wedge$ is not a nicely placed set to ensure that this example does not obviously follow from the lifting properties enjoyed by quotients of L-embedded spaces by L-embedded subspaces.

It is not difficult to show that if $\tilde{P}$ is a $\mathcal{U}$-projection, also P is; what if $\tilde{P}$ is just a "good" projection? We are going to show that P is "good" too; we observe that if $\tilde{Q}(\tilde{G}) = G$ and $E \in \Sigma$ we have for any $\epsilon > 0$ the existence of a finite number of pairwise disjoint elements $(A_i)_{i=1}^p$ from Σ such that

$$\|P_E G\|(S) \leq \epsilon + \sum_{i=1}^p \|(P_E G)(A_i)\| = \epsilon + \sum_{i=1}^p \|(P_E \tilde{Q}(\tilde{G}))(A_i)\| =$$

$$\epsilon + \sum_{i=1}^p \|(\tilde{Q}(\tilde{G}))(A_i \cap E)\| = \epsilon + \sum_{i=1}^p \|Q[\tilde{G}(A_i \cap E)]\| \leq \epsilon + \sum_{i=1}^p \|\tilde{G}(A_i \cap E)\| =$$

$$\epsilon + \sum_{i=1}^{p} \|(P_E\tilde{G})(A_i)\| \le \epsilon + \|P_E\tilde{G}\|(S)$$

which gives

$$\|P_E G\|(S) \le \|P_E\tilde{G}\|(S) \qquad \forall E \in \Sigma.$$

Similarly, we may show that

$$\|G - P_E G\|(S) \le \|\tilde{G} - P_E\tilde{G}\|(S) \qquad \forall E \in \Sigma.$$

Now choose a sequence $(\tilde{G}_n) \in G$ such that

$$\frac{1}{n} + \|G\|(S) \ge \|\tilde{G}_n\|(S) \qquad \forall n \in \mathbb{N}$$

and calculate as follows, remembering that any P_E is an L-projection,

$$\|G\|(S) = \|P_E G\|(S) + \|G - P_E G\|(S) \le \|P_E\tilde{G}_n\|(S) + \|\tilde{G}_n - P_E\tilde{G}_n\|(S) = \|\tilde{G}_n\|(S) \le \frac{1}{n} + \|G\|(S).$$

It clearly follows that

$$\|P_E\tilde{G}_n\|(S) = \|\tilde{G}_n\|(E) \longrightarrow \|P_E G\|(S) = \|G\|(E)$$

uniformly on $E \in \Sigma$. Thus we have

$$\|P(G)\|(E) = \|\tilde{P}(\tilde{G}_n)\|(E) \le L\|\tilde{G}_n\|(E) \qquad \forall E \in \Sigma, n \in \mathbb{N}$$

from which easily follows that

$$\|P(G)\|(E) \le L\|G\|(E) \qquad \forall E \in \Sigma.$$

□

As remarked in the Introduction the projections we are considering are also "good" in the sense that we may use them to construct a further projection from $L(L_1(\mu), K(Z,X))$ onto $R(L_1(\mu), K(Z,X))$ thanks to a procedure followed in our paper [7]; we briefly describe the main idea in [7]: it is known (see [2], Chapter III, specially pp. 62 and 84) that there is a 1-1 correspondence (that is *not* an isomorphism) between the space $L(L_1(\mu), X)$ (resp. $R(L_1(\mu), X)$) and a not closed subspace of the space $cabv(\mu, X)$ (resp. $L_1(\mu, X)$), exactly the subspace of those measures for which there is a constant $C > 0$ such that

$$\|G(E)\| \le C\mu(E) \qquad \forall E \in \Sigma; \tag{3}$$

more precisely to any $T \in L(L_1(\mu), X)$ we may associate the measure (called the representing measure of T) $G \in cabv(\mu, X)$ defined by the equality

$$G(E) = T(\chi_E) \qquad \forall E \in \Sigma.$$

We observe that if G is the representing measure of some $T \in L(L_1(\mu), X)$ and P is the projection of $cabv(\mu, X)$ onto $L_1(\mu, X)$, then PG does not necessarily determine an element in $R(L_1(\mu), X)$, since if G satisfies (3) it seems that there is no reason why even PG must satisfy a similar condition.

But, if, as in our case, the projection $P : cabv(\mu, X) \to L_1(\mu, X)$ is "good" also PG clearly verifies (3); so that we easily have the following result

Theorem 4. *Let X be such that $L_1(\mu, X)$ is complemented in $cabv(\mu, X)$ by a "good" projection. Then $R(L_1(\mu), X)$ is complemented in $L(L_1(\mu), X)$. In particular, if Z, X verify the assumptions of Theorem 1, then $R(L_1(\mu), K(Z, X))$ is complemented in the bigger space $L(L_1(\mu), K(Z, X))$.*

A result similar to the second part of the statement of Theorem 4 was obtained in [8] under the assumption "X=$L^1[0,1]$" (from which we took great advantage in the proof of Theorem 2 from [8], Section 2), that is now completely superfluous as the present results show.

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Received 13 March, 2003