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Banach spaces in which Dunford-Pettis sets are relatively compact [1])

By

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Introduction. Let E be a Banach space and X a bounded subset of E. X is called a Dunford-Pettis set if for any weak null sequence $(x_n^*) \subset E^*$ one has $\lim_n \sup_X |x_n^*(x)| = 0$. This note is devoted to a study of the family of Banach spaces with the property that their Dunford-Pettis subsets are relatively compact; we shall say that such a space has the (DPrcP). Our interest in this class of Banach spaces is motivated by the following fact: in the paper [4] we proved that a dual Banach space with the Weak Radon-Nikodym Property ([8], in short (WRNP)) has the (DPrcP); since any Banach space with the (DPrcP) has the so called Compact Range Property ([8], in short (CRP)), it turns out that the result from [4] can be reversed, so obtaining a new characterization of the (WRNP) in dual spaces; this result makes the (WRNP) in dual spaces easier to be handled: for instance, we are able to answer a question by Ruess, [12], when defining a research program concerning projective tensor products of Banach spaces. There is another reason that could as well motivate our study: we state that dominated operators from special $C(K, E)$ spaces taking values in a Banach space with the (DPrcP) are Dunford-Pettis; if E is finite dimensional it is well-known that this is always verified, but when the dimension of E is infinite the above result is no longer true. When looking for hypotheses on E and F making dominated operators Dunford-Pettis we realize that this happens if E has the Dunford-Pettis property ([2]) and F the (DPrcP); and these are in a sense the best hypotheses one can consider. All of these facts are contained in Section 1. Section 2 contains some more examples of Banach spaces with the (DPrcP) as well as some permanence results.

Main results. Before starting, we note that in the paper we use, without any warning, the well-known equivalence "E does not contain $l^1 \Leftrightarrow E^*$ has the (WRNP)" ([14], 7.3.8). From the definition of the (DPrcP) it follows easily that Schur spaces have this property, because it is well-known that Dunford-Pettis sets are weakly precompact, i.e. each sequence has a weak Cauchy subsequence. Moreover, in [4] it was shown that dual Banach spaces with the (WRNP) have the (DPrcP), a property inherited by closed

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subspaces. So it is very easy to prove that there exist Banach spaces with the (DPrcP) but without the (WRNP) or the Schur property. The first result contains the announced characterization of the (WRNP) in dual Banach spaces.

Theorem 1. *Let E be a Banach space with the* (DPrcP). *Then E has the* (CRP). *If moreover E is a dual space, then it has the* (WRNP) *iff it has the* (DPrcP).

Proof. Take an E-valued measure μ defined on a σ-field $\sum$ of subsets of a set S, having finite variation. It is known ([14], 7.1.2) that there is a Gelfand integrable function $f: S \to E^{**}$ for which there is $M > 0$ such that $|x^* f(s)| \leqq M \|x^*\|$ a.e. on S, where the exceptional set depends upon x^*, and such that, for all $A \in \sum$

$$\langle \mu(A), x^* \rangle = \int_A x^* f(s)\, d|\mu| \qquad x^* \in E^*.$$

Now, consider a weak null sequence $(x_n^*) \subset E^*$. It is clear that $x_n^* f(s) \to 0$ pointwise. Since there is a null set S^0 such that $|x_n^* f(s)| \leqq M \sup_n \|x_n^*\|$, $s \notin S^0$, we get easily that

$$\sup_A \langle \mu(A), x_n^* \rangle \to 0.$$

This means that the range of μ is a Dunford-Pettis set in E and so it is relatively compact. Assume, now, that E is a dual space. If E has the (DPrcP), it has the (CRP) by the previous proof. If the predual of E contained l^1, E should contain $L^1[0, 1]$ ([9]); but $L^1[0, 1]$ doesn't have the (CRP) ([14], 7.2.4) a property usually inherited by subspaces. This contradiction shows that E has the (WRNP). Since the converse is in [4] we are done.

As stated in the Introduction, an application of Theorem 1 allows us to answer a question by Ruess in [12], 6.4. First we prove the following lemma, where $K_{W^*}(E^*, F)$ denotes the Banach space of all compact, weak*-weak continuous operators from E^* into F with the usual operator norm.

Lemma 2. *If E and F have the* (DPrcP) *and H is a Dunford-Pettis subset of $K_{W^*}(E^*, F)$, then we have*

a) $H(x^*) = \{h(x^*): h \in H\}$ (resp. $H^*(y^*)$) *is relatively compact in F (resp. in E) for all $x^* \in E^*$ (resp. $y^* \in F^*$).*

b) $H(B_{E^*}) = \{h(x^*): h \in H, x^* \in B_{E^*}\}$ (resp. $H^*(B_{F^*})$) *is weakly precompact in F (resp. in E).*

Proof. Since the map $h \to h^*$ from $K_{W^*}(E^*, F)$ into $K_{W^*}(F^*, E)$ is a (surjective) isometry, thus preserving the property of being a Dunford-Pettis set, it is enough to prove the first assertions of (a) and (b), respectively.

a) If $x^* \in E^*$ we consider the bounded, linear operator assigning to each h its value at x^*, from $K_{W^*}(E^*, F)$ into F; hence, $H(x^*)$ is a Dunford-Pettis set in F and so it is relatively compact.

b) Let $(h_n(x^*))$ be a sequence in $H(B_{E^*})$. Since H is weakly precompact, we can suppose that (h_n) is a weak Cauchy sequence (otherwise we pass to a subsequence). Furthermore, since E_0 = closed linear span of $\{h_n^*(F^*): n \in \mathbb{N}\}$ is separable, we can suppose that

$(x^*_{n|E_0})$ is weak* converging in $B_{E_0^*}$ (otherwise, we pass to a subsequence of the x_n^*'s and then to the corresponding subsequence of the h_n's). For $n, m \in \mathbb{N}$ and $y^* \in F^*$ we have

$$\begin{aligned}\langle h_n(x_n^*) - h_m(x_m^*), y^*\rangle &= \langle h_n(x_n^*) - h_m(x_n^*), y^*\rangle + \langle h_m(x_n^*) - h_m(x_m^*), y^*\rangle \\ &\leqq \|(h_n - h_m)^*(y^*)\| + \langle x_n^* - x_m^*, h_m^*(y^*)\rangle \to 0\end{aligned}$$

as n, m go to infinity because, as in a), $(h_n - h_m)^*(y^*)$ goes to θ in norm and $(x^*_{n|E_0})$ is weak* converging, while the set $\{h_n^*(y^*): n \in \mathbb{N}\}$ is relatively compact in E_0 by virtue of a). This completes the proof.

In the next result we shall use the following other characterization of the (WRNP) in dual spaces that we obtained in [4]: E^* has the (WRNP) iff any (L)subset of E^* is relatively compact. (A (L) subset X is one verifying the following limit relation $\lim_n \sup_X |x_n(x^*)| = 0$ for any weak null sequence (x_n) in E). We denote by $L(E, H)$ (resp. $K(E, H)$) the Banach space of all bounded, linear (resp. bounded, linear, compact) operators from E into H with the usual operator norm.

Theorem 3. *Let E, F contain no copies of l^1. If $L(E, F^*) = K(E, F^*)$ then $E \otimes_\pi F$ doesn't contain copies of l^1.*

Proof. By virtue of Theorem 1 it is enough to show that $(E \otimes_\pi F)^* = L(E, F^*) = K(E, F^*)$ has the (DPrcP). Let H be a Dunford-Pettis subset of $(E \otimes_\pi F)^*$. Since $K(E, F^*) = K_{W^*}(E^{**}, F)$ via the mapping $h \to h^{**}$, we can use Theorem 1.5 of [11] to prove that H is relatively compact. So it will be enough to prove that the following are true: i) $H^{**}(x^{**})$ is relatively compact, for all $x^{**} \in E^{**}$ (this is true by virtue of Lemma 2, a)); ii) $H^{**}(B_{E^{**}})$ is relatively compact in F^*. Since B_E is w^* dense in $B_{E^{**}}$ it is enough to prove that $H(B_E)$ is relatively compact. So let $(h_n(x_n))$ be a sequence in $H(B_E)$; we shall prove it is a (L) subset of F^*. To this aim, let us consider a weak null sequence $(y_n) \subset F$; we claim that the sequence $(x_n \otimes y_n)$ is a weak null sequence in $E \otimes_\pi F$. Indeed, if $h \in (E \otimes_\pi F)^* = K(E, F^*)$, we have

$$|h(x_n \otimes y_n)| \leqq \|h^*(y_n)\| \to 0$$

because of the compactness of h. This means that

$$h_n(x_n)(y_n) \to 0;$$

the last limit relation easily implies that $(h_n(x_n))$ is a (L) subset of F^* and so a relatively compact subset of F^* (as a consequence of the recalled characterization from [4]). We are done.

Theorem 3 is an improvement over Theorem 4.4 from [13] obtained under the following more restrictive hypotheses: F doesn't contain l^1 and E^* has the Radon-Nikodym property and the Approximation property. The following corollaries are immediate from Theorem 3, so we don't give the proof

Corollary 4. *Let E, F contain no copies of l^1 and $L(E, F^*) = K(E, F^*)$. Then $E^* \otimes_\varepsilon F^*$ has the* (DPrcP).

Corollary 5. *Let E^*, F contain no copies of l^1 and $L(E^*, F^*) = K(E^*, F^*)$. Then the space $N^1(E, F)$ of all nuclear operators from E into F doesn't contain copies of l^1.*

In some special cases the hypothesis $L(E, F^*) = K(E, F^*)$ in Theorem 3 is even necessary for $E \otimes_\pi F$ to contain no copies of l^1 as the following result shows; it follows along the lines of reasoning for Corollary 4.5 in [13], but it actually is a wide generalization of that result obtained under the following hypotheses: E has a shrinking unconditional basis and F doesn't contain copies of l^1. The difference between Corollary 4.5 in [13] and our Corollary 6 below resides in the fact that thanks to Theorem 3 above we can use in its complete generality a result (used even in [13]) due to Diestel and Morrison ([1]) about the containment of l^∞ by spaces of operators

Corollary 6. *Assume E and F contain no copies of l^1. If E or F^* has a finite dimensional unconditional Schauder decomposition, the following facts are equivalent*

i) *$E \otimes_\pi F$ doesn't contain copies of l^1*
ii) *$E \otimes_\pi F$ doesn't contain complemented copies of l^1*
iii) *$L(E, F^*) = K(E, F^*)$.*

We omit the proof similar to that of Corollary 4.5 in [13]. We also note that thanks to a result from [5] the assumption on F^* can be substituted with the following one: F^* is a complemented subspace of a Banach space Z having an unconditional Schauder decomposition (Z_n) with $L(E, Z_n) = K(E, Z_n)$ for all $n \in \mathbb{N}$. Indeed, we showed in [5] that if $L(E, F^*) \neq K(E, F^*)$ then c_0 embeds into $K(E, F^*)$. Other results implying the presence of a copy of c_0 in $K(E, F^*)$ as soon as $L(E, F^*) \neq K(E, F^*)$ can be found in the paper [6]. For anyone of them, we can obtain a result like Corollary 6 above.

Theorem 7. *Let E have the Schur property and F the* (DPrcP). *Then, the Banach space $K_{W^*}(E^*, F)$ has the* (DPrcP).

Proof. Using Theorem 1.5 in [11] and Lemma 2 we obtain our thesis immediately, since E has the Schur property.

Corollary 8. *Let E, F be as in Theorem 7. Then $E \otimes_\varepsilon F$ has the* (DPrcP).

Corollary 9. *Let E have a dual E^* with the Schur property, F have the* (DPrcP). *Then, $K(E, F)$ has the* (DPrcP). *If F is a dual Banach space, i.e. $F = Z^*$, then $E \otimes_\pi Z$ doesn't contain copies of l^1.*

Proof. We have already noted that $K(E, F) = K_{W^*}(E^{**}, F)$; so it enough to note that, when $F = Z^*$, any operator $T: E \to F$ maps the unit ball of E, that is a Dunford-Pettis set, in a relatively compact set.

Corollary 9 improves greatly a result in [13], Corollary 4.6, obtained under the hypotheses: E^* has the Schur property and Z has a shrinking unconditional basis.

Corollary 10. *Let F have the* (DPrcP). *Then the space $l^1[F]$ of all unconditionally converging series in F, equipped with the norm $\|(y_n)\| = \sup\left\{\sum_{n=1}^{\infty} |y^*(y_n)| : y^* \in B_{F^*}\right\}$ has the* (DPrcP).

Proof. It is well-known that $l^1[F]$ is isometrically isomorphic to $K(c_0, F)$.

The second application of the family of Banach spaces with the (DPrcP) is to dominated operators on $C(K, E)$ spaces, as announced in the Introduction. For the definition of this kind of operators we refer to the book [3], CH. III, § 19.3.

Theorem 11. *Let E have the Dunford-Pettis property and F the* (DPrcP). *If K is a compact Hausdorff space, then any dominated operator T from $C(K, E)$ into F is Dunford-Pettis.*

Proof. We need a representation Theorem for dominated operators that can be found in [3], CH. III, § 19.3, Thm. 5. It says that for any dominated operator $T: C(K, E) \to F$ there is a function G from K into $L(E, F^{**})$ such that

a) $\|G(t)\| = 1$ μ. a.e. in K
b) for each $y^* \in F^*$ and $f \in C(K, E)$ the function $\langle G(\cdot) f(\cdot), y^* \rangle$ is μ-integrable and moreover

$$\langle T(f), y^* \rangle = \int_K \langle G(t) f(t), y^* \rangle \, d\mu \quad f \in C(K, E)$$

where μ is the least regular Borel measure dominating T. In order to prove our thesis, we shall consider a weak null sequence (f_n) in $C(K, E)$ and we shall show that $(T(f_n))$ is a Dunford-Pettis subset of F. Let us consider a weak null sequence $(y_n^*) \subset F^*$. For each $t \in K$, $G^*(t) y_n^* \xrightarrow{w} \theta$ in E^*. On the other hand, $(f_n(t))$ is a weak null sequence in E. Hence, by virtue of the Dunford-Pettis property in E, we have

$$\langle G(t) f_n(t), y_n^* \rangle = \langle f_n(t), G^*(t) y_n^* \rangle \to 0$$

pointwise on K. Furthermore, we note that there exists a constant $M > 0$ such that $|\langle G(t) f_n(t), y_n^* \rangle| \leqq M$ for a.a. t in K and all $n \in \mathbb{N}$. Thanks to the nature of μ we may conclude that

$$\lim_n \langle T(f_n), y_n^* \rangle = \lim_n \int_K \langle G(t) f_n(t), y_n^* \rangle \, d\mu = 0$$

a limit relation that finishes our proof.

We note that the assumption on E in the above theorem cannot be dropped at all; indeed, if a Banach space E doesn't enjoy the Dunford-Pettis property, it is well-known that there is a reflexive Banach space F and a non Dunford-Pettis operator $S: E \to F$; hence, the dominated operator $T: C([0, 1], E) \to F$ defined by

$$T(f) = \int_{[0,1]} S f(t) \, dm$$

is not Dunford-Pettis (use constant functions). If we try to change the hypothesis on F in Theorem 11, leaving unchanged that one on E, we cannot say anything about dominated T. It is enough to consider the cases $E = F = c_0$ or $E = F = L^1[0, 1]$ defining T as above (S = identity).

We finish this section with the following natural

Question. If E and F have the (DPrcP), does $K_{W*}(E^*, F)$ have the same property, provided $K_{W*}(E^*, F) = L_{W*}(E^*, F)$?

In case of a positive answer to this question, Thms. 3, 7 and 13 would be nothing but special cases of it.

Further examples and permanence properties. In light of Theorem 11 it becomes interesting to have results pointing out more Banach spaces with the (DPrcP). The final part of the note is devoted to this problem.

Theorem 12. *Let E be a Banach lattice with the* (WRNP). *Then E has the* (DPrcP).

Proof. First of all, note that E has the Radon-Nikodym property ([14], 7.5.2). Now, consider a separable subspace Y of E. It is known ([7]) that there is a closed sublattice Z of E that is separable and contains Y. A beautiful result by Talagrand ([14], 7.5.4) shows that Z is isometrically isomorphic to a dual Banach lattice, because Z inherites the Radon-Nikodym property by E. Now, we can apply our Theorem 1 to show that Z has the (DPrcP) and so even Y has the (DPrcP). Hence, we have proved that any separable subspace of E has the (DPrcP). Now, observe that E doesn't contain c_0 and so it is an order continuous Banach lattice, from which it follows that it is a separably complemented Banach space. Let (x_n) be a Dunford-Pettis sequence in E. From the above remarks it follows that (x_n) is contained in a complemented separable subspace Y of E, possessing the (DPrcP). Since one sees very easily that (x_n) is Dunford-Pettis even in Y, we have that (x_n) is relatively compact in (Y and so in) E. We are done.

Theorem 13. *Let E be a Banach space such that E^* has the* (DPrcP) *and F be a Banach lattice with the* (WRNP). *If $L(E, F) = K(E, F)$, then $K(E, F)$ has the* (DPrcP).

Proof. Let (h_n) be a Dunford-Pettis sequence in $K(E, F)$. Since the closed span Y of $\{h_n(x): x \in E, n \in N\}$ is separable, there is a closed subspace Z of F that is separable and complemented in F and contains Y (see Theorem 12). It is clear that $K(E, Z)$ is complemented in $K(E, F)$ and so (h_n) is Dunford-Pettis in $K(E, Z)$. As in Theorem 12, we choose a separable closed sublattice H containing Z and isometrically isomorphic to a dual Banach space. From our hypotheses, it follows that $L(E, H) = K(E, H)$. Hence, in this setting, we may apply Theorem 3. $K(E, H)$ has the (DPrcP), that is inherited by $K(E, Z)$. This gives the relative compactness of (h_n). We are done.

The next results can be obtained with a straightforward (but long) proof; so we shall only give sketches of their proofs

Proposition 14. *Let Γ be a set and $1 \leqq p < \infty$. If $(E_\gamma)_\Gamma$ is a family of Banach spaces with the* (DPrcP), *then $l^p(\Gamma, E_\gamma)$ has the same property.*

Proof. Let us assume first that Γ is countable, i.e. $\Gamma = \mathbb{N}$. Let (x_n) be a Dunford-Pettis sequence in $l^p(E_k)$, where $x_n = (x_n^i)$ for all $n \in \mathbb{N}$. Arguing by contradiction it is not difficult to show that

$$(1) \qquad \lim_m \sup_n \sum_{i=m}^{\infty} \|x_n^i\|^p = 0.$$

Now, observe that (x_n^i) is a Dunford-Pettis set in E_i and hence a relatively compact subset of E_i. By passing to a subsequence if necessary, we can assume (x_n) is such that there is $x^i \in E_i$, for all $i \in \mathbb{N}$, with $\lim_n x_n^i = x^i$. From this last limit relation we can easily see that $x = (x^i)$ is in $l^p(E_i)$. Moreover, since for all $s \in \mathbb{N}$, one has

$$\|x_n - x\| = \left(\sum_{i=1}^{s} \|x_n^i - x^i\|^p\right)^{1/p} + \left(\sum_{i=s+1}^{\infty} \|x_n^i - x^i\|^p\right)^{1/p}$$
$$\leqq \left(\sum_{i=1}^{s} \|x_n^i - x^i\|^p\right)^{1/p} + \left(\sum_{i=s+1}^{\infty} \|x_n^i\|^p\right)^{1/p} + \left(\sum_{i=s+1}^{\infty} \|x^i\|^p\right)^{1/p}$$

it is a straightforward task to prove that $x_n \to x$, using (1) and the fact that $\lim_n x_n^i = x^i$ and $x \in l^p(E_i)$. When Γ is not necessarily countable, if (x_n) is a Dunford-Pettis sequence in $l^p(E)$, it is easy to show that there is a countable subset Γ_0 of Γ such that (x_n) can be considered a Dunford-Pettis sequence in $l^p(\Gamma_0, E_{\gamma_0})$, that is isomorphic to $l^p(E_i)$. The first part of the Proposition concludes our proof.

Proposition 15. *Let (S, Σ, μ) be a finite measure space and $1 < p < \infty$. If E is a closed subspace of a dual space Z^* with the* (DPrcP), *then $L^p(E)$ has the* (DPrcP).

Proof. If $p^{-1} + q^{-1} = 1$, $L^q(Z)$ doesn't contain copies of l^1 ([10]) and hence $(L^q(Z))^*$ has the (DPrcP) by Theorem 1. Since $L^p(E)$ is a closed subspace of $(L^q(Z))^*$ we are done.

We observe that thanks to Proposition 14 the last result can be extended very easily to case of a σ-finite measure space.

The last fact we present is a characterization of the (DPrcP) by means of the behaviour of certain operators

Proposition 16. *The following facts are equivalent:*

a) *E has the* (DPrcP)

b) *any conjugate operator T from E^* into any F^* that is Dunford-Pettis is compact*

c) *the same as b) with $F = l^1$.*

Proof. a) $\Rightarrow$ b): Let $T: E^* \to F^*$ be a Dunford-Pettis operator. Consider a weak null sequence $(x_n^*) \subset E^*$ and $y \in B_F$; we have

$$|x_n^*(T(y))| \leqq \|T^*(x_n^*)\| \to 0 \quad \text{uniformly on } y.$$

Hence $T(B_F)$ is a Dunford-Pettis set in E and so, from a), T is compact. b) $\Rightarrow$ c) is obvious. c) $\Rightarrow$ a): Assume X is a non relatively compact, Dunford-Pettis subset of E. Since X is weakly precompact and $X - X = \{x - y : x, y \in X\}$ is still a Dunford-Pettis subset of E, we can find a sequence $(x_n) \subset E$ that is weak null, Dunford-Pettis and for which $0 < \inf_n \|x_n\| \leqq \sup_n \|x_n\| \leqq 1$. Define $S: E^* \to l^\infty$ by putting

$$S(x^*) = (x^*(x_n)) \quad x^* \in E^*$$

(note that actually S takes its values in c_0). S is a conjugate operator. To this aim, consider a weak* null net $(x_\alpha^*) \subset E^*$ with $\|x_\alpha^*\| \leqq 1$, and choose an element $y \in l^1$, $\|y\| \leqq 1$, $y = (y_n)$. For each α we have

$$S(x_\alpha^*)(y) = \sum_{n=1}^{\infty} x^*(x_n)(y_n).$$

Given $\varepsilon > 0$ there is $n_0 \in \mathbb{N}$ such that $\sum_{n=n_0+1}^{\infty} |y_n| < \varepsilon/2$. So we have

$$|S(x_\alpha^*)(y)| \leqq \sum_{n=1}^{n_0} |x_\alpha^*(x_n)| + \varepsilon/2 \quad \text{for all } \alpha.$$

Since (x_α^*) is weak* null and ε is arbitrary it is easy to conclude that $(S(x_\alpha^*))$ is weak* null, too, i.e. S is a conjugate operator. Now, we show that S is a Dunford-Pettis operator. Let (x_n^*) be a weak null sequence in E^*. Since (x_n) is a Dunford-Pettis sequence in E, we have $\sup_n |x_k^*(x_n)| \to 0$ as $k \to \infty$, i.e. $\|S(x_n^*)\| \to 0$. So S is compact, because of c). If $T: l^1 \to E$ is such that $T^* = S$, T is compact, too; since, clearly, T maps the unit basis of l^1 onto (x_n) we reach a contradiction that concludes the proof.

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