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A REMARK ON MY PAPER "CONVERGENCE OF THE MANN–ISHIKAWA ITERATIVE PROCESS FOR NONEXPANSIVE MAPPINGS"

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IN MY paper ([1], see Notes), I have shown the following result:

"Let E be a strictly convex Banach space. Let C be a convex and weakly compact subset of E. Let f, $f: C \to C$, be a nonexpansive mapping such that: (i) f is demiclosed, i.e. $x_n - f(x_n) \xrightarrow{s} 0, x_n \xrightarrow{w} y$ imply $y = f(y)$; (ii) there exists an increasing function φ, from R^+ into R^+, which satisfies $\varphi(0) = 0$, $\lim_{r \to +\infty} \varphi(r) = +\infty$ and such that

$$((i-f)(x) - (i-f)(y), x-y)_+ \geqslant [\varphi(\|x\|) - \varphi(\|y\|)][\|x\| - \|y\|]$$

where i denotes the identity mapping on E and

$$(p, q)_+ = \sup\{q^*: q^* \in J(q)\}, J(q) = \{q^*: q^* \in E^*, q^*(q) = \|q^*\|^2 = \|q\|^2\}.$$

Then, the sequence $\{x_n\}$ defined by

$$x_{n+1} = (1-t_n)x_n + t_n f(x_n) \qquad n \in N,$$

$\{t_n\} \subseteq [0, b] \subseteq [0, 1[, \Sigma t_n = +\infty$, converges strongly to a fixed point of f, if E satisfies the following condition $(H) x_n \xrightarrow{w} x$, $\|x_n\| \to \|x\|$ imply that $x_n \xrightarrow{s} x$."

The proof of this result is based upon the fact that the existence of a unique fixed point of f follows from (ii) and the strict convexity of E. Purpose of this note is to show that even when E is not strictly convex, then $\{x_n\}$ converges strongly to a (nonnecessarily unique) fixed point of f.

Indeed, from weak compactness of C, an $\{x_{h(n)}\}$ and a $y \in C$ exist for which $x_{h(n)} \xrightarrow{w} y$; using (i) we have $y = f(y)$ since (see [2]) $x_n - f(x_n) \xrightarrow{s} 0$. Using (ii), we obtain like in [1] that $\|x_n\| \to \|y\|$; and so $x_{h(n)} \xrightarrow{s} y$, from (H). Now, we observe that, for each $n \in N$, one has $\|x_{n+1} - y\| \leqslant \|x_n - y\|$; then, $d \geqslant 0$ exists for which $\lim_n \|x_n - y\| = d$; this implies that $d = \lim_n \|x_{h(n)} - y\| = 0$. This completes the proof.

At the end, we observe that the Banach space $l_1 \times l_2$ equipped with the norm

$$\|(x, y)\|_{l_1 \times l_2} = \|x\|_{l_1} + \|y\|_{l_2} \qquad (x, y) \in l_1 \times l_2$$

is not strictly convex and it satisfies (H). Moreover, in $l_1 \times l_2$, the well-known Opial's condition holds true [3]; hence (i) is verified; the above example shows that the present result is strictly more general than the previous one in [1].

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