On the Banach Spaces with the Property (V*) of Pelczynski (*).

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Summary. – *We consider the Banach spaces with the property* (V*) *of Pelczynski giving a sufficient condition for a Banach space to have this property as well as a characterization of Banach lattices with the same property. Several other results are given which are concerning relationships among that property and other famous isomorphic properties of Banach spaces. Also a characterization of Banach spaces with property* (V*) *using Schauder decompositions is given. Some result concerning lifting of that property from a Banach space* E *to* $L^1(\mu, E)$ *is presented, too.*

0. – Introduction.

Let E be a Banach space and X be a bounded subset of E. We say that X is a (V*) set iff for any weakly unconditionally converging series $\sum x_n^*$ in E^* one has

$$\lim_n \sup_X |x_n^*(x)| = 0$$

(we recall that a series $\sum x_n^*$ is weakly unconditionally converging if $\sum |x^{**}(x_n^*)| < \infty$ for all $x^{**} \in E^{**}$ ([1]); this is equivalent to $\sum |x(x_n^*)| < \infty$ for all $x \in E$).

Following [17] we say that E has the property (V*) (of Pelczynski) iff any its (V*) subset is relatively weakly compact.

This property was introduced by Pelczynski in [17] as a dual property, in a sense, of the property (V) (see [17] for this definition); indeed Pelczynski showed that if a Banach space E has property (V) then E^* has property (V*). This could be seen as a good reason to study property (V*); moreover we observe that the following result is true (we recall that an operator $T: E \to F$, E, F two Banach spaces, is named unconditionally converging if it maps weakly unconditionally converging series into unconditionally converging ones).

PROPOSITION. – *Let E, F be two Banach spaces. If E has property* (V*) *of Pelczynski any conjugate unconditionally converging operator from E^* into F^* is weakly compact.*

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PROOF. – Let $S^*: E^* \to F^*$ be a conjugate operator which is unconditionally converging. We consider a sequence $(y_n) \subset B_F$, the unit ball of F, and we show that $(S(y_n))$ must be relatively weakly compact. If $\sum x_n^*$ is a weakly unconditionally converging series in E^* we have

$$\langle S(y_n), x_n^* \rangle = \langle y_n, S^*(x_n^*) \rangle \to 0$$

since $\|S^*(x_n^*)\| \to 0$. Hence $(S(y_n))$ is a (V*) set in E and so it is relatively weakly compact. We are done.

The first result of this note is a characterization of (V*) sets by means of operators with values in l^1. We also give a useful sufficient condition for a Banach space E to have property (V*) and a simple proof of the converse result stated, without proof, in [13] for the first time. As a consequence we obtain a result on weak compactness of operators defined on $C(K, E)$ spaces as well as a characterization of Banach lattices with property (V*) and other results relating that property to other famous properties of Banach spaces always in the setting of Banach lattices. Section 2 contains a characterization of Banach spaces with property (V*) using Schauder decompositions and some results concerning the lifting of property (V*) from a Banach space E to the usual space $L^1(\mu, E)$ of Bochner integrable functions on a finite measure space $(S, \mathcal{F}, \mu)$.

1. – Characterization of (V*) sets and other results.

Our first result is a necessary and sufficient condition for a set to be a (V*) set. For our aim we recall that there is a one-to-one correspondence between operators from E into l^1 and weakly unconditionally converging series $\sum x_n^*$ in E^* (see [5]) given by $T(x) = (x_n^*(x))$ for $x \in E$.

THEOREM 1.1. – *A bounded subset X of E is a* (V*) *set iff any $T: E \to l^1$, linear and continuous, maps X into a relatively compact subset of l^1.*

PROOF. – If X is a (V*) set in E, $T(X)$ is a (V*) set in l^1, a space with property (V*) (see [17]) and Schur property. Hence $T(X)$ is relatively compact. Conversely, let X be a subset of E which is mapped into a relatively compact subset of l^1 by any operator T from E into l^1. We consider a weakly unconditionally converging series $\sum x_n^*$ in E^* and the corresponding operator T. Since $T(X)$ is relatively compact, a well known result ([6]) on the compactness in l^1 gives that $\lim_n \sup_X |x_n^*(x)| = 0$. Hence X is a (V*) set. The proof is complete.

It is easy to see that Propositions 5 and 6 of [17] are consequences of Theorem 1.1. Another useful consequence of that result is the following theorem

THEOREM 1.2. – *Let E be a Banach space verifying the following assumption: if a bounded subset X is not relatively weakly compact there is a sequence $(x_n) \subset X$ equivalent to the unit basis of l^1 such that $\overline{\text{span}}\,(x_n)$ is complemented in E. Then E has property* (V*).

PROOF. – Let X be a bounded (V*) subset of E which is not relatively weakly compact. Let (x_n) be as in our assumption and P be the existing projection from E onto $\overline{\text{span}}\,(x_n)$. We also denote with j the existing isomorphism between l^1 and $\overline{\text{span}}\,(x_n)$. Obviously, $j \circ P$ maps (x_n) into the unit basis of l^1. Theorem 1.1 implies that the basis of l^1 should be relatively compact. A clear contradiction.

We observe that both A^* (the dual of the disk algebra A) and L^1/H_0^1 verify the assumption of Theorem 1.2 (see [18]).

Theorem 1.2 has the following converse obtained for the first time in [13]; there the authors didn't furnish a proof, but only said that one can get it by using the difficult techniques of Theorem 13 of the paper [7]. We are able to present a simpler proof of it based upon our Theorem 1.1 and other well known results concerning l^1.

THEOREM 1.3 (Godefroy, Saab [13]). – *Let E be a Banach space with property* (V*). *Then if (x_n) is a bounded, but not relatively weakly compact, sequence in E, then there is a subsequence $(x_{k(n)})$ equivalent to the unit basis of l^1 so that its closed linear span is complemented in E.*

PROOF. – Since (x_n) is bounded and not relatively weakly compact it cannot be a (V*) set. Theorem 1.1 gives the existence of an operator $T\colon E \to l^1$ so that $(T(x_n))$ is not relatively (weakly) compact in l^1. Hence $(T(x_n))$ has a subsequence, say $(T(x_{k(n)}))$, equivalent to the unit basis of l^1 (see [5]). Further $\overline{\text{span}}\,(T(x_{k(n)}))$ can be supposed complemented in l^1 by virtue of Theorem 3.3 of [19] applied to l^1 (otherwise we pass to a subsequence). So we have that

i) $(x_{k(n)})$ and $(T(x_{k(n)}))$ are equivalent to the unit basis of l^1

ii) $\overline{\text{span}}\,(T(x_{k(n)}))$ is complemented in l^1 by a projection Q

iii) $T|_{\overline{\text{span}}\,(x_{k(n)})}$ is an isomorphism.

Hence $P = T^{-1}|_{\overline{\text{span}}\,(x_{k(n)})}$ is the required projection.

COROLLARY 1.4. – *Let $L^1(\mu, E)$ have property* (V*). *Any nonreflexive subspace of it contains a complemented copy of l^1.*

The same proof of Theorem 1.3 can be used to obtain the following result

THEOREM 1.5. – *Let (f_n) be abounded sequence in $L^1(\mu, E)$ with E not containing l^1. Then only one of the two mutually exclusive facts is true:*

i) (f_n) *is uniformly integrable* (*a fact equivalent to the conditional weak compactness, by* [2])

ii) *there is a subsequence* $(f_{k(n)})$ *of* (f_n) *equivalent to the unit basis of* l^1 *having a* $\overline{\text{span}}$ *complemented in* $L^1(\mu, E)$.

Another result, concerning weakly compact operators on $C(K, E)$ spaces, follows from Theorem 1.3. Here K is a compact Hausdorff space and $C(K, E)$ denotes the usual Banach space of continuous functions. In the paper [14] Grothendieck showed that if $E = R$ and F is weakly sequentially complete then any $T: C(K) \to F$ is weakly compact; this result was improved in [17] by assuming E reflexive and F not containing c_0. In the paper [12] GAMLEN obtained a similar result supposing F weakly sequentially complete (a more restrictive assumption than the non containment of c_0); in this way he was able to weaken the assumption on E; he supposed that E^* has the Radon Nikodym property. Recently this result has been generalized by Fierro BELLO ([11]) to the case of E not containing copies of l^1. Now we have the following theorem

THEOREM 1.6. – *Let E be a Banach space not containing complemented copies of l^1. If F has property* (V*) *then any T from $C(K, E)$ into F is weakly compact.*

PROOF. – If a not weakly compact operator T exists there is a sequence (f_n') so that $(T(f_n'))$ is not a weakly Cauchy sequence. Using a proof like that of Theorem 1.3 we obtain a subsequence (f_n) equivalent to the unit basis of l^1 so that $\overline{\text{span}}\,(f_n)$ is complemented in $C(K, E)$. A result of SAAB and SAAB ([20]) now implies that E has a complemented copy of l^1, a contradiction which concludes the proof.

We observe that in case F is a Banach lattice the above theorem is an improvement of the cited results from [11], [12], [14], [17]. Indeed in such a framework F doesn't contain copies of c_0 iff it is weakly sequentially complete iff F has property (V*) (for the last equivalence see Theorem 1.7 below). We observe that our result is not longer true if one assumes F weakly sequentially complete. The Banach space X constructed in [3] is an infinite dimensional Banach space with the Schur property (hence it is weakly sequentially complete) and it doesn't contain complemented copies of l^1 since it is a separable $\mathcal{L}^\infty$-space. If any $T: C(K, X) \to X$ were weakly compact, the same would happen for the identity operator from X into itself. So X would be reflexive and, via the Schur property, finite dimensional. The above remarks on X also show that it is a Banach space with Schur property not having property (V*).

After this digression we return to consider consequences of our previous results concerning Banach spaces with property (V*). Theorem 1.2 is useful to characterize Banach lattices with property (V*).

THEOREM 1.7. – *Let E be a Banach lattice. Then E has property* (V*) *iff it is weakly sequentially complete.*

PROOF. – If E has property (V*) it is weakly sequentially complete. Conversely a result due to Niculescu ([16]) shows that E verifies the assumption of Theorem 1.2.

REMARK. – Theorem 1.7 was discovered at the same time by E. SAAB and P. SAAB in their paper [21] with entirely different techniques.

Now we are going to present some implication of Theorem 1.7 relating property (V*) and other isomorphic properties of Banach lattices. We need the following definitions due to Grothendieck ([14]): *j*) a Banach space E has the Reciprocal Dunford Pettis (in symbols (RDP)) property iff any completely (i.e. Dunford Pettis) operator $T: E \to F$ is weakly compact; *jj*) a Banach space E has the Dieudonné (in symbols (D)) property iff any weakly completely continuous operator $T: E \to F$ is weakly compact.

THEOREM 1.8. – *Let E be a Banach lattice. E has the* (RDP) *property iff E^* has property* (V*).

PROOF. – Theorem 2.1 of [16] states that a Banach lattice E has the (RDP) property iff it doesn't contain complemented copies of l^1. This fact is true iff E^* doesn't contain copies of c_0 (see [1]), i.e. iff E^* is weakly sequentially complete. Theorem 1.7 concludes the proof.

REMARK 2. – In the paper [15] the author introduced the (RDP*) property showing that

i) if E has the (RDP) property, then E^* has the (RDP*) property;

ii) if E^* has the (RDP) property, then E has the (RDP*) property

without giving results or counterexamples on the converse. Theorem 1.8 allows us to show that ii) cannot be reversed. Indeed in [24] an example of weakly sequentially complete Banach lattice with E^{**} not weakly sequentially complete is given. Then E has (property (V*) via Theorem 1.7 and so) the (RDP*) property. If E^* had the (RDP) property, then Theorems 1.7 and 1.8 would imply that E^{**} has to be weakly sequentially complete. In passing we observe that the space X constructed in [3] easily implies that i) cannot be reversed.

COROLLARY 1.9. – *Let E be a Banach lattice with the (D) property. Then E^* has property* (V*).

Proof. – It is easy to see that if E has the (D) property it has the (RDP) property. The above results seem to be interesting since we have

i) if E has property (V) then E^* has property (V*) (see [17]);

ii) if E has property (V) it has property (D) and hence the (RDP) property.

Hence Theorem 1.8 and Corollary 1.9 are improvements of i) in the setting of Banach lattices whereas the same implications are not longer true in general Banach spaces: the famous space J of James doesn't contain a copy of l^1 and hence it has the (D) property (see [8]) but it doesn't have property (V) since its dual is not weakly sequentially complete. In a sense we can say that the three quoted properties are closer in Banach lattices. However ([10]) there is a Banach lattice E which doesn't contain l^1 (and hence has the (D) property) and an operator $T: E \to c_0$ which is unconditionally converging without being weakly compact (and so E doesn't have (V) property). By virtue of Corollary 1.9 this space furnishes a positive answer to the following question put by PELCZYNSKI in [17]: Does a Banach space E exist so that E^* has property (V*) and E doesn't have property (V)?

REMARK 3. – This question was also solved in [21] independently; the example given there is not a Banach lattice.

At the end of the section we present two other consequences of Theorems 1.7 and 1.8.

COROLLARY 1.10. – *Let E be a Banach lattice. Then $C(K, E)$ has the* (RDP) *property iff E has.*

PROOF. – First of all we observe that $C(K, E)$ is a Banach lattice as well as $rcabv(Bo(K), E^*)$, its dual space. Theorems 1.7 and 1.8 imply that E^* is weakly sequentially complete. A result by Talagrand ([23] and [24]) implies that also the space $rcabv(Bo(K), E^*)$ is weakly sequentially complete. An appeal to Theorems 1.7 and 1.8, again, concludes the proof.

COROLLARY 1.11. – *Let E be a Banach lattice with the Radon Nikodym property. Then E has property* (V*).

PROOF. – It is known that E doesn't contain a copy of c_0. Hence E is weakly sequentially complete. Now Theorem 1.7 works.

The converse of Corollary 1.11 isn't true as the space $L^1([0, 1])$ shows. Moreover the same corollary cannot be extended to general Banach spaces as the James space J proves.

2. – More on the property (V*) of Pelczynski.

The first result of this section furnishes a characterization of Banach spaces with property (V*) which relies on the concept of unconditional Schauder decomposition (see [22]). A sequence (E_n) of closed subspaces of a Banach space E is said a Schauder decomposition if any $x \in E$ can be written in a unique way as sum of a series $\sum x_n$, $x_n \in E_n$. This decomposition is said unconditional if for each $x \in E$ the corresponding series converges unconditionally.

The following result clearly shows that the class of Banach spaces with property (V*) is quite small

THEOREM 2.1. – *Let E be a Banach space. Then E has property* (V*) *iff either E is reflexive or* i) *E is weakly sequentially complete and* ii) *E has an unconditional Schauder decomposition (E_n) with any E_n having property* (V*).

PROOF. – We suppose that E has property (V*) and is not reflexive. Obviously i) is true. It remains to show ii). Since E is not reflexive there is a bounded not relatively weakly compact sequence in E. Hence E contains a completemented copy F of l^1, by Theorem 1.3. An appeal to Proposition 15.12 of [22] concludes the proof, after recalling that any closed subspace of E has property (V*). In passing we observe that if P is the projection from E onto F a decomposition can be obtained as if follows: $E_1 = \ker P$, $E_{n+1} = \{\lambda y_n : \lambda \in R\}$ for $n \in N$ where (y_k) is a basis for F. To show the converse implication we first observe that if E is reflexive, then obviously it has property (V*). Now suppose that i) and ii) are true. Let X be abounded (V*) set in E. We shall show that for any $x^* \in E^*$ we have

$$\lim_n \sup_X |\langle S_n(x) - x, x^* \rangle| = 0$$

where $S_n = \sum_{i=1}^{n} P_i$ and P_i is the existing projection from E onto E_i, $i \in N$. We consider the mapping T from E into l^1 defined by putting

$$T(x) = (\langle S_{n+1}(x) - S_n(x), x^* \rangle) \qquad x \in E .$$

One can easily see that T is well defined, linear and continuous, using the unconditionality of (E_n). Hence $T(X)$ is relatively compact in l^1 by Theorem 1.1. This fact implies that the following limit relation is true

$$\lim_n \sup_X \sum_{i=n}^{\infty} |\langle S_{i+1}(x) - S_i(x), x^* \rangle| = 0 .$$

Since $S_n(x) \to x$ strongly in E, for any $x \in E$, we obtain

$$\lim_n \sup_X |\langle S_n(x) - x, x^* \rangle| = 0 .$$

Now we observe that any $P_n(X)$ is a (V*) set and hence it is relatively weakly compact set in E_n. If (x_k) is a sequence in X we can suppose that $(S_n(x_k))$ is a weak Cauchy sequence for any $n \in N$ (by passing to a subsequence if necessary). The following inequalities

$$|\langle x_{k'} - x_{k''}, x^*\rangle| \leq |\langle x_{k'} - S_n(x_{k'}), x^*\rangle| + \\ + |\langle S_n(x_{k'}) - S_n(x_{k''}), x^*\rangle| + |\langle S_n(x_{k''}) - x_{k''}, x^*\rangle| \qquad k', k'', n \in N$$

and the above remarks allow us to conclude that (x_k) is a weak Cauchy sequence in E. Weak sequential completeness of E concludes the proof.

REMARK 4. – A first version of the sufficient part of the above result was obtained under the more restrictive assumption « any E_n is reflexive ». Prof. L. DREWNOWSKI observed that the same proof worked under the hypothesis « any E_n has property (V*) ». We take this opportunity to thank him very much.

The last part of the paper concerns with the property (V*) in $L^1(\mu, E)$. The first result of this type is an easy consequence of Theorem 1.7 and a result of Talagrand on weak sequential completeness of $L^1(\mu, E)$ (see [23]).

THEOREM 2.2. – *Let E be a Banach lattice with property* (V*). *Then $L^1(\mu, E)$ has the same property.*

The following result makes use of Theorem 2.1.

THEOREM 2.3. – *Let E have an unconditional Schauder decomposition (E_n) with reflexive summands. If E is weakly sequentially complete (and hence has property* (V*) *by Theorem* 2.1) *then $L^1(\mu, E)$ has property* (V*).

PROOF. – Let X be a (V*) set in $L^1(\mu, E)$. It is not difficult to see that it is uniformly integrable. The operators P_n and S_n (see the proof of Theorem 2.1) extend to operators on $L^1(\mu, E)$ in an obvious manner; we continue to denote by P_n and S_n the extensions. Let L belong to $(L^1(\mu, E))^*$ and we consider an operator T_L from $L^1(\mu, E)$ into l^1 defined by

$$T_L(f) = \big(\langle S_{n+1}(f) - S_n(f), L\rangle\big) \qquad f \in L^1(\mu, E) .$$

As in [9] we can prove that T_L is well defined, bounded and linear. Hence we obtain (as in Theorem 2.1)

$$\lim_n \sup_X |\langle S_n(f) - f, L\rangle| = 0 .$$

Moreover the set $\{\int_A f(s)\, d\mu \colon f \in X\}$ is easily seen to be relatively weakly compact in E, for any $A \in \Sigma$. Summing up the following facts are true

j) X is bounded and uniformly integrable;

jj) the set $\{\int_A f(s)\,d\mu\colon f\in X\}$ is relatively weakly compact, for any $A\in\Sigma$;

jjj) for any $L\in(L^1(\mu,E))^*$, $\lim_n \sup_X |\langle S_n(f)-f, L\rangle| = 0$.

Theorem 6 of [9] now works to get that X is conditionally weakly compact. Weak sequential completeness of $L^1(\mu,E)$ (see [23]) concludes the proof.

REMARK 5. – At the beginning of the paper [21] the authors put the following question: Let $(\Omega,\mathcal{F},\lambda)$ be a probability measure space and let X be a closed subspace of a Banach space Y with an unconditional basis. Does the Banach space $L^1(\lambda,X)$ have property (V*) whenever X has?

They move from this question to investigate property (V*) and answer positively the question considered. Using our techniques of section 2 and Theorem 1.1 we are able to answer positively the following more general question: Let $(\Omega,\mathcal{F},\lambda)$ be a probability measure space and let X be a closed subspace of a Banach space Y with an unconditional Schauder decomposition with reflexive summands. Does the Banach space $L^1(\lambda,X)$ have property (V*) whenever X has?

Let Y and X be as above. If M is a (V*) set in $L^1(\lambda,X)$ then it is a (V*) set in $L^1(\lambda,Y)$ via Theorem 1.1. The proof of Theorem 2.3 may be now applied to show that M is conditionally weakly compact (in $L^1(\lambda,Y)$ and hence) in $L^1(\lambda,X)$, which is weakly sequentially complete ([23]). So M is relatively weakly compact.

Now we extend some of our previous results to the case of an uncountable family of summands

LEMMA 2.4. – *Let E be a Banach space such that for any its separable subspace F there is a complemented closed subspace Z of E so that* i) *Z contains F*, ii) *Z has property* (V*). *Then E has property* (V*).

PROOF. – Let X be a (V*) set in E and (x_n) be a sequence in X. We take $F=\overline{\text{span}}\,(x_n)$ and Z as in our hypotheses. Let P from E onto Z the existing projection. We consider an operator T from Z into l^1 and the operator $T\circ P\colon E\to l^1$. Theorem 1.1 implies that $(T\circ P)(X)$ is relatively compact as well as the sequence $((T\circ P)(x_n))$. Since $P|_Z=$ identity on Z, the sequence $(T(x_n))$ actually is relatively compact in l^1. The arbitrarity of T gives that (x_n) is a (V*) set in Z and so it is relatively weakly compact. The proof is over.

THEOREM 2.5. – *Let E be a weakly sequentially complete Banach space with a* (*not necessarily countable*) *unconditional Schauder decomposition $(E_i)_{i\in I}$* (*see* [22]) *where any E_i has property* (V*). *Then E has the same property.*

PROOF. – Let F be a separable closed subspace of E and (x_n) be a dense sequence in F. We have $x_n=\sum_{i\in I} x_i^{(n)}$ for any $n\in N$. It is known that

$$\text{supp}\,(x_n)=\{i\in I\colon x_i^{(n)}\neq 0\}$$

is countable (see [22]) for any $n \in N$. This implies that F can be embedded into a closed subspace Z of E having an unconditional Schauder decomposition $(Z_i)_{i\in I}$ where only countable many Z_i's are equal to suitable E_i's whereas the remaining ones are equal to the null subspace $\{\theta\}$. By taking advantage of the unconditionality of $(E_i)_{i\in I}$ we can reordered the Z_i's in such a way that Z can be considered isomorphic to a Banach space Y with a countable unconditional Schauder decomposition. An appeal to Theorem 2.1 gives that Z has property (V*). Since Z is obviously complemented in E, Lemma 2.4 finishes the proof.

We present now two corollaries of the above obtained result

COROLLARY 2.6. – *If* $(E_i)_{i\in I}$ *is a (possibly uncountable) family of Banach spaces with property* (V*), *then the space* $l^1(I, E_i)$ *has the same property.*

For the proof of this result we have to use Theorem 2.5.

COROLLARY 2.7. – *If a Banach space* E *verifies the assumptions of Theorem* 2.5, *but with* E_i *reflexive for all* $i \in I$, *then* $L^1(\mu, E)$ *has property* (V*).

PROOF. – Let A be a separable closed subspace of $L^1(\mu, E)$. It is known that there exists a separable closed subspace F of E such that $L^1(\mu, F)$ contains A (see [6]). Let Z and P be as in Theorem 2.5. By virtue of the nature of Z we can affirm that $L^1(\mu, Z)$ is isomorphic to a suitable $L^1(\mu, Y)$, Y being a Banach space isomorphic to Z as in Theorem 2.5. Hence ($L^1(\mu, Y)$ and so) $L^1(\mu, Z)$ has property (V*), via Theorem 2.3. Now we can extend P to a projection from $L^1(\mu, E)$ onto $L^1(\mu, Z)$ in an obvious manner. In this way all of the assumptions of Lemma 2.4 are satisfied and the proof is complete.

REFERENCES

[1] C. BESSAGA - A. PELCZYNSKI, *On bases and unconditional convergence in Banach spaces*, Studia Math., **17** (1958), pp. 151-164.

[2] J. BOURGAIN, *An averaging result for* l^1 *sequences and application to weakly conditionally compact set in* L^1_X, Israel J. Math., **32** (1979), pp. 289-298.

[3] J. BOURGAIN - F. DELBAEN, *A special class of* $\mathcal{L}^\infty$*-spaces*, Acta Math., **145** (1980), pp. 155-176.

[4] J. DIESTEL - J. J. UHL jr., *Vector measures*, Math. Surveys, **15** AMS (1977).

[5] J. DIESTEL, *Sequences and series in Banach spaces*, Graduate Texts in Math., Springer-Verlag, 1984.

[6] N. DUNFORD - J. T. SCHWARTZ, *Linear Operators, I*, Interscience, 1958.

[7] G. A. EDGAR, *An ordering for the Banach spaces*, Pacific J. Math., **108** (1983), pp. 83-98.

[8] G. EMMANUELE, *On the Dieudonné property*, Comm. Math., to appear.

[9] G. EMMANUELE, *On weak compactness in Banach spaces of E-valued measures and E-valued Bochner integrable functions, when E has an unconditional Schauder decomposition*, Revue Roumanie Math. Pures Appl., to appear.

[10] J. FIGIEL - N. GHOUSSOUB - W. B. JOHNSON, *On the structure of nonweakly compact operators on Banach lattices*, Math. Annalen, **257** (1981), pp. 317-334.

[11] C. FIERRO BELLO, *On weakly compact and unconditionally converging operators in spaces of vector valued continuous functions*, preprint.

[12] J. L. B. GAMLEN, *On a theorem of Pelczynski*, Proc. Amer. Math. Soc., **44** (1974), pp. 283-285.

[13] G. GODEFROY - P. SAAB, *Lès propriétés* (V) *et* (V*) *de A. Pelczynski: quelques nouveaux exemples*, Compte Rendu Acad. Sci. Paris, **303** (1986), pp. 503-506.

[14] A. GROTHENDIECK, *Sur les applications linéaires faiblement compactes d'espaces du type $C(K)$*, Canad. J. Math., **5** (1953), pp. 129-173.

[15] T. LEAVELLE, *The reciprocal Dunford-Pettis property*, Annali Mat. Pura Appl., to appear.

[16] C. P. NICULESCU, *Weak compactness in Banach lattices*, J. Operator Theory, **9** (1981), pp. 217-231.

[17] A. PELCZYNSKI, *Banach spaces on which every unconditionally converging operator is weakly compact*, Bull. Acad. Pol. Sci., **10** (1962), pp. 641-648.

[18] A. PELCZYNSKI, *Banach spaces of analytic functions and absolutely summing operators*, CBMS 30, AMS 1977.

[19] H. P. ROSENTHAL, *On relatively disjoint families of measures with some applications to Banach space theory*, Studia Math., **37** (1970), pp. 13-36.

[20] E. SAAB - P. SAAB, *A stability property of a class of Banach spaces not containing a complemented copy of l^1*, Proc. Amer. Math. Soc., **84** (1982), pp. 44-46.

[21] E. SAAB - P. SAAB, *On Pelczynski's properties* (V) *and* (V*), Pacific J. Math., **125** (1986), pp. 205-210.

[22] I. SINGER, *Bases in Banach spaces, II*, Springer-Verlag, 1981.

[23] M. TALAGRAND, *Weak Cauchy sequences in $L^1(E)$*, Amer. J. Math., **106** (1984), pp. 703-724.

[24] M. TALAGRAND, *Quand l'espace des measures a variation bornée est il faiblement séquentiellement complete*, Proc. Amer. Math. Soc., **90** (1984), pp. 285-288.

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